Uncertain Vibration Analysis of Laminated Structure-A Fuzzy Finite Element Approach

A Thesis Submitted in Partial Fulfilment
of the Requirements for the Award of the Degree of

Bachelor of Technology
in
Mechanical Engineering

by

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111ME0327

Under the supervision of:
Prof. Subrata Kumar Panda

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National Institute of Technology, Rourkela
Rourkela-769008, Odisha, INDIA
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DEPARTMENT OF MECHANICAL ENGINEERING

NATIONAL INSTITUTE OF TECHNOLOGY

ROURKELA, ODISHA-769008

CERTIFICATE

This is to certify that the thesis entitled "**Uncertain Vibration Analysis of Laminated Structure-A Fuzzy Finite Element Approach**" by ***Umesh Panigrahy (111ME0327)***, has been carried out under my supervision in fulfillment of the requirements for the degree of **Bachelor of Technology** in **Mechanical Engineering** during session 2014-2015 in the *Department of Mechanical Engineering, National Institute of Technology, Rourkela*.

To the best of my knowledge, the matter embodied in this thesis has not been submitted to any other University/Institute for the award of any degree or diploma.

Date:

Place:

Prof. Subrata Kumar Panda

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Rourkela, Odisha-769008

Acknowledgement

Firstly thanks to the Almighty God, without whose blessings, I wouldn't have been writing this "acknowledgment". I am fortunate enough to be involved in an exciting and challenging research project work on — **Uncertain Vibration Analysis of Laminated Structure-A Fuzzy Finite Element Approach**. This enriched my life, gave me an opportunity for working in a new environment of ANSYS and MATLAB.

I would like to express my foremost honor to my supervisor **Prof. S. K. Panda**, for his brilliance guidance, endless support and valuable suggestions. He is not only an honest person but also a wonderful supervisor. I consider myself lucky enough to be able to work under his guidance. He is the most genuine and dynamic person I have ever known.

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Finally, I express my sincere gratitude to my parents for their constant encouragement and support at all phases of my life.

UMESH PANIGRAHY

Abstract

Free vibration behavior of flat panel laminated composite with and without random material properties are investigated in this work. The laminated structure has been modelled mathematically using the higher-order model taking the randomized material properties through the fuzzy based tool. The desired governing equation is obtained with the help of Hamilton's principle and discretizing through the suitable finite element steps. Fuzzy-finite element method is used with randomized material property to obtain the free vibration responses. The model is evaluated by comparing the various responses with that of the published literature. Finally, the effect of different geometrical and material parameters on the vibration responses are evaluated by solving some numerical examples.

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CHAPTER 1

INTRODUCTION

1.1 Introduction

Composite materials are the uprising materials that have its applications in many engineering fields. Differing from conventional materials, the mechanical properties of composite materials like the elastic modulus or shear modulus which are not precisely known, may be imprecise, vague, linguistic, incomplete or qualitative. This is because of the complex processes involved ,may vary due to environmental conditions like temperature, pressure and various other parameters like when they are subjected to uncertainties associated with uncontrolled aspects of its manufacturing processes. Thus, many specifications of composite material are subject to uncertainties and are imprecise. Fuzzy finite method is a new approach to analyze random data. For the past few decades research work were held for analyzing random data. Fuzzy techniques and various theories of composites were developed since the past few years but the idea of using fuzzy analysis in composite materials remains unfathomed. Information like, “Young’s modulus of the material is between 5.5 GPa to 8.3 GPa”, and “shear modulus varies within 10%”, cannot be managed successfully by probabilistic and deterministic approaches. When we use approximated values of the parameters to find the response parameters and the material characteristics of composite material, we cannot assure reliability and the accuracy of results. In finding the probability variations of uncertain parameters, the probabilistic approach needs large amount of information, which in many cases are impossible or unrealistic. Thus, as the probability distribution of random parameters are not known, the probabilistic method cannot be applied. There are some cases where the mechanical specifications of many composite materials are known in linguistic terms only. Hence, in this present work the concept of fuzzy is used for modeling the randomness confronted in various composite materials. The theory of fuzzy set is used to model the uncertainties of material properties with less effort.

1.2 Literature Review

The mechanical responses (vibration, buckling, bending etc.) of the laminated structure have been studied with and without random material properties by many researchers (Nigam and Narayanan [1], Ibrahim [2], Manohar and Ibrahim [3], Chen *et al*. [4], Liu *et al*. [5], Chandrashekhar and Ganguli[6], Lal *et al*. [7], Onkar and Yadav[8], Kant and Swaminathan[9], Singh *et al*. [10]).

Composite structure analysis using uncertain material properties based on fuzzy finite element method are limited. Liu and Rao [11] found out that for fuzzy composite material its characteristics will vary within ranges, (which is not deterministic values), and for every range, the interval of confidence for each α can be found out. Singh *et al*. [12] studied the free vibration behavior of laminated composite panels of random material property using the probabilistic approach. The fuzzy finite approach is introduced by Rao and Berke [13], Kaufmann and Gupta [14], Akpan *et al*. [15] to analyze imprecisely defined systems. Li and Rao [16] analyzed uncertain structural systems with the help of interval analysis. Reddy [17] developed the higher order model for the laminated composite plates. Senthilnathan et al. [18], Whitney and Pagano [19] studied the bending and buckling behavior of laminated composite plates. Kant and

Swaminathan [20] developed a model based on the higher order theory and obtain the vibration responses for laminated plates. Katariya and Panda [21] studied the free vibration behaviour of laminated composite panels under thermal and mechanical loading using finite element model.

1.3 Objective of the Work:

Uncertain vibration analysis of laminated composites using Fuzzy Finite Method has not received any attention till now. The main objective of the present work is to find out the natural-frequency of laminated composite with and without uncertainties in its material properties using Fuzzy Finite Approach. This approach is valid and provides better and quicker results than probabilistic approach. The numerical results of natural frequencies are obtained for simply supported cross-ply-symmetric laminates with different thickness ratio (a/h). The present results are compared with the available published literature on probabilistic approach for the accuracy.

CHAPTER 2

THEORITICAL FORMULATION

2.1 Fuzzy Logic:

In classical theory, crisp values are used as input and the characteristic value is either zero or one, depending on the output i.e. we get the output correctly or we don't get it. But in fuzzy, inputs are not crisp values, rather they are in linguistic forms. Here, each input has a value between zero and one as its confidence limit and depending on that we get the output. Properties of fuzzy sets have been well defined.

For convenience sake, a fuzzy set (A) in universal set (U) is defined by the membership function A that takes values in the interval [0, 1]. The fuzzy set is represented as follows:

$$A = \mu_A\left(x_1\right)/x_1 + \ \ldots + \mu_A\left(x_n\right)/x_n$$

where $\mu_A(x_i)/x_i$ is called as a singleton and is a pair "grade of membership" element, i varies from 1 to n.

The fuzzy tool converts the crisp values into linguistic terms i.e. whether it is a high value , low value or very low or medium value.

α-cut of the fuzzy set A is the set A_α that includes various elements of the universal set of U which has a membership grade in A more than or equal to a particular value of α .This is defined as:

$A_\alpha = \{\mu_A(x) \geq \alpha, \forall x \in X\}$.

The various fuzzy operations that are used are:

- Complement: The complement of a set contains those elements which are not included in the given set. For example, when there is a set of handicapped men, its complement becomes the set of NOT handicapped men.

  Let A be a fuzzy set, then the complement $\overline{A}$ is represented as:

$$\overline{\mu}_A(x) = 1 - \mu_A(x)$$

- Containment: It is a set which contain all other sets. Subset is a small set of the containment set. For example, the set of handicapped men includes all handicapped men; largely handicapped men is only a subset of handicapped men. But, the set of handicapped men is just a subset of the former set-men.

- Intersection: Fuzzy intersection set contains the lower membership worth of all the sets of each element. The intersection of any two fuzzy sets *A* and *B* in the universal set X:

$$\mu_{A\cap B}(x) = \min\,[\mu_A(x), \mu_B(x)] = \mu_A(x) \cap \mu_B(x), \quad \text{where } x \in X.$$

- Union: The union is the inverse of intersection. That is, union contains the largest membership worth of all the element in either of the sets. The union of two fuzzy sets A and B on universal set X is:

$$\mu A \cup B(x) = \max\,[\mu A(x), \mu B(x)] = \mu A(x) \cup \mu B(x), \quad \text{where } x \in X$$

Fuzzy-arithmetic operations contain fuzzy-addition, fuzzy-multiplication, fuzzy-subtraction and fuzzy-division. Fuzzy operations are represented using (*), where * represents deterministic arithmetic-operations like +, -, ×, /. Hence, + represents the deterministic addition whereas (+) denotes the fuzzy-addition. Likewise, *x* in general denotes a deterministic-number where as (*x*) represents a fuzzy input or output. The features of fuzzy arithmetic operations differ from that of the deterministic arithmetic-operations. The operations like fuzzy-addition and fuzzy-multiplication follow commutative, associative and distributive law of sets but the operations like fuzzy-subtraction and fuzzy-division do not, as $A(-)B(+)B \neq A$, and $[A(/)B](\cdot)B \neq A$. Moreover, a fuzzy one (1) is interpreted as a fuzzy-number where the value one has a membership value of one, but the numbers on either side of one may be different. They can take any value depending on the level of confidence.

The fuzzy tool follows a pattern for finding out the output. It includes four steps. They are:

1. Fuzzification: In this step, the crisp inputs are converted into linguistic form and represented by membership functions. There are various types of membership functions available like triangular, trapezoidal, Gaussian etc. The membership functions are chosen based on the requirement of how accurate we want the output to be. The Gaussian membership-function provides the best output of all the membership functions.
2. Rule formation and evaluation: Here we form the rules on the basis of which the output changes. The rules are made mostly based on past experience, available formulas and the dependencies of various parameters on the output. The rules are made based on "and", "or" and "not" operators. The "and" operator gives the maximum of all the outputs. While, the "or" operator gives the minimum of all the outputs. Then the rules are evaluated.
3. Aggregation of rules: after evaluating the rules, the outputs are all aggregated together and the final fuzzified output was obtained.
4. Defuzzification: This is the last step of fuzzy logic. Here, the obtained output is again converted back to crisp value to obtain the output of the given problem.

The mamdani approach of fuzzy logic was used. These all steps were followed in the fuzzy tool box.

2.2 Finite Element Model:

For approximating a 3D elastic problem into a 2D beam problem, we expand the displacement functions (u, v, w) at a point (x, y and z) terms of thickness co-ordinate by using Taylor series. Due to the changing of transverse shear stress throughout the beam thickness, the displacement fields [20] are expanded to cubic power of thickness co-ordinate as follows:

$$u(x,y,z) = u_0(x,y) + z.\theta_x(x,y) + z^2.\phi_x(x,y) + z^3.\xi_x(x,y)$$
$$u(x,y,z) = v_0(x,y) + z.\theta_y(x,y) + z^2.\phi_y(x,y) + z^3.\xi_y(x,y)$$
$$w(x,y,z) = w_0(x,y) + z.\theta_z(x,y)$$

The relations between strain and displacement are:

$$\varepsilon_x = \varepsilon_{x_0} + z\kappa_x + z^2\varepsilon^*_{x_0} + z^3\kappa^*_x$$
$$\varepsilon_y = \varepsilon_{y_0} + z\kappa_y + z^2\varepsilon^*_{y_0} + z^3\kappa^*_y$$
$$\varepsilon_z = \varepsilon_{z_0} + z\kappa_z + z^2\varepsilon^*_{z_0} + z^3\kappa^*_z$$
$$\gamma_{xy} = \varepsilon_{xy_0} + z\kappa_{xy} + z^2\varepsilon^*_{xy_0} + z^3\kappa^*_{xy}$$
$$\gamma_{yz} = \varepsilon_{yz_0} + z\kappa_{yz} + z^2\varepsilon^*_{yz_0} + z^3\kappa^*_{xy}$$
$$\gamma_{xz} = \varepsilon_{xz_0} + z\kappa_{xz} + z^2\varepsilon^*_{xz_0} + z^3\kappa^*_{xz}$$

where,

$$\varepsilon_{x_0} = \frac{\partial u_0}{\partial x}, \qquad \kappa_x = \frac{\partial \theta_x}{\partial x}, \qquad \varepsilon^*_{x_0} = \frac{\partial \phi_x}{\partial x}, \qquad \kappa^*_x = \frac{\partial \xi_x}{\partial x},$$

$$\varepsilon_{y_0} = \frac{\partial v_0}{\partial y}, \qquad \kappa_y = \frac{\partial \theta_y}{\partial y}, \qquad \varepsilon^*_{y_0} = \frac{\partial \phi_y}{\partial y}, \qquad \kappa^*_y = \frac{\partial \xi_y}{\partial y},$$

$$\varepsilon_{z_0} = \theta_z, \qquad \kappa_z = 2\phi_z, \qquad \varepsilon^*_{z_0} = 3\xi_z, \qquad \kappa^*_z = 0,$$

$$\varepsilon_{xy_0} = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x}, \qquad \kappa_{xy} = \frac{\partial \theta_x}{\partial y} + \frac{\partial \theta_y}{\partial x}, \qquad \varepsilon^*_{xy_0} = \frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x},$$

$$\kappa^*_{xy} = \frac{\partial \xi_x}{\partial y} + \frac{\partial \xi_y}{\partial x}, \qquad \varepsilon_{yz_0} = \theta_y + \frac{\partial w_0}{\partial y}, \qquad \kappa_{yz} = 2\phi_y + \frac{\partial \theta_z}{\partial y},$$

$$\varepsilon^*_{yz_0} = 3\xi_y + \frac{\partial \phi_z}{\partial y}, \qquad \kappa^*_{yz} = \frac{\partial \xi_z}{\partial y}, \qquad \varepsilon_{xz_0} = \theta_x + \frac{\partial w_0}{\partial x},$$

$$\kappa_{xz} = 2\phi_x + \frac{\partial \theta_z}{\partial x}, \qquad \varepsilon^*_{xz_0} = 3\xi_x + \frac{\partial \phi_z}{\partial y}, \qquad \kappa^*_{xz} = \frac{\partial \xi_z}{\partial x}.$$

The above relations are the in-plane strains and curvatures. The parameters φ_x, φ_y, φ_z, ξ_x, ξ_y, ξ_z are the higher-order transverse cross-sectional deformation modes in the Taylor series expansion. For a lamina, the stress-strain relation with respect to the fiber matrix co-ordinate axis (1, 2, 3) can be given as:

$$
\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{12} \\ \tau_{23} \\ \tau_{13} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{12} \\ \gamma_{23} \\ \gamma_{13} \end{Bmatrix}.
$$

Here (ε_1, ε_2, ε_3, γ_{12}, γ_{23}, γ_{13}) are the strain and (σ_1, σ_2, σ_3, τ_{12}, τ_{23}, τ_{31}) are the stresses components corresponding to the lamina co-ordinates (1, 2, 3). C_{ij} is the compliance matrix w.r.t lamina axis (1, 2, 3). The stress-strain relations in laminate co-ordinates (x, y, z) of the lamina are:

$$
\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & Q_{14} & 0 & 0 \\ Q_{12} & Q_{22} & Q_{23} & Q_{24} & 0 & 0 \\ Q_{13} & Q_{23} & Q_{33} & Q_{34} & 0 & 0 \\ Q_{14} & Q_{24} & Q_{34} & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_{55} & Q_{56} \\ 0 & 0 & 0 & 0 & Q_{56} & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}.
$$

The elasticity matrix [D] can be obtained as:

$[D] = [T]^T [Q_{ij}] [T]$

[T] is the thickness co-ordinate matrix. So, D-matrix for the HSDT model is given by:

$$[D]=\begin{bmatrix} A_{ij} & B_{ij} & D_{ij} & E_{ij} & 0 & 0 & 0 & 0 \\ B_{ij} & D_{ij} & E_{ij} & F_{ij} & 0 & 0 & 0 & 0 \\ D_{ij} & E_{ij} & F_{ij} & G_{ij} & 0 & 0 & 0 & 0 \\ E_{ij} & F_{ij} & G_{ij} & H_{ij} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & AA_{ij} & BB_{ij} & DD_{ij} & EE_{ij} \\ 0 & 0 & 0 & 0 & BB_{ij} & DD_{ij} & EE_{ij} & FF_{ij} \\ 0 & 0 & 0 & 0 & DD_{ij} & EE_{ij} & FF_{ij} & GG_{ij} \\ 0 & 0 & 0 & 0 & EE_{ij} & FF_{ij} & GG_{ij} & HH_{ij} \end{bmatrix}.$$

In this case,

$$\left(A_{ij}, B_{ij}, D_{ij}, E_{ij}, F_{ij}, G_{ij}, H_{ij}\right) = \int_{-h/2}^{h/2} Q_{ij}\left(1, Z, Z^2, Z^3, Z^4, Z^5, Z^6\right) dZ \rightarrow i, j = 1,2,3,4$$

And

$$\left(AA_{ij}, BB_{ij}, DD_{ij}, EE_{ij}, FF_{ij}, GG_{ij}, HH_{ij}\right) = \int_{-h/2}^{h/2} Q_{ij}\left(1, Z, Z^2, Z^3, Z^4, Z^5, Z^6\right) dZ \rightarrow i, j = 5,6.$$

where,

[A_{ij}]- Extensional stiffness matrix

[B_{ij}]- Stretching-bending coupling matrix

[D_{ij}]- Flexural stiffness matrix.

The final form of governing differential equation of composite flat panels can be obtained using Hamilton's principle. This result in

$$\delta\int_{t_1}^{t_2} L dt = 0$$

where, $L = T - \left(U_{S.E.} + W\right)$

To analyse the free vibration behaviour of laminated composite flat panel, the governing equation can be presented as follows:

$$\left\{K_S - \omega^2 \left[M\right]\right\}\left\{\delta\right\} = 0$$

where, $\left[M\right] = \int_{dA} \left[N_i\right]^T \left[m\right]\left[N_i\right] dA$ and. $\left\{\delta\right\}$ and $\left[K_S\right]$ are the displacement vector and the stiffness matrix, respectively.

CHAPTER 3
RESULTS AND DISCUSSIONS

3.1 Introduction

Various examples are solved for to check effectiveness of the present proposed and developed fuzzy finite element model for accurate prediction of the nondimensional fundamental frequency of laminated-composite flat panels. The effect of the modular ratios, the thickness ratios and the boundary conditions on the natural frequency of the laminated composite panel is investigated. The present results obtained are compared to those available published literature for the validation of the present model. In the present analysis it is assumed that all the layers have the same thickness, mass of density and orthotropic material properties in the material principal axes. The laminated composite flat panel is considered for the present free vibration analysis and being analyzed. The mesh configuration for the finite element model can be seen in Fig. 1.

The present analysis is carried out using two different models:

Model 1: ANSYS model developed using ANSYS Parametric Design Language (APDL) Code

Model 2: Based on Higher-order shear deformation theory (HSDT) developed using MATLAB code

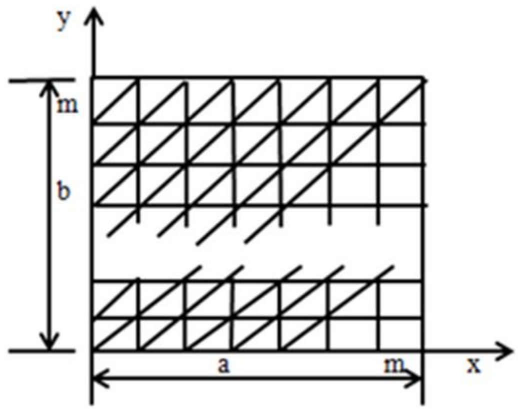

Figure 1: The mesh configuration for the finite element model

The lamina properties used for the free vibration of laminated composites are:

M1: E_1/E_2= 40; $G_{12} = G_{13} = 0.6E_2$; G_{23}=0.5E_2; υ_{12} = 0.25;$\rho = 1$; E_2 =6.92 GPa

M2: E_1/E_2= open; $G_{12} = G_{13} = 0.6E_2$; G_{23}=0.5E_2; υ_{12} = 0.25;$\rho = 1$; E_2 =6.92 GPa

M3:E_1/E_2= open; $G_{12} = G_{13} = 0.6E_2$; G_{23}=0.5E_2; υ_{12} = 0.25; E_1 =276.8 GPa

M4: E_1/E_2= 40; $G_{12} = G_{13}$ = open; G_{23}=0.5E_2; υ_{12} = 0.25; $\rho = 1$

M5:E_1/E_2= 40; $G_{12} = G_{13} = 0.6E_2$; G_{23}=0.5E_2;; υ_{12} = open; $\rho = 1$

M6: All open

The following nondimensional property is used for the analysis:

$$\bar{\omega} = (\omega a^2 / h)(\sqrt{\rho / E_2})$$

3.2 Convergence and Validation

In this section, the convergence and validation is presented for different geometrical and material conditions of the developed flat panel model. The results are obtained using the same type of conditions of available published literatures.

A simply supported (SSSS) cross-ply ($0^0/90^0/90^0/0^0$) laminated composite flat panel with a/h=10 is considered for the analysis, and the convergence has been obtained and plotted in the Figure 2. The present results are obtained using both models [22], [23] and presented in Figure 2.

It is observed from the Figure that a (5×5) mesh is sufficient to get the desired frequency response of the proposed model. The same mesh size is used for the further analysis of plate.

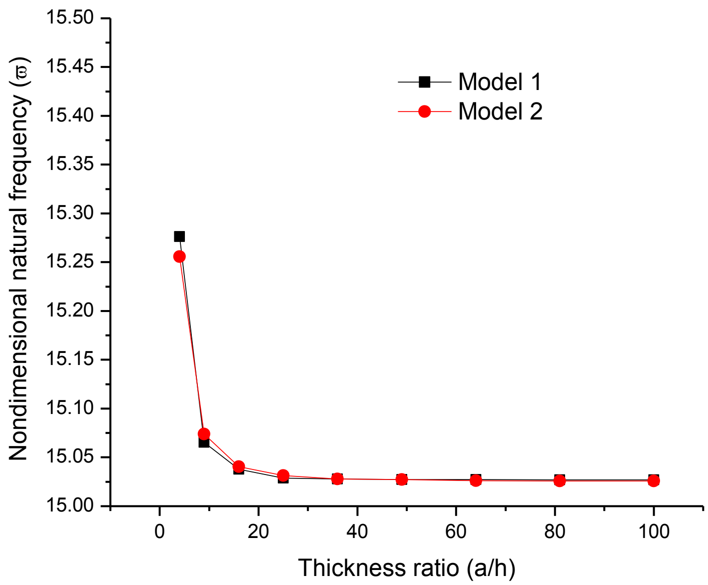

Figure 2: Convergence of nondimensional natural frequency

A simply-supported cross-ply ($0^0/90^0/90^0/0^0$) laminated flat panel is analyzed for four different thickness ratios (*a*/*h*=10, 20, 50 and 100) and compared with the available published literature and results are plotted in Figure 3. It is observed from the figure that the responses are well converging.

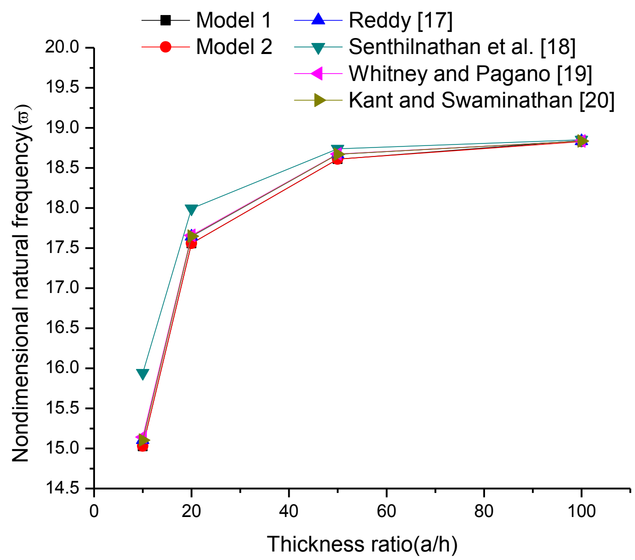

Figure 3: Effect of thickness ratios on natural (nondimensional) frequency of plates (M1)

3.3 Numerical Examples:

In this section, different numerical have been solved for a simply-supported cross-ply ($0^0/90^0/90^0/0^0$) laminated flat panel using different material properties. The numerical are solved by using Model 2 which is developed using the MATLAB [23]. The values are randomized using Fuzzy Tool box (Mamdani Fuzzy Method). For that each of the properties were randomized by using twenty percent variation from the deterministic values. During the process of randomization the alpha value was found out which gives the confidence interval of each property. This gives the range of inputs after randomizing 20% variation in the property values. The rules were created that govern the randomization of the material properties. These rules were created based on many data available. The figures indicate the changes in the natural frequency.

The fundamental frequency (nondimensional) responses are obtained by taking M2 and a/h=10 for SSSS condition, which is presented in Figure 4. It is observed from the figure that the responses obtained are increasing with an increase E_1. Similarly, in the next case, the vibration responses are obtained for different E_2. In Figure 5, the natural frequency (nondimensional) responses are obtained by taking M3 and a/h=10 for SSSS condition. It is noted from the figure that the responses obtained are decreasing with an increase E_2. Further the vibration responses are obtained for laminated composite plate by varying G_{12} and υ_{12}. The natural frequency (nondimensional) is obtained for SSSS condition taking M4 and a/h=10, and the responses are plotted in Figure 6. It is observed that the frequencies (nondimensional) are increasing as the G_{12} of material property increases. In Figure 7 the variation of υ_{12} property is presented by taking M5 material property and SSSS condition for laminated composite plate. It is seen from the figure that the frequency responses are increasing, remaining constant and again increasing as the υ_{12} property increases.

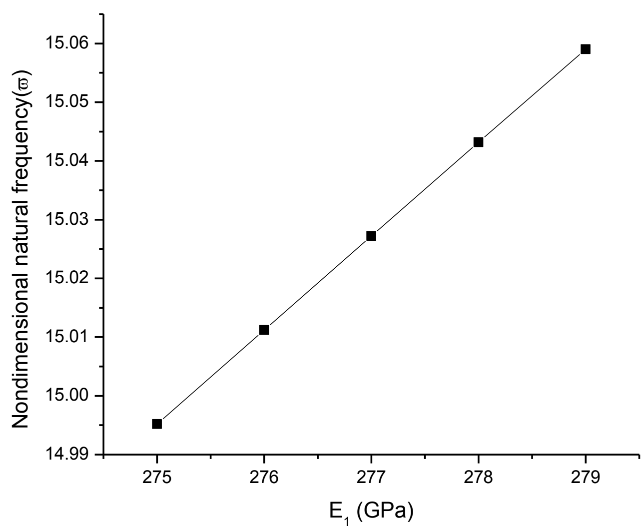

Figure 4: Effect of E_1 on natural (nondimensional) frequency of plates (M2)

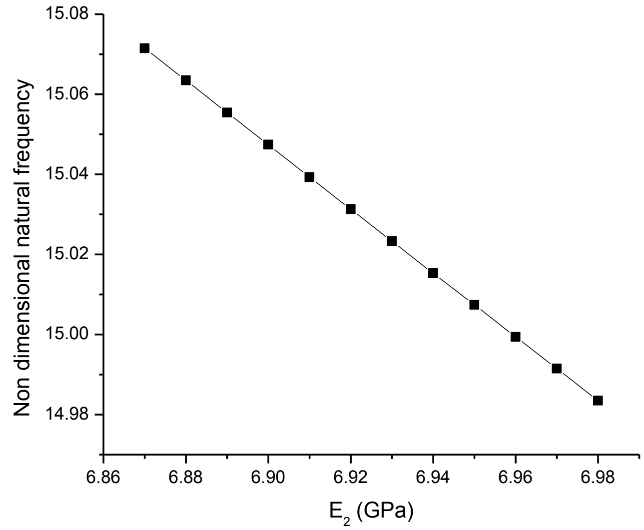

Figure 5: Effect of E_2 on natural (nondimensional) frequency of plates (M3)

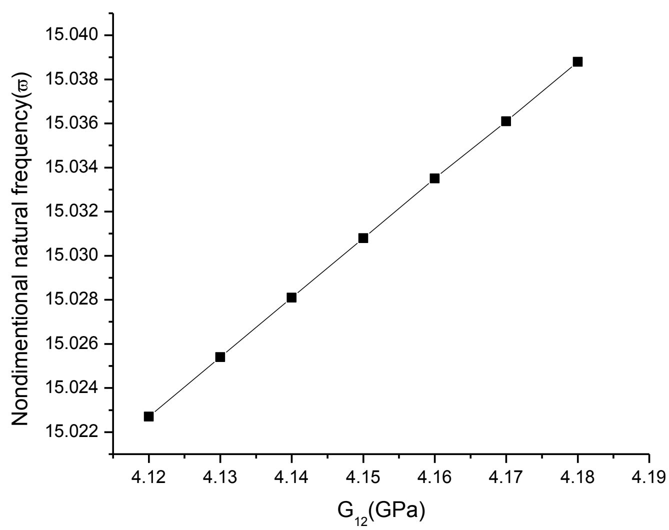

Figure 6: Effect of G_{12} on natural (nondimensional) frequency of plates (M4)

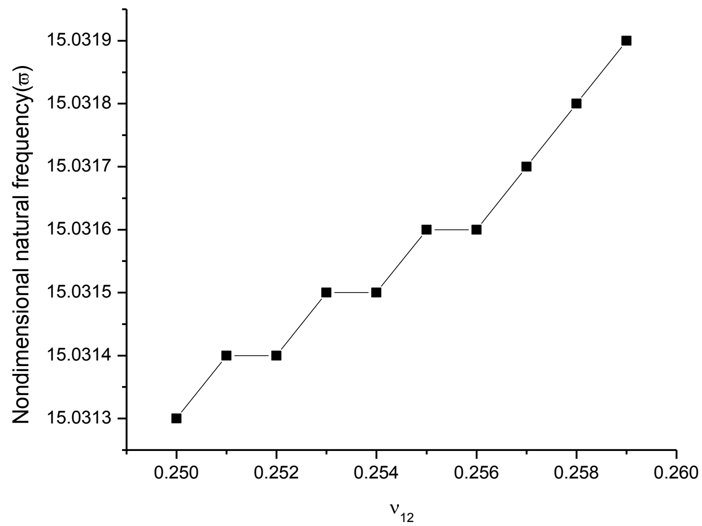

Figure 7: Effect of υ_{12} on natural (nondimensional) frequency of plates (M5)

Finally, the natural (nondimensional) frequency is obtained for simply supported cross-ply laminated plate by taking material property M6 i.e. random properties. The responses are presented in the table 1. The results show that the variation in natural frequency is very less even when the material properties were randomized.

Table 1: Frequency responses of laminated composite plates (M6)

E_1	E_2	υ_{12}	G_{12}	**Frequency**
275	6.87	0.25	4.12	15.0307
275	6.88	0.251	4.125	15.0273
275	6.89	0.252	4.13	15.0207
275	6.9	0.253	4.135	15.0141
275	6.91	0.254	4.14	15.0075
275	6.92	0.255	4.145	15.0009
275	6.93	0.256	4.15	14.9943
275	6.94	0.257	4.155	14.9878
275	6.95	0.258	4.16	14.9812
275	6.96	0.259	4.165	14.9806
275	6.97	0.26	4.17	14.9724
275	6.98	0.259	4.18	14.9628
276	6.87	0.25	4.12	15.0501
276	6.88	0.251	4.125	15.0434
276	6.89	0.252	4.13	15.0368
276	6.9	0.253	4.135	15.0302
276	6.91	0.254	4.14	15.0236
276	6.92	0.255	4.145	15.017

276	6.93	0.256	4.15	15.0104
276	6.94	0.257	4.155	15.0038
276	6.95	0.258	4.16	14.9972
276	6.96	0.259	4.165	14.9812
276	6.97	0.26	4.17	14.9801
276	6.98	0.259	4.18	14.9788
276	6.93	0.252	4.165	15.0141
276	6.95	0.251	4.17	14.9994
277	6.87	0.25	4.12	15.0661
277	6.88	0.251	4.125	15.0594
277	6.89	0.252	4.13	15.0528
277	6.9	0.253	4.135	15.0462
277	6.91	0.254	4.14	15.0396
277	6.92	0.255	4.145	15.033
277	6.93	0.256	4.15	15.0264
277	6.94	0.257	4.155	15.0198
277	6.95	0.258	4.16	15.0132
277	6.96	0.259	4.165	15.0099
277	6.97	0.26	4.17	15.0001
277	6.98	0.259	4.18	14.9948
277	6.93	0.252	4.165	15.0301
277	6.95	0.251	4.17	15.0154
278	6.87	0.25	4.12	15.082
278	6.88	0.251	4.125	15.0754
278	6.89	0.252	4.13	15.0687
278	6.9	0.253	4.135	15.0621
278	6.91	0.254	4.14	15.0555
278	6.92	0.255	4.145	15.0489
278	6.93	0.256	4.15	15.0423
278	6.94	0.257	4.155	15.0357
278	6.95	0.258	4.16	15.0292
278	6.98	0.259	4.18	15.0107
278	6.93	0.252	4.165	15.0461
278	6.95	0.251	4.17	15.0314
279	6.87	0.25	4.12	15.0979
279	6.88	0.251	4.125	15.0901
279	6.89	0.252	4.13	15.0846
279	6.9	0.253	4.135	15.078
279	6.91	0.254	4.14	15.0714
279	6.92	0.255	4.145	15.0648
279	6.93	0.256	4.15	15.0582
279	6.94	0.257	4.155	15.0516
279	6.95	0.258	4.16	15.045
279	6.98	0.259	4.18	15.0266
279	6.93	0.252	4.165	15.062
279	6.95	0.251	4.17	15.0473

CHAPTER 4

CONCLUSIONS

The natural-frequency of simply supported laminated composite plates is found out by using two models namely, HSDT and ANSYS. The results obtained for the deterministic case in this study were compared with the available research and this was found to be accurate.

Solutions for fuzzy analysis of laminated composite plate were not available in the published literature. Here, in fuzzy approach each of the characteristic properties is randomized using fuzzy tool. This randomization was done for 20% variation in the different material properties. It can be seen that the variation in nondimensional natural frequency of the laminated composite plate is different for different material property. The natural frequency varies more by changing E_1 property. The variation in natural frequency is very small for the change in G_{12} or υ_{12} property. When each of the material properties is varied, the nondimensional natural frequency does not vary much. This new result obtained for which no literature was available before to the best of author's knowledge. This shows that the random properties does not affect the natural frequency to a large extent. It varies maximum by 0.4% which is a good result. It also easier and simpler way of finding out the natural frequency of random material property, and also time saving.

Future Scope of the Work

- In the present work, only linear model has been considered which can be further extended for nonlinear.
- This work can be extended for the nonlinear free/forced vibration behavior of laminated composite/sandwich structures by using fuzzy-finite element method.
- An experimental study on vibration of laminated composite panels will give better understanding.
- This work can be further extended to find the various parameters of uncertain properties with less effort by using fuzzy logic rather than probabilistic approach.

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