Estimation of Parameters in Exponentiated-Weibull Distribution in Presence of Mask Data

A Project Report Submitted by

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Declaration

I hereby declare that the thesis entitled " Estimation of Parameters in Exponentiated-Weibull Distribution in Presence of Mask Data " submitted for the M.Sc degree is a revise work carried out by me and the thesis has not formed elsewhere for the award of any degree,fellowship or diploma.

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This is to certify that the thesis entitled " Estimation of Parameters in Exponentiated-Weibull Distribution in Presence of Mask Data "which is being submitted by Mr. Mitragupta Mohanta, Roll No.413MA2070, for the award of the degree of Master of Science from National Institute of Technology, Rourkela, is a record of bonafide research work, carried out by him under my supervision. The results embodied in this thesis are modified and have not been submitted to any other university or institution for the award of any degree or diploma.

To the best of my knowledge, Mr. Mitragupta Mohanta bears a good moral character and is mentally and physically fit to get the degree.

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Abstract

When the lifetime data from the series system are masked, we consider the reliability estimation of exponentiated-Weibull distribution based on different masking level. We consider a series system of two independent component which follows exponentiated- Weibull distribution. Maximum Likelihood Estimators (mle) of the unknown parameters are derived for different masking level. Finally, numerical simulation studies are done to obtain the values of the mles. We also obtain the percentile errors of the estimators.

Contents

Chapter 1

1 Introduction

Exponentiated-Weibull distribution has very nice physical interpretation. If there are n components in parallel system and lifetime of each component are independently and identically distributed as exponentiated-Weibull, then system life is also Exponentiated-Weibull distributed. Exponentiated-Weibull distribution arises as a mixture of exponential distributions. The probability density function (pdf) for exponentiated-Weibull distribution is given by

$$f(x|\theta,\alpha) = \alpha\theta\exp(x^{-\alpha})((1-exp(x^{-\alpha}))^{\theta-1}x^{\alpha-1};\ \alpha, x, \theta > 0. \tag{1}$$

Its survival function (sf) is given by

$$R(x) = 1 - (1 - \exp(x^{-\alpha})). \tag{2}$$

There are two special cases of Exponentiated-Weibull distribution: (i) $\alpha = 1$ gives exponentiated exponential distribution and (ii) $\theta = 1$ gives Weibull distribution. Practically, many products have more than one cause of failure. A multi-component system can fail due to failure of any of the system components. Component life distributions are then estimated from system failure data. Sometimes some of the failure times are observed without a complete knowledge of the cause of system failure. This is known as masking of failure cause.

Chapter 2

2 Literature Review and Summary

The exponentiated-Weibull family was introduced by Mudholkar and Srivastava (1993) as extension of the Weibull family, contains distribution with bathtub shaped and unimodal failure rate. The applications of the exponentiated-Weibull distribution in reliability and survival studies were illustrated by Mudholkar et al. (1995). Its properties have been studied in more detail by Mudholkar and Hutson (1996). The Weibull family and the exponentiated-exponential family are particular cases of this family. The distribution has been compared with the two-parameter Weibull and gamma distributions with respect to failure rate. Miyakawa (1984) studied the problem of a two-component series system when the system components have constant failure rates. Usher and Hodgson (1988) extended Miyakawa's (1984) results to a three-component series system using the same assumptions. Sarhan and El-Gohary (2003) studied the problem of a series system of two independent components each has a Pareto distribution based on masked. There are some cases where the cause of failure is completely unknown that is, completely masking. And in some cases the cause is unknown. In this case we are able to isolate the cause to an subset of system components. This type of masking is called partial masking. Xin and Yi (2012) studied the problem of estimation of generalized rayleigh component reliability in parallel system using dependent masked data. Huairui Guo,Ferenc Szidarovszky and Pengying Niu studied the problem on estimating component reliabilities from incomplete system failure Data

Here we will study the problem of estimating parameters included in the lifetime distributions of the individual components in a series system with J independent components. Life time of each component follows exponentiated-Weibull distribution. Here we will discussed three cases. In first case we will assume that θ is unknown parameter and α is known. In second case we will assume thatθ is known and α is unknown. And in third case we will discuss a special case that is, by putting $\alpha = 1$ and then we will estimate θ_j. Then Maximum likelihood estimator for the parameters θ_1 and θ_2 and the reliability functions of system components R_1 and R_2 are obtained. And at last weakness of maximum likelihood estimator is also discussed. A large simulation study is done in order to give some conclusions. In this purpose we use different masking level.

Chapter 3

3 Maximum Likelihood Estimators

In this section we derive the maximum likelihood estimators of the unknown parameters of exponentiated-Weibull distribution based on masked data. In the following we consider different cases.

3.1 Maximum likelihood estimator of θ when α is known

To obtain maximum likelihood estimator of θ when α is known, we assume the following assumptions.

3.1.1 Assumptions: A

A.1 The system consists of J independent components which are arranged in series.
A.2 n identical systems are in the life test. The test is terminated when all systems failed.
A.3 The random variables $T_{i,j}, j = 1, 2, \ldots, n$ are independent with $T_{1,j}, T_{2,j}, \ldots, T_{n,j}$ are identically, independently exponentiated-Weibull distributed. Denote the hazard rate function $h_j(t)$, pdf $f_j(t)$ and reliability function $R_j(t)$. The random variable $T_{i,j}$ denotes the life time of component j in system i.
A.4 The parameter α is known whereas the parameter θ_j is unknown for $j = 1, 2, ..., n$.
A.5 The observable quantities for the system i on the life test are its life time T_i and a set S_i of system components that may cause the system i fails.
A.6 Masking is s independent of the cause of failure.

3.1.2 Derivation

Using group A assumptions and the the likelihood function which is in Ref[7] the mle is given by

$$L(data, \theta) = \prod_{i=1}^{n} \left(\sum_{j \in S_i} f_j(t_i) \prod_{l=1, l \neq j} R_l(t_i) \right). \tag{3}$$

where $(\theta = \theta_1, \theta_2, ..., \theta_j)$

By using the relation between $h_j(t), R_j(t)$ and $f_j(t)$ $(f_j(t) = h_j(t) R_j(t))$ in Eq. (3), then likelihood function $L(data, \theta)$ can be written as

$$L(data, \theta) = \prod_{i=1}^{n} \left(\sum_{j \in S_i} f_j(t_i) \prod_{l=1}^{J} R_l(t_i) \right). \tag{4}$$

Now using (1) and (2), the above equation becomes

$$L(data,\theta)=\prod_{i=1}^{n}\left(\sum_{j\in S_i}\frac{\alpha\theta_j t_i^{(\alpha-1)}e^{-t_i{}^{\alpha}}(1-e^{-t_i{}^{\alpha}})^{(\theta_j-1)}}{1-(1-e^{-t_i{}^{\alpha}})^{\theta_j}}\sum_{l=1}^{j}(1-(1-e^{-t_i{}^{\alpha}})^{\theta_j})\right). \quad (5)$$

Let $E_i=(1-(1-e^{-t_i{}^{\alpha}})$. Then above equation becomes

$$L(data,\alpha)=\left[\prod_{i=1}^{n}\alpha t_i^{\alpha-1}e^{t_i^{\alpha}}\right]\left[\prod_{i=1}^{n}\sum_{j\in S_i}\frac{\theta_j E_i^{\theta_j-1}}{1-E_i^{\theta_j-1}}\right]\prod_{i=1}^{n}\left[\prod_{l=1}^{J}(1-E_i^{\theta_j})\right]. \quad (6)$$

The log-likelihood function is

$$l=log(L(data,\theta))=\sum_{i=1}^{n}log(\alpha t_i^{\alpha-1}e^{t_i{}^{\alpha}})+\sum_{i=1}^{n}log(\sum_{j\in S_i}\frac{\theta_j E_i^{\theta_j-1}}{1-E_i^{\theta_j-1}})+\sum_{i=1}^{n}log(1-E_i^{\sum_{j=1}^{J}\theta_j}). \quad (7)$$

The partial derivatives of l w.r.t. $\theta_l, l=1,2,...,J$

we get

By solving above nonlinear equation the maximum likelihood estimates of $\theta_j, j=1,2,...,J$ can be obtained. As it seems, this system in its general form has no closed form solution. So we study the problem when it consists of two component. For this case we need following notations. Let $n_j, j=1,2$ be the number of observations when component J causes system failure. Here n_j be the number of observations when S_i={j}. Let n_{12} when be the number of observations when the cause of system failure was masked. that is, n_{12} be the number when of observation when S_i={1,2}. Hence

$$n=n_1+n_2+n_{12}.$$

For this case the likelihood function becomes

$$L(data,\theta)=nlog(\alpha)+(\alpha-1)\sum_{i=1}^{n}t_i+n_1log(\theta_1)+n_2log(\theta_2)+n_{12}log(\theta_1+\theta_2)+(\theta_1+\theta_2-1)\sum_{i=1}^{n}E_i. \quad (8)$$

Now the likelihood function reduces to

$$\frac{n_j}{\theta_j}+\frac{n_{12}}{\theta_1+\theta_2}+\sum_{i=1}^{n}E_i=0; j=1,2. \quad (9)$$

By solving above equations for θ_1 and θ_2, one can obtain the mle of θ_j which is given by

$$\theta_j=-\frac{n_j}{\sum_{i=1}^{n}E_i}\left[1+\frac{n_{12}}{n_1+n_2}\right]. \quad (10)$$

The mle of the reliability of components $j, j=1,2$ can be obtained at $t=t_0$ as

$$R_j=\left(1-(1-e^{-t_0^{\alpha}})\right)^{-\frac{n_j}{(n_1+n_2\sum_{i=1}^{n}E_i)}}. \quad (11)$$

3.2 Maximum likelihood estimator of α when θ is known

In this case we will assume following assumptions and then obtain the maximum likelihood estimators of α for fixing θ.

3.2.1 Assumption:B

B.1 The system consists of J independent components which are arranged in series.

B.2 n identical systems are in the life test. The test is terminated when all systems failed.

B.3 The random variables $T_{i,j}, j = 1, 2, ..., n$ are independent with $T_{1,j}, T_{2,j}, ..., T_{n,j}$ are identical.And having Exponentiated-Weibull distribution is with hazard rate function $h_j(t)$, pdf $f_j(t)$ and reliability function $R_j(t)$. The random variable $T_{i,j}$ denotes the life time of component$j$ in system i.

B.4 The parameter θ is known where as the parameter α_j is unknown for$j = 1, 2, ..., n$.

B.5 The observable quantities for the system$i$ on the life test are its life timeT_i and a set S_i of systems components that may cause the system i fails.

B.6 Masking is s independent of the cause of failure.

3.2.2 Derivation

Using group B assumption and the the likelihood function which is in Ref[7] the mle is given by

$$L(data, \alpha) = \prod_{i=1}^{n} \left(\sum_{j \in S_i} f_j(t_i) \prod_{l=1, l \neq j} R_l(t_i) \right). \tag{12}$$

Where $(\alpha = \alpha_1, \alpha_2, ..., \alpha_J)$ By using the relation between $h_j(t), R_j(t)$ and $f_j(t)$ i.e. $f_j(t) = h_j(t)R_j(t)$ in Eq(12) then likelihood function $L(data, \alpha)$ reduces to

$$L(data, \alpha) = \prod_{i=1}^{n} \left(\sum_{j \in S_i} f_j(t_i) \prod_{l=1}^{J} R_l(t_i) \right). \tag{13}$$

Now using the relation between $f_j(t), h_j(t)$ and $R_j(t)$ in above equation we get

$$L(data, \alpha) = \prod_{i=1}^{n} \left(\sum_{j \in S_i} \frac{\theta \alpha_j t_i^{(\alpha_j - 1)} e^{-t_i{}^{\alpha_j}} (1 - e^{-t_i{}^{\alpha_j}})^{(\theta-1)}}{1 - (1 - e^{-t_i{}^{\alpha_j}})^{\theta}} \right) \prod_{l=1}^{J} \left(1 - (1 - e^{-t_i{}^{\alpha_j}})^{\theta - 1} \right). \tag{14}$$

Above equation can be written as

$$L(data, \alpha) = \theta \prod_{i=1}^{n} \left(\sum_{j \in S_i} \frac{\alpha_j t_i^{(\alpha_j - 1)} e^{-t_i{}^{\alpha_j}} (1 - e^{-t_i{}^{\alpha_j}})^{(\theta-1)}}{1 - (1 - e^{-t_i{}^{\alpha_j}})^{\theta}} \right) \prod_{l=1}^{J} \left(1 - (1 - e^{-t_i{}^{\alpha_j}})^{\theta - 1} \right). \tag{15}$$

The log-likelihood function is

$$l = logL(data, \alpha) = log(\theta) + \sum_{i=1}^{n} log \left[\sum_{j \in S_i} \frac{\alpha_j t_i^{(\alpha_j - 1)} e^{-t_i{}^{\alpha_j}} (1 - e^{-t_i{}^{\alpha_j}})^{(\theta-1)}}{1 - (1 - e^{-t_i{}^{\alpha_j}})^{\theta}} \right] + \theta \sum_{l=1}^{J} \left(1 - (1 - e^{-t_i{}^{\alpha_j}})^{\theta - 1} \right). \tag{16}$$

As it seems, this system in its general form has no closed form solution. Therefore, we study the problem when the system consists of two components. We study the problem when it consists of two component.

For this case we need following notations. Let $n_j, j = 1, 2$ be the number of observation when component J causes system failure.Here n_j be the number of observations when S_i={j}. Let n_{12} when be the number of observations when the cause of system failure was masked.i.e. n_{12} be the number when of observation when S_i={1,2}.

Hence $n = n_1 + n_2 + n_{12}$. So the above equation reduces to

$$l = logL(data, \alpha) = log\theta + n_1 log\alpha_1 + n_2 log\alpha_2 + n_{12} log(\alpha_1 + \alpha_2) + (\theta - 1) \sum_{i=1}^{n} log(1 - e^{-t_i^{(\alpha_1+\alpha_2)}}) + \sum_{i=1}^{n} t_i^{\alpha_1+\alpha_2-1}. \tag{17}$$

Now differentating above equation w.r.t.$\theta_j; j = 1, 2$ then we get

$$\frac{dl}{d\alpha_j} = \frac{n_j}{\alpha_j} + \frac{n_{12}}{\alpha_1 + \alpha_2} + \sum_{i=1}^{n} \frac{e^{-t_i^{(\alpha_1+\alpha_2)}} t_i^{(\alpha_1+\alpha_2)} log(t_i)}{1 - e^{-t_i^{(\alpha_1+\alpha_2)}}} + \sum_{i=1}^{n} t_i^{\alpha_1+\alpha_2-2}. \tag{18}$$

Where $j = 1, 2$

Now taking fixing $t = t_o$ and then solving above Eq(18) numerically we get the mle of $\theta_j, j = 1, 2$.

Accordingly we can calculate the estimates f reliability function by using the value of $\theta_j, j = 1, 2$.

In this case we will discuss a general case by taking $\alpha = 1$ and θ as a variable. Here we have to estimate $\theta_j; j = 1, 2, ..., J$. Now putting $\alpha = 1$ Eq(1) reduces to

$$f(x|\theta) = \theta e^t (1 - e^t)^{(\theta-1)}; \theta > 0 t > 0. \tag{19}$$

It is became Exponential Distribution with survival function (sf)

$$R(t) = 1 - (1 - e^{-t})^{\theta}. \tag{20}$$

and hazard rate

$$h(t) = \frac{\theta e^t (1 - e^t)^{(\theta-1)}}{1 - (1 - e^{-t})^{\theta}}. \tag{21}$$

3.3 A Particular Case: Maximum likelihood estimator of θ when $\alpha = 1$

For this case we will consider the following assumptions

3.3.1 Assumption:C

C.1 The system consists of J independent components which are arranged in series.

C.2 n identical systems are in the life test. The test is terminated when all systems failed.

C.3 The random variables $T_{i,j}, j = 1, 2, ..., n$ are independent with $T_{1,j}, T_{2,j}, ..., T_{n,j}$ are identical.And having exponentiated-Weibull distribution is with hazard rate function $h_j(t)$,pdf $f_j(t)$ and reliability function $R_j(t)$. The random variable $T_{i,j}$ denotes the life time of component$j$ in system i.

C.4 In this case α=1 is known where as the parameter θ_j is unknown for$j = 1, 2, ..., n$.

C.5 The observable quantities for the system$i$ on the life test are its life timeT_i and a set S_i of systems components that may cause the system i fails.

C.6 Masking is s independent of the cause of failure.

3.3.2 Derivation

According to group C assumption and the the likelihood function which is in Ref[1] the mle is given by

$$L(data, \theta) = \prod_{i=1}^{n} \left(\sum_{j \in S_i} f_j(t_i) \prod_{l=1, l \neq j} R_l(t_i) \right). \tag{22}$$

where $(\theta = \theta_1, \theta_2, ..., \theta_j)$

By using the relation between $h_j(t), R_j(t)$ and $f_j(t)$ i.e. given by $f_j(t) = h_j(t)R_j(t)$ in Eq(3) then likelihood function $L(data, \theta)$ can be written as

$$L(data, \theta) = \prod_{i=1}^{n} \left(\sum_{j \in S_i} f_j(t_i) \prod_{l=1}^{J} R_l(t_i) \right). \tag{23}$$

Now using the relation between $f_j(t), h_j(t)$ and $R_j(t)$ in Eq(23) we get

$$L(data, \theta) = \prod_{i=1}^{n} \left(\sum_{j \in S_i} \frac{\theta_j e^{-t_i}(1 - e^{-t_i})^{(\theta_j - 1)}}{1 - (1 - e^{-t_i})^{\theta_j}} \sum_{l=1}^{j} (1 - (1 - e^{-t_i})^{\theta_j} \right). \tag{24}$$

Let $F_i = (1 - e^{-t_i})$

This can be written as

$$L(data, \theta) = \left[\prod_{i=1}^{n} e^{-t_i} \right] \left[\prod_{i=1}^{n} \sum_{j \in S_i} \frac{\theta_j F_i^{\theta_j - 1}}{1 - F_i^{\theta_j}} \right] \left[\prod_{i=1}^{n} 1 - F_i^{\theta_j} \right]. \tag{25}$$

Now the log-likelihood function is

$$l = logL(data, \theta) = \sum_{i=1}^{n} log(e^{-t_i}) + \sum_{i=1}^{n} log \left(\sum_{j \in S_i} \frac{\theta_j F_i^{\theta_j - 1}}{1 - F_i^{\theta_j - 1}} \right) + \sum_{i=1}^{n} log \left(1 - E_i^{\sum_{j=1}^{J} \theta_j} \right). \tag{26}$$

Now taking partial derivative of $\theta_j; j = 1, 2, ..., J$

we get

$$\frac{dl}{d\theta_l} = \frac{1}{\sum_{j \in S_i} \theta_j} + \sum_{i=1}^{n} logF_i + \sum_{i=1}^{n} \frac{\sum_{j \in S_i} F_i^{\theta_j - 1} logF_i}{\sum_{j \in S_i} 1 - F_i^{\theta_j - 1}} - \sum_{i=1}^{n} \frac{E_i^{\sum_{j=1}^{J} \theta_j} logE_i}{1 - E_i^{\sum_{j=1}^{J} \theta_j}}. \tag{27}$$

By setting $\frac{dl}{d\theta_l}$=0 for $l = 1, 2, ..., J$ then we get the likelihood equation as

$$\frac{1}{\sum_{j\in S_i}\theta_j}+\sum_{i=1}^{n} logF_i+\sum_{i=1}^{n}\frac{\sum_{j\in S_i}F_i^{\theta_j-1}logF_i}{\sum_{j\in S_i}1-F_i^{\theta_j-1}}-\sum_{i=1}^{n}\frac{E_i^{\sum_{j=1}^{J}\theta_j}logE_i}{1-E_i^{\sum_{j=1}^{J}\theta_j}}=0 \tag{28}$$

As it seems, this system in its general form has no closed form solution. Therefore, we study the problem when the system consists of two components. We study the problem when it consists of two component.For this case we need following notations.Let $n_j, j = 1, 2$ be the number of observation when component J causes system failure. Here n_j be the number of observations when S_i={j}.Let n_{12} when be the number of observations when the cause of system failure was masked.i.e. n_{12} be the number when of observation when S_i={1,2}.

Hence $n = n_1 + n_2 + n_{12}$.

In this case, the likelihood function became

$$l = n_1 log\theta_1 + n_2 log\theta_2 + n_{12} log(\theta_1 + \theta_2) + (\theta_1 + \theta_2 - 1)\sum_{i=1}^{n} logF_i. \tag{29}$$

Now Eq(28)reduces to

$$\frac{dl}{d\theta_j}=\frac{n_j}{\theta_j}+\frac{n_{12}}{(\theta_1+\theta_2)}+\sum_{i=1}^{n} logF_i, j = 1, 2. \tag{30}$$

Now solving above equation for θ_1 and θ_2 we get the mle of $\theta_j; j = 1, 2$ as following form

$$\theta_j=\frac{n_j}{\sum_{i=1}^{n} logF_i}\left[1+\frac{n_{12}}{n_1+n_2}\right]. \tag{31}$$

The mle of the reliability of component $j, j = 1, 2$ can be obtained by putting Eq(31) to Eq(20)and taking $t = t_0$.

$$R_j(t_0) = 1-(1-e^{t_0})^{[\frac{n_j}{\sum_{i=1}^{n} logF_i}[1+\frac{n_{12}}{n_1+n_2}]]}. \tag{32}$$

Weaknesses of mle obtained arises when the available data is completely masking. This is because $n_j = 0$ for $j = 1, 2$ and $n = n_{12}$.

Chapter 4

4 Simulation study

To show how one can utilize the obtained theoretical result,we present a numerical result. Here we consider that the system consists of two independent component each has Exponentatied Weibull distributed life time. To simulate data 30 (n=30) independent and identically systems are put in a life test.Life time of each component and sub sets of component that may cause system failure. The true case of system failure is minimum life time of two component. These data are generated when $n_1 = 7$,$n_2 = 8$ and 15_{12}. i.e. incase of 50 percent masking level. These data are generated when life time of system component are distributed by Exponentiated-Weibull Distribution with $\alpha = 1.5$ and $\theta = 1.8$.

Table 1

	System I	t_i	S_i	System I	t_i	S_i	System I	t_i	S_i
	1	0.675034	{1}	11	0.70436	{2}	21	0.842916	{1}
	2	0.86287	{1}	12	0.521334	{1}	22	0.951125	{2}
	3	0.921073	{2}	13	0.716008	{2}	23	0.244378	{1,2}
	4	0.928226	{1,2}	14	0.840256	{1,2}	24	1.32869	{1,2}
	5	1.76424	{1,2}	15	0.739697	{1}	25	0.577484	{2}
	6	0.820141	{1,2}	16	1.33113	{1,2}	26	0.272778	{2}
	7	0.519421	{1}	17	0.558821	{2}	27	0.955981	{2}
	8	0.680626	{2}	18	0.179795	{1,2}	28	0.427919	{1,2}
	9	0.709189	{1}	19	0.935202	{2}	29	0.939167	{1}
	10	1.05523	{1,2}	20	1.18923	{1,2}	30	1.07191	{1,2}

Table 1

Table 2 gives the mle value of θ_1 ,θ_2 ,$R_1(0.5)$,$R_2(0.5)$ and percentage error assocated with mle of that parameter. The reliability is evaluated at $t_0 = 0.5$. Exact value of θ_1=10.62345 and θ_2=11.7284. Percentage error is given by

$$PE_\theta = \frac{|Estimatedvalue - Exactvalue|}{Exactvalue} \times 100 \tag{33}$$

Table 2

ML	n_1	n_2	n_{12}	MLE_{θ_1}	MLE_{θ_2}	PE_{θ_1}	PE_{θ_2}	R_1	R_2	PE_{R_1}	PE_{R_2}
0	12	18	-	8.4157	12.62360	20.78148	7.632296	2.31999	3.53371	19.8101	9.3647
10	12	15	3	9.3508	11.68852	11.9792	0.34045	2.547397	3.21826	11.9498	0.39814
30	9	11	10	9.4670	11.5716	10.8854	1.3373	2.57717	3.18085	10.9207	1.5776
50	7	8	15	9.818360	11.22098	7.57798	4.32682	2.66933	3.0712623	7.7353	4.9476
70	2	3	25	8.41737	12.62379	20.79996	7.63391	2.32037	4.770094	19.79702	47.6292

Table 2

In this case we consider that the system consists of two independent component each has Exponentially distributed life time. To simulate data 30 (n=30) independent and identically systems are put in a life test. Life time of each component and sub sets of component that may cause system failure. The true case of system failure is minimum life time of two component. These data are generated when $n_1 = 10$,$n_2 = 10$ and $n_{12} = 10$. i.e. incase of 30 percent masking level. These data are generated when life time of system component are distributed Exponentially with $\theta = 1.2$.

	System I	t_i	S_i	System I	t_i	S_i	System I	t_i	S_i
	1	1.27689	{2}	11	2.78881	{1}	21	1.82854	{1,2}
	2	1.51091	{1,2}	12	1.66234	{2}	22	1.95596	{1,2}
	3	1.34815	{2}	13	1.60002	{1}	23	1.54333	{1}
	4	2.34522	{1,2}	14	3.85008	{1,2}	24	1.3787	{1,2}
	5	1.566	{2}	15	2.16058	{1}	25	1.43812	{2}
	6	1.22788	{1}	16	1.35983	{1,2}	26	1.21454	{1,2}
	7	1.4005	{2}	17	1.31498	{1}	27	1.24843	{1}
	8	1.51634	{1,2}	18	1.26849	{1}	28	1.48883	{1}
	9	1.4604	{2}	19	1.46923	{2}	29	2.04077	{2}
	10	1.31546	{1}	20	1.67261	{2}	30	1.26953	{1,2}

Table 3

Table 4 gives the mle value of θ_1 ,θ_2 ,$R_1(0.02)$,$R_2(0.02)$ and percentage error assocatied with mle of that parameter. The reliability is evaluated at $t_0 = 0.02$. Exact value of θ_1=15.23816 and θ_2=6.2843.

	ML	n_1	n_2	n_{12}	MLE_{θ_1}	MLE_{θ_2}	PE_{θ_1}	PE_{θ_2}	R_1	R_2	PE_{R_1}	PE_{R_2}
	0	20	10	-	14.98334	7.49167	1.6722	19.212	1.3494	1.6164	1.7263	20.698
	30	15	5	10	16.85826	5.61875	10.6187	10.5906	1.40097	1.11893	5.4898	16.4478
	50	12	3	15	17.98001	4.49500	17.993	28.4726	1.4327	1.0941	7.8289	18.3019
	70	3	2	25	13.4850	8.9900	11.5050	43.0549	1.309571	1.19697	1.2479	10.6205

Table 4

5 Reference

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