EXPERIMENTAL AND NUMERICAL STUDY ON TUNED
MASS DAMPER IN CONTROLLING VIBRATION OF FRAME
STRUCTURES

A Thesis Submitted in partial fulfillment of the requirements for

the award of Degree of

MASTER OF TECHNOLOGY

In

STRUCTURAL ENGINEERING

By

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DEPARTMENT OF CIVIL ENGINEERING
NATIONAL INSTITUTE OF TECHNOLOGY
ROURKELA, ODISHA-769008
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ROURKELA, ODISHA-769008

2015
This is to certify that the thesis entitled, “EXPERIMENTAL AND NUMERICAL STUDY ON TUNED MASS DAMPER IN CONTROLLING VIBRATION OF FRAME STRUCTURES” submitted by Padmabati Sahoo in partial fulfilment of the requirements for the award of Master of Technology Degree in Civil Engineering with specialization in “Structural Engineering” at National Institute of Technology, Rourkela is an authentic work carried out by her under my supervision and guidance. To the best of my knowledge, the matter embodied in this Project review report has not been submitted to any other university/ institute for award of any Degree or Diploma.

Date: 

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Dedicated to

My Parents, and to each and every teacher, who taught us from alphabets to whatever till date. And to friends who have been there for us from genesis to apocalypse.
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PADMABATI SAHOO
ABSTRACT

Day by day, the numbers of taller and lighter structures are continuously increasing in the construction industries which are flexible and having a very low damping value. Those structures can easily fail under structural vibrations induced by earthquake and wind. Therefore several techniques are available today to minimize the vibration of the structure, out of which concept of using TMD is a newer one. There are large numbers of studies on theoretical investigation of behaviour of buildings with tuned mass dampers under various impacts. However, the experimental studies in this area are quite limited. In this thesis, a one-storey and a two-storey building frame models are developed for shake table experiment under sinusoidal excitation to observe the response of the structure with and without TMD. The TMD is tuned to the structural frequency of the structure keeping the stiffness and damping constant. Various parameters such as frequency ratio, mass ratio, tuning ratio etc. are considered to observe the effectiveness and robustness of the TMD in terms of percentage reduction in amplitude of the structure. Then the responses obtained are validated numerically using finite element method. From the study it is observed that, TMD can be effectively used for vibration control of structures.
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1. INTRODUCTION

1.1 Introduction:

Earthquake is a compartment of structural analysis which involves the computation of the response of a structure subjected to earthquake excitation. This is required for carrying out the structural design, structural assessment and retrofitting of the structures in the regions where earthquakes are prevalent.

Now a day number of tall buildings are going on increasing which are quite flexible and having very low damping value to minimize increasing space problems in urban areas. These structures should be designed to oppose dynamic forces through a combination of strength, flexibility and energy absorption such that it may deform beyond elastic limit when subjected to severe earthquake motion. To make these structures free from earthquake and wind induced structural vibration, various techniques has been adopted which can be broadly classified into 4 categories. (i) Active control, (ii) Passive control, (iii) Semi-active control and (iv) Hybrid control.

1.1.1 Active control devices:

These devices use an external power source which operates control actuators to apply forces to the structures. Some signals are sent to the actuators which are a function of responses of the structure. Requirement of equipments are more in active control strategies than passive control thereby increasing the cost and maintenance of such systems. Active tuned mass damper, active tuned liquid column damper and active variable stiffness damper are some of the examples of active control devices. Applications - AMD on Kyobashi Seiya Building, Duox on ANDO Nighikicho, Trigon on Shinjuku Tower.
1.1.2. Passive control devices:

It is a device which imparts forces that are developed in response to the motion of the structures. By absorbing some of the input energy, it reduces the energy dissipation demand on the structure. Therefore no external power source is required to add energy to the structural system. Base isolation, tuned mass dampers (TMD), tuned liquid dampers (TLD), metallic yield dampers, viscous fluid dampers are some of the examples of passive control devices.


1.1.3. Semi-active control devices:

It is a controllable passive control system where the external energy requirement is less than that of active devices. It unites the optimistic aspects of passive and active control devices. These devices generate forces as a result of the motion of the structure and cannot add energy to the structural system. Variable orifice dampers, variable friction dampers, variable stiffness damper, and controllable fluid dampers are some of the examples of semi active control devices.

Applications- Kajima Shizuoka Building in Shizuoka, Japan, Walnut Creek Bridge in Oklahoma, 11-storey building CEPCO Gifu Japan, Keio University School of Science and Technology Tokyo in Japan. Dongting Lake Bridge in Hunan, China.

1.1.4. Hybrid control Devices:

These devices combine the passive, active or semi-active devices to achieve higher level of performance. Since a portion of the control objective is accomplished by the passive system, less active control effort, implying less power resource, is required. A side benefit of hybrid systems is that, in the case of a power failure, the passive components of the control still offer some
degree of protection, unlike a fully active control system. Examples of hybrid control devices include hybrid mass damper and hybrid base isolation.

Applications- Sendagaya INTES building in Tokyo

1.2. Tuned mass damper:
Tuned mass damper is a passive control device connected to the structure like a secondary mass to reduce the dynamic response of the structure and increases the damping capacity. It has been widely used for vibration control in many mechanical engineering systems. Recently many theories have been adopted to reduce vibration in civil engineering structures because of its easy and simple mechanism. To obtain optimum response the natural frequency of the secondary mass is always tuned to that of primary structure such that when that particular frequency of the structure get excited, the TMD will resonate out of phase with the structural motion. The excess amount of energy built up in the structure is transformed to the secondary mass and dissipated due to relative motion developed between them at a later stage.

1.3. Practical implementation:
Till date many tuned mass dampers has been installed worldwide. Centre point Tower in Sydney, Australia is the first structure in which TMD was installed.

TMD is also installed in two buildings of United state. One is Citicorp Centre in New York City with its height 279 m, placed on the 63rd floor in topmost point of the structure, having a mass of 366 mg, with a linear damping from 8-14%, reducing the amplitude of the building by 50%.

Two dampers are installed in 60th floor of John Hancock Tower, Boston to reduce wind reduce structural vibration, each having its weight 2700 kN with a lead filled steel box 5.2 m square and 1m deep sliding on a 9m long steel plate.
Berlin TV tower, one of the tallest structures of Germany constructed between 1965 and 1969 with its height of 368 meters is installed with a tuned mass damper. The entrance of observation deck is 6.25 m above ground with 2 kone lifts for transport of visitors. Weight of the sphere is 4800 tones and diameter is 32 m. There is a steel stairway with 986 steps.

Burj Al Arab, a luxury hotel in Dubai, is the 3rd tallest hotel in the world is installed with 11 tuned mass damper. 39% of the total height is made up of non-occupiable space. There is helipad 210 m above sea level which provides an opportunity to arrive or depart from Burj Al Arab by helicopter and admire the city from a different perspective.

Perk Tower, Chicago, 257 m tall with 70 floors is the eleventh tallest building in Chicago, the 39th tallest building in the United States, and the eighty-third tallest in the world and the 1st structure in United state to be designed with a tuned mass damper to counteract the wind effect on the structure. It is a 300 ton massive steel pendulum damper hanging from 4 cables inside a cage which stabilizes the building from swaying in the wind.

Taipai 101, Taiwan was the world’s tallest building from 2004 to 2010, consists of 101 floors and 5 underground floor, equipped with a steel pendulum that serves as a tuned mass damper suspended from the 92nd to the 87th floor. The pendulum sways to offset movements in the building caused by strong gusts. It is the largest damper sphere in the world, consists of 41 circular steel plates of varying diameters, each 125 mm thick, welded together to form a 5.5 m diameter sphere. Two additional tuned mass dampers, each weighing 6 tonnes (7 short tons), are installed at the tip of the spire which help prevent damage to the structure due to strong wind loads.

Spire of Dublin (Monument of Light), 121.2 m in height is the largest stainless steel monument located in Dublin, Ireland. It has an elongated cone of diameter 3 m at the base and narrowing to
15 cm at the top. It is constructed from eight hollow tubes of stainless steel and equipped with a tuned mass damper to counteract sway.

The Akashi Kaikyo Bridge (Pearl Bridge) located in Japan has had the longest span of any suspension bridges in world. It has 3 spans with central span 1991 m and two others each 960 m long, designed to allow the structure to withstand winds of 286 km/Hr and earthquake of magnitude 8.5. A tuned mass damper is designed to operate at a resonance frequency of the bridge to increase the damping value.

One wall centre (Sheraton Vancouver Wall Centre Hotel) with height 157.8 m, 48 storey hotel in Canada is installed with tuned water damper at the highest point of the building consisting of two water tanks and the tanks are designed to counteract the harmonic frequency of the building.

John Hancock tower (Hancock Place) is a 60 storey tower in Boston equipped with two 300 ton weight mass dampers on 58th floor made up of steel box, filled with lead. Both the weights are rested on steel plates which are lubricated for free sliding of weights. But the weight is attached to the steel frame of the building by means of springs and shock absorbers. When the Hancock sways, the weight tends to remain still, allowing the floor to slide underneath it. Then, as the springs and shocks take hold, they begin to tug the building back. The effect is like that of a gyroscope, stabilizing the tower. The reason there are two weights, instead of one, is so they can tug in opposite directions when the building twists. The cost of the damper was $3 million. The dampers are free to move a few feet relative to the floor.
Figure 1.1 Practical implementation of tuned mass damper
2. LITERATURE REVIEW

2.1. Literature Review:

Till date TMD has been studied by many researchers. The thought of TMD was first used by Frahm in 1909 to diminish the undulating motion of ships as well as ship hull vibrations. Later Hartog in 1940 developed analytical model for vibration controlling power of TMD. Later he optimized TMDs parameter for sinusoidal excitations. Fahim et al. (1997) considered different parameters like mass ratio, frequency ratio, damping ratio etc to obtain the optimum parameters which are used to compute the response of various single degree of freedom and multi degree of freedom structures with TMD at different earthquake excitation. The optimum parameters obtained are helpful in reducing the displacement and acceleration response significantly.

Wu et al. (1999) considered soil structure interaction in seismic response of tuned mass damper when fixed on a flexible based structure. A frequency independent model is used which covers a wide range of soil and structural characteristics. A stationary excitation is given to the model structure and the responses are used to measure the performance of TMD. It was observed that strong soil structure interaction considerably defeats the seismic effectiveness of TMD systems. Reduction in maximum response of the structure reduces with decrease in soil shear wave velocity. For any structure over soft soil, TMD structure is less effective in reducing the response due to high damping characteristics of soil structure system. The model is also subjected to NS component of the 1940 El Centro, California earthquake to observe the effectiveness of TMD in a realistic environment.
Nagashima et al. (2001) developed a hybrid mass damper (HMD) and applied it to a 36 storey high rise building with a bi-axial eccentricity, located in Tokyo. The system uses a gear type pendulum which make the natural period of the auxiliary mass relatively long minimizing the height of the device and a linear actuator which ensures smooth and noiseless operation of the system. Transverse torsion coupled vibration is controlled by two HMD systems and various feedback control techniques has been developed to consume the capacity of HMD system for any external excitation. Free vibration tests as well as control of wind vibrations of the building induced by the Typhoon 9810 in 1998 were used to verify the performance of the control system. It was observed that the acceleration responses of the building were reduced to 63% and 47% of the corresponding uncontrolled accelerations. Setareh (2001) studied the application of semi-active tuned mass dampers to base excitation systems. A single-degree-of-freedom system was subjected to sinusoidal base excitations and a damper was used to reduce the vibration. A new class of damper known as ground hook tuned mass damper (GHTMD) was used and optimum design parameters were obtained based on the minimization of steady state amplitude response of the main structure for different mass ratio and damping ratio. Frequency response was compared with and without TMD. In this paper, a design guide was presented based on non dimensional values to find optimum parameters of GHTMD. Li and Liu (2002) manufactured an active multiple tuned mass damper (AMTMD) for structures subjected to ground acceleration keeping the stiffness and damping constant and varying the mass. Vibration in the structure is controlled by mode reduced order method. A numerical searching technique is used to demarcate the effect of optimum dynamic parameters on the strength of AMTMD. The parameters include the frequency spacing, damping ratio and frequency ratio and acceleration feedback gain coefficient. They compare the results of MTMD
and AMTMD and concluded that it can effectively reduce the vibration of the structure under ground acceleration. The AMTMD can also increase the performance of MTMD and more effective than ATMD.

Samali et al. (2003) described a five storey model using an active tuned mass damper by Fuzzy Logic Controller and linear quadratic regulator under earthquake excitation and a comparison is made. The effect of mass ratio and frequency ratio is conducted using fuzzy controller because of its ability to handle any non linear behavior of the structure. Chen and Wu (2003) numerically observed the effect of multiple tuned mass damper (MTMD) and compared the results with tuned mass damper (TMD). A three storey building frame was subjected to white noise excitation and tested in shake table. The results observed that multiple tuned mass dampers are more effective than tuned mass dampers in reducing the floor acceleration. The experimental and numerical results are compared and dynamic properties of the structure are validated successfully.

Li (2003) numerically observed the performance of multiple active-passive tuned mass dampers (MAPTMD) to prevent vibration of single degree of freedom structures subjected to ground acceleration with a uniform distribution of natural frequency. The MAPTMD generates a controlling force by keeping the displacement and velocity response gain and changing the acceleration response gain. Conclusion has been made that maximum tuning frequency ratio of MAPTMD decreases with increasing mass ratio and the effectiveness increases with the increase in mass ratio.

Chen and Wu (2004) studied experimentally to reduce the seismic responses of a three-storey building structure by using multiple tuned mass dampers. They identified various dynamic properties of both structure and damper from free and forced vibration analysis. The structure was analyzed numerically with and without dampers and tested on shake table under white noise.
excitation. Damper parameters are studied. Ghosha and Basu (2004) observed the effect of soil structure interaction and concluded that when the soil becomes stiff, it allows the foundation to move relative to the surrounding soil which changes the soil foundation system from that of the fixed base. In such a case a conventional TMD loses its effectiveness in controlling the response of the structure to base excitation. So to avoid the effect of SSI, it is necessary to tune the damper to the fundamental frequency of the structure–foundation system. It is also essential to provide damping in the TMD greater than the critical damping to ensure response reduction of the structure. Chouw (2004) studied the behavior of soil structure interaction with tuned mass dampers during near source earthquake at two different places varying the natural frequency of the dampers. They used the ground motions at the stations SCG and NRG of the 1994 Northridge earthquake for their study and concluded that soil structure interaction and ground motion can increase or decrease the effect of TMD.

Kwok and Samali (2006) carried out some experimental verifications of both active and passive TMD and compare the results with parametric study which are very useful in selection of optimal TMD parameters. 40-50% reduction in wind induced response & an additional damping of 3-4% of critical damping by using passive system and $\frac{2}{3}$rd reduction in wind induced response & an additional damping of 10% of critical damping by using active system was obtained from their experimental investigation.

Saidi et al. (2007) developed a Tuned mass damper using viscoelastic material and concluded that TMD is effective when tuned to the natural frequency over a narrow band. They also describe the process of estimation of viscous damping of a damper made up of viscoelastic material. For any given floor mass, damping and stiffness a damper can be an economical and simple solution for retrofitting floors with excessive vibrations. Ueng et al.(2007) studied the
practical design issues of tuned mass dampers for torsionally coupled buildings under earthquake loadings and determined the optimal PTMD system parameters by minimizing the mean square displacement response ratio on the top floor of buildings with and without PTMDs.

Wong (2008) studied dissipation of seismic energy in inelastic structures with tuned mass dampers. By using the force analogy method, an inelastic structure is modeled which is chosen as the base of plastic energy dissipation analysis in the structure. Energy response reduction after using TMD is also studied by using plastic energy spectra for various levels of structural yielding. The use of TMD increases the capacity of the structure to accumulate huge amounts of energy inside the TMD that will be released afterward in the form of damping energy. It reduces the plastic energy dissipation and increases the damping energy dissipation.

Alexander and Schilder (2009) proposed the performance of nonlinear tuned mass damper. A two degree of freedom system with a cubic nonlinearity is modeled. The nonlinearity is originated from geometric arrangement of two pairs of springs. One pair helps in providing linear stiffness whereas the other pair rotates as they extend and helps in hardening spring stiffness. In this paper a software AUTO has been used to study numerically the periodic response of a nonlinear tuned mass damper and the optimum design parameters has been observed. Lourenco et al.(2009) performed some experimental work taking a pendulum tuned mass damper with advantageous over conventional TMD. They did some simulation study considering the three dimensional behavior of pendulum mass and found that the frequency can be re tuned by changing the cable length.

Lin et al.(2010) studied the vibration control of seismic structures using semi-active friction multiple tuned mass dampers. In this paper a semi active friction type multiple tuned mass damper (SAF-MTMD) is developed to control vibration in seismic structures. Since a friction
type mass damper is same as a conventional mass damper if the static frictional force inactivates the mass damper. Various friction mechanisms have been used to activate all the mass units of friction type multiple tuned mass damper during earthquake. A comparison study is made with a passive friction type multiple tuned mass dampers and concluded that SAF-MTMD effectively reduces the seismic motion particularly at a larger intensity. Islam and Ahsan (2012) optimized Tuned mass damper parameters using evolutionary operation algorithm and determined the optimum parameters of TMD in reducing the top storey response of the structure by using an evolutionary algorithm. They used El Centro NS earthquake to develop a computer program and found a higher percentage of reduction on the roof of a ten storey structure using TMD with the application of EVOP.

2.2. Objective and scope of present work:

The objective of the present work is to study experimentally and numerically the application of TMD to control vibration of both single and multi storey frame structure under sinusoidal excitation.

The scope of the work includes experimental modeling and analysis of single and multi-storey building frame under horizontal excitation with and without TMD considering different parameters like mass ratio, frequency ratio, tuning ratio. Linear time history analysis is carried out using finite element method and STAAD Pro with and without TMD under sinusoidal ground acceleration. The Newmark Beta method is used to solve the dynamic equations for the structure-TMD system.
3. METHODOLOGY

3.1. Forced vibration analysis of a frame model:

The equation of motion for the model subjected to external dynamic force \( R(t) \) can be written in matrix form as,

\[
\begin{bmatrix}
  m_1 & 0 \\
  0 & m_2
\end{bmatrix}
\begin{bmatrix}
  \ddot{u}_1 \\
  \ddot{u}_2
\end{bmatrix} +
\begin{bmatrix}
  c_1 + c_2 & -c_2 \\
  -c_2 & c_2
\end{bmatrix}
\begin{bmatrix}
  \dot{u}_1 \\
  \dot{u}_1
\end{bmatrix} +
\begin{bmatrix}
  k_1 + k_2 & -k_2 \\
  -k_2 & k_2
\end{bmatrix}
\begin{bmatrix}
  u_1 \\
  u_2
\end{bmatrix} =
\begin{bmatrix}
  R_1 \\
  R_2
\end{bmatrix}
\]

It can be written as,

\[
[M]\{\ddot{U}\} + [C]\{\dot{U}\} + [K]\{U\} = R(t)
\]

Here the external force \( R(t) \) can be distributed into three mechanisms of the structure. First by the stiffness component, second by damping component and third by mass component. So the dynamic response of the structure to the excitation can be expressed by the displacement \( u(t) \), velocity \( \dot{u}(t) \) and acceleration \( \ddot{u}(t) \).

\([M] = \text{Global mass matrix of the frame model}\)
\([C] = \text{Global damping matrix of the frame model}\)
\([K] = \text{Global stiffness matrix of the frame structure}\)
\([U] = \text{Global nodal displacement vector}\)
\(\{\dot{\text{U}}\}\) = Global nodal velocity vector

\(\{\ddot{\text{U}}\}\) = Global nodal acceleration vector

\(R(t)\) = Applied external force

A static analysis can be done using a simple linear equation \([A]\{x\} = \{B\}\). Because in such an analysis time does not play any role. But dynamic analysis follows a complex governing equation like \([M]\{\ddot{\text{U}}\} + [C]\{\dot{\text{U}}\} + [K]\{\text{U}\} = R(t)\) which depends on time.

Therefore, when a single degree of freedom system is subjected to a ground acceleration which varies arbitrarily with time, analytical solution of the equation of motion is usually not possible. Such problems can be solved by numerical time stepping methods to integrate the differential equations. It can be done by two approaches.

(i) Implicit integration

(ii) Explicit integration

Implicit solution is one in which the calculation of current quantities in one time step are based on the quantities calculated in the previous time step. This is called Euler Time Integration Scheme. In this scheme even if large time steps are taken, the solution remains stable. This is also called an unconditionally stable scheme. This algorithm requires the calculation of inverse of stiffness matrix, since in this method we are directly solving for \(\{\text{U}\}\) vector and calculation of an inverse is a computationally intensive step. It is used in Newmarks beta method.

In an explicit analysis, instead of solving for \(\{\text{U}\}\), we go for solving \(\{\text{U}''\}\). Thus we bypass the inversion of the complex stiffness matrix, and we just have to invert the mass matrix \([M]\). In case lower order elements are used, which an explicit analysis always prefers, the mass matrix is also a lumped matrix, or a diagonal matrix, whose inversion is a single step process of just making the diagonal elements reciprocal. Hence this is very easily done. But disadvantage is that the
Euler Time integration scheme is not used in this, and hence it is not unconditionally stable. So we need to use very small time steps. It is used in Wilson-theta method

3.2. Solution of forced vibration analysis by Newmark-Beta method:

Newmark-Beta method is an unconditionally stable time stepping implicit method primarily used for analysis of structural dynamics problems based on the following two equations. It is also known as constant average acceleration method.

\[
\dot{U}_{t+\Delta t} = \dot{U}_t + \left[(1-\delta) \dot{U}_t + (\delta) \ddot{U}_{t+\Delta t}\right] \Delta t \\
U_{t+\Delta t} = U_t + (\ddot{U}_t) \Delta t + \left[(0.5-\alpha) \dot{U}_t + (\alpha) \ddot{U}_{t+\Delta t}\right] \Delta t^2
\]

The two parameters \( \delta \) and \( \alpha \) characterize the acceleration variation over a time step and verify the stability and integration accuracy characteristics of the method.

\( \delta = 0.5 \quad \alpha = 0.25 \), for constant average acceleration method.

\[
M_{t+\Delta t} \ddot{U} + C_{t+\Delta t} \dot{U} + K_{t+\Delta t} U = R_{t+\Delta t}
\]

Equation (2) is solved for \( \ddot{U}_{t+\Delta t} \) in terms of \( U_{t+\Delta t} \) and substituted in equation (1) to get equations for \( \dot{U}_{t+\Delta t} \) and \( U_{t+\Delta t} \) in terms of unknown \( U_{t+\Delta t} \). After putting these two equations in equation (3), we can obtain \( U_{t+\Delta t} \) using which \( \dot{U}_{t+\Delta t} \) and \( \ddot{U}_{t+\Delta t} \) can be calculated.

An implicit integration scheme solution is obtained using the equation,

\[
K_{t+\Delta t} U = R_{t+\Delta t}
\]

3.3. Step-by-step solution using Newmark integration method:

1. Formation of stiffness matrix \( K \), mass matrix \( M \) and damping matrix \( C \) whichever is required.
2. Selection of time steps \( \Delta t \).
3. Calculation of constants

\[ a_0 = \frac{1}{\alpha \Delta t^2}, \quad a_1 = \frac{\delta}{\alpha \Delta t}, \quad a_2 = \frac{1}{\alpha \Delta t}, \quad a_3 = \frac{1}{2\alpha} - 1 \]

\[ a_4 = \frac{\delta}{\alpha} - 1, \quad a_5 = \frac{\Delta t}{2} \left( \frac{\delta}{\alpha} - 2 \right), \quad a_6 = \Delta t (1 - \delta), \quad a_7 = \delta \Delta t \]

4. Initialization of \( U_0, \dot{U}_0 \) and \( \ddot{U}_0 \).

5. Formation of effective stiffness matrix .

\[ K^\wedge K^\wedge = K + a_0 M + a_1 C \]

\[ K^\wedge K^\wedge = K + a_0 M , \text{ neglecting damping in the structure.} \]

6. For each time step,

- Calculation of effective load vector at time \( t+\Delta t \).

\[ R^\wedge t+\Delta t = R^\wedge t+\Delta t + M (a_0 U_t + a_2 \dot{U}_t + a_3 \ddot{U}_t) + C (a_1 U_t + a_4 \dot{U}_t + a_5 \dddot{U}_t) \]

\[ R^\wedge t+\Delta t = R^\wedge t+\Delta t + M (a_0 U_t + a_2 \dot{U}_t + a_3 \ddot{U}_t) , \text{ neglecting damping in the structure.} \]

- Solution for displacement response at time \( t+\Delta t \).

\[ (K^\wedge K^\wedge) U_{t+\Delta t} = R^\wedge_{t+\Delta t} \]

- Calculation of velocity and acceleration response at time \( t+\Delta t \).

\[ \dot{U}_{t+\Delta t} = a_0 (U_{t+\Delta t} - U_t) - a_2 \dot{U}_t + a_3 \ddot{U}_t \]

\[ \ddot{U}_{t+\Delta t} = \dot{U}_t + a_6 \ddot{U}_t + a_7 \dddot{U}_{t+\Delta t} \]
4. EXPERIMENTAL STUDY

4.1. Introduction:

Tuned mass damper is a low cost seismic protection technique which is implemented in many tall building and tower in the world without interrupting the use of the building. Thus till now various research works have been conducted to discover the effect of TMD to reduce the seismic shaking of the structure numerically. But experimental works under this field is quite limited.

The motive of this study is to reduce the response by attaching a tuned mass damper to the structure under sinusoidal loading and also to obtain the effect of various parameters such as mass ratio, frequency ratio, tuning ratio etc. on response of the structure. Ratio of damper mass to the mass of the structure is known as mass ratio, ratio of excitation frequency to the fundamental frequency of the structure is known as frequency ratio and the ratio of damper tuning frequency to structural frequency is known as tuning ratio.

For this experiment, shaking table test is conducted to study the dynamic behavior of a single and a double frame structure with and without TMD where it is subjected to sinusoidal ground motion. The structure is rigidly attached to the shaking table platform. The weight of the structure may be regarded as concentrated at the roof level. Since a sinusoidal motion consists of a single frequency, it will provide a better understanding of the behavior of TMD-structure system. The fundamental frequency of the structure is determined from free vibration analysis.
Force vibration analysis is carried out by exciting the frame at various frequencies and the response is recorded.

Signal study is usually divided into time and frequency domains; each domain gives a different outlook and insight into the nature of the vibration.

Time domain analysis starts by analyzing the signal as a function of time. A signal analyzer can be used to develop the signal. The time history analysis plots give information that helps describe the behavior of the structure. Its behavior can be characterized by measuring the maximum vibration level.

Frequency analysis also provides valuable information about structural vibration. Any time history signal can be transformed into the frequency domain. The most common mathematical technique for transforming time signals into the frequency domain is called the Fourier Transform. Fourier Transform theory says that any periodic signal can be represented by a series of pure sine tones. In structural analysis, usually time waveforms are measured and their Fourier Transforms are computed. The Fast Fourier Transform (FFT) is a computationally optimized version of the Fourier Transform. With test experience, one can gain the ability to understand structural vibration by studying frequency data.
4.2. Experimental set up:

The laboratory equipment consists of,

i. A unidirectional shaking table

ii. Vibrating analyzer

iii. Control panel

iv. Accelerometer

v. PC loaded with NV gate software

vi. Frame model with and without secondary mass
4.2.1. Unidirectional shake table:

The unidirectional shaking table is nothing but a $1\text{m} \times 1\text{m}$ sliding stand regulated by an induction motor which operates a screw to impose horizontal motion. The screw sequentially operates a circular ball nut which is united to the sliding stand. The motor is driven by power supply. The sliding surface has 81 tie down points located on a 100x100 mm grid. The shake table can operate frequencies ranging from 0 to 20 Hz which can be control from the control panel. There is a system to fix up the maximum displacement of the shake table. It has the capacity to produce simple harmonic motion (sine wave forms). The maximum displacement of the table is 100 mm ($\pm$ 50mm) with amplitude resolution of 5 mm. The maximum payload of the shake table is 100 kg.

4.2.2. Vibrating analyzer:

The vibrating analyzer provides a solution in the form of structural testing equipment for vibration measurement and analysis. It helps in converting the electrical signal to measured digital signal for analysis after which a variety of analysis and displays can be possible. The most common processing on dynamic data is to perform FFT analysis to convert the data to the frequency domain from where most of the data can be viewed. Thus it provides a fast, easy and accurate way to use time and frequency domain measurements for structural tests.

4.2.3. Accelerometer:

It is attached to the frame at the location where the acceleration needs to be measured. Further the data can be transformed to determine the displacement by a software known as NV gate loaded in PC. It has the provision to get the velocity and displacement response by integrating the acceleration data.
4.2.4. Control panel:

The control panel is managed by an input voltage of 440 volts. It can control the excitation frequency of the shake table ranging from 0-20 Hz.

4.2.5. TMD structure model:

A one-storey and a two-storey building frame are developed for this experiment. The frame is supported by four columns of circular cross section of diameter 7.7 mm. The height of the column is 70 cm for single storey and 50 cm each for double storey. The roof of the frame is a rectangular iron plate of size 50 cm x 40 cm weighting 15.44 kg which is connected to the columns by nuts. An accelerometer is attached to the model to record the storey acceleration and displacement. The TMD is made up of various square iron plates of size 12.6 x 12.6 cm each having a weight of 0.707 kg, attached at the centre of roof plate by a circular rod. The frame is subjected to free vibration analysis to know the fundamental frequency of the structure. Then the damper is designed by tuning it to that frequency to obtain maximum response reduction at various mass ratios.

During the experiments, the frame as shown in figure 4.2 (a) is subjected to lateral harmonic excitation defined by the expression, \( x = x_0 \omega^2 \sin(2\pi ft) \) where, \( x_0 \) and \( f \) are the amplitude and frequency of excitation respectively which are the two varying parameters.
Weight of the plate=15.44 Kg

Height of column (l)= 70 cm

Diameter of column (D)= 0.77 cm

Fundamental frequency of frame (f) = 1.75 Hz (obtained from free vibration analysis)

Displacement Amplitude (x₀) = 0.5 cm

Stiffness of frame (k)= \( mw^2 = 1864.84 \text{ N/m} \) \((w = 2\pi f)\)

Stiffness of each column=k/4=466.62 N/m

Moment of inertia of column (I) = \( \frac{\Pi D^4}{64} = 1.7 \times 10^{-10} m^4 \)

Modulus of elasticity of steel frame (E) = \( \frac{k \times l^3}{4 \times 12I} \) (from the equation \( \frac{k}{4} = \frac{12EI}{l^3} \))

\[ = \frac{466.62 \times 0.7^3}{12 \times 1.7 \times 10^{-10}} \]
\[ =7.8 \times 10^{10} \frac{N}{m^2} \]

4.3. Time-domain analysis for single storey frame:

The frame is excited under sinusoidal excitation at various exciting frequencies ranging from 0.18 Hz to 2.97 Hz and the signals obtained are studied both in time and frequency domain which gives two different outlooks to examine the nature of vibration. The maximum displacement and acceleration response for each excitation frequency is obtained from corresponding time domain plots. Displacement and acceleration time history signal of the frame at various mass ratios with and without TMD are plotted for a frequency ratio of 0.8 and 1.0 in figure 4.3 to 4.6.

![Figure 4.3](image1.png)

Figure 4.3 Time histories of structural displacement with and without TMD at frequency ratio = 0.8

![Figure 4.4](image2.png)

Figure 4.4 Time histories of structural acceleration with and without TMD at frequency ratio = 0.8
Figure 4.5 Time histories of structural displacement with and without TMD at frequency ratio = 1.0

Figure 4.6 Time histories of structural acceleration with and without TMD at frequency ratio = 1.0

In each case, displacement and acceleration response at the top of the frame is observed taking different mass ratios. From figure 4.3 and 4.4, it is observed that for frequency ratio of 0.8, optimum displacement and acceleration response of the frame without TMD are 26 mm and 2.1 m/s$^2$ which are reduced gradually with increase in mass ratio from 0.10 to 0.25. At mass ratio of 0.25, the value of maximum displacement and acceleration is found to be 5 mm and 0.5 m/s$^2$ respectively.

Figure 4.5 and 4.6 explains the displacement and acceleration response at a frequency ratio of 1.0 (state of resonance). In this case, optimum displacement and acceleration response of the frame without TMD are found to be 70 mm and 8 m/s$^2$ respectively which are reducing abruptly after
attachment of TMD. At mass ratio of 0.25, the value of maximum displacement and acceleration is found to be 10 mm and 1 m/s² respectively.

From the above observations, it can be concluded that optimum reduction in peak displacement and acceleration for a particular frequency ratio is obtained at high mass ratio and the reduction is maximum when the frequency ratio is unity i.e, when the frame is subjected to a excitation frequency equal to the fundamental frequency of the structure.

4.3.1 Beating effect in structure mass damper system:

When the damping is very high in the secondary system, the combined system basically behaves as a SDOF system and the transfer of energy takes place in the coupled system which can stimulate vibrations in the primary structure instead of suppressing them. However, beyond a certain level of damping in the TMD, this beat phenomenon ceases and the structural response resembles SDOF decay. It basically appears when the exciting frequency nears the fundamental frequency of the structure.

Mechanically, we know that beats occur when two frequencies are close together. The difference between the two modal frequencies is defined as beat frequency.

Beat frequency,  \( f_{\text{beat}} = |f_1-f_2| \)

Where, \( f_1 \) = fundamental frequency of the structure.

\( f_2 \) = excitation frequency given to the structure.

Beat period is the reciprocal of beat frequency and can be expressed as,

\( T_{\text{beat}} = \frac{1}{f_{\text{beat}}} \)

Some experimental results are obtained to explain the beat phenomenon in combined structure-mass damper system showing in figure 4.7 to 4.10. Table 1 shows different beating parameters for a frequency ratio of 0.6, 0.8, 1.0 and 1.3.
Table 1 Parameters of beating phenomenon

| Frequency ratio | $f_1$ | $f_2$ | Beat frequency ($f_{beat} = |f_1 - f_2|$) | Beat period ($T_{beat} = 1/f_{beat}$) |
|-----------------|-------|-------|------------------------------------------|-----------------------------------|
| 0.6             | 1.75  | 1.05  | 0.7                                      | 1.43                              |
| 0.8             | 1.75  | 1.4   | 0.35                                     | 2.86                              |
| 1.0             | 1.75  | 1.75  | 0                                        | $\infty$                         |
| 1.3             | 1.75  | 2.29  | 0.54                                     | 1.85                              |

Figure 4.7 Time histories of displacement responses at a frequency ratio = 0.6

Figure 4.8 Time histories of displacement responses at a frequency ratio = 0.8
From the figure 4.7 to 4.10, it can be observed that beating effect becomes more prominent at a higher beat frequency. As the forcing frequency approaches to the fundamental frequency, the beating effect is going on decreasing. At the state of resonance, no beating is found since the beating period becomes infinitely long. Again it is appearing after resonance up to certain extent.

**4.4. Frequency domain analysis for single storey frame:**

The frame is excited under sinusoidal excitation containing various exciting frequency ranging from 0.18 Hz to 2.97 Hz and the signals obtained are studied both in time and frequency domain which gives two different outlooks to examine the nature of vibration. The maximum amplitude
response for each excitation frequency is obtained from corresponding time domain plots and presented graphically against the frequency ratio as shown in figure 4.11.

![Graph showing displacement response of structure for varying frequency ratios without TMD](image)

Figure 4.11 Displacement response of structure for varying frequency ratios without TMD

Figure 4.11 shows the amplitude response of the frame subjected to sinusoidal ground motion without TMD, from which it can be observed that initially when the forcing frequency is lower than the resonance frequency, the peak structural amplitude increases with increase in excitation frequency. At the state of resonance when $f/f_1=1$, the amplitude becomes maximum (70 mm) and again it starts decreasing with increase in forcing frequency. So a single acme point is observed from the frequency domain analysis curve.

4.4.1 Effect of frequency ratio on structural amplitude response:

A mass damper is attached to the primary frame as shown in figure 4.2 (b) to observe the reduction in response taking different mass ratio ranging from 0.1 to 0.25. In each case, the damper is tuned to the fundamental frequency of the primary frame by varying the length of the damper which is tabulated in table 2. The amplitude response of the structure for different forcing frequency ratio with various mass ratios are plotted in figure 4.12 to 4.15. In this study,
various frequency ratios starting from 0.1 to 1.7 and mass ratio ranging from 0.1 to 0.25 are considered and the corresponding amplitude responses are observed.

Table 2 Different parameters of single storey frame model with TMD

<table>
<thead>
<tr>
<th>Mass ratio (m_d/m)</th>
<th>Stiffness of damper (k_d) (N/m)</th>
<th>Damper length (L_d) (cm)</th>
<th>Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>1st mode (f_1)</td>
</tr>
<tr>
<td>0.25</td>
<td>459.57</td>
<td>44.23</td>
<td>1.25</td>
</tr>
<tr>
<td>0.20</td>
<td>367.65</td>
<td>47.65</td>
<td>1.3</td>
</tr>
<tr>
<td>0.15</td>
<td>275.74</td>
<td>52.44</td>
<td>1.35</td>
</tr>
<tr>
<td>0.10</td>
<td>183.83</td>
<td>60</td>
<td>1.4</td>
</tr>
</tbody>
</table>

Mass ratio= m_d/m

Stiffness of damper(k_d) = m_d w^2 (in N/m)

Damper length (L_d)= \left( \frac{3EI}{k_d} \right)^{\frac{1}{3}} (in cm)

Figure 4.12 Displacement response of structure for varying frequency ratios with TMD at a mass ratio of 0.25
Figure 4.13 Displacement response of structure for varying frequency ratios with TMD at a mass ratio of 0.20

Figure 4.14 Displacement response of structure for varying frequency ratios with TMD at a mass ratio of 0.15
Figure 4.15 Displacement response of structure for varying frequency ratios with TMD at a mass ratio of 0.10

Figure 4.12 to 4.15 shows the reduction in amplitude response of the frame after attaching TMD which is tuned to the fundamental frequency of the primary structure at various mass ratios. It can also be observed that, after attaching a mass damper, two peaks are observed corresponding to the two modal frequencies of the new structure. The first peak at lower frequency ratio is less than that at higher frequency ratio. This is because of the fact that at lower frequency of excitation, flexible structures (damper) are more affected than rigid structures (frame) and vice versa. Therefore at lower frequency ratio, the damper moves faster than the frame thereby reducing the amplitude response of the frame. But at higher frequency ratio, the frame being rigid, it moves faster than the damper.
4.4.2 Effect of mass ratio on structural response for single storey frame:

Figure 4.16 describes the effect of various mass ratios in reducing the response, when the damper is tuned to the fundamental frequency of the primary structure.

![Graph showing displacement response of structure for varying frequency ratios with different mass ratio](image)

Figure 4.16 Displacement response of structure for varying frequency ratios with different mass ratio

It can be noticed that when the structure is subjected to sinusoidal excitation, maximum reduction is occurring at a frequency ratio of 0.9 which is nearer to the point of resonance. So the optimum TMD tuning frequency ratio obtained here is 0.9. With increase in mass ratio, the peak displacement is going on decreasing up to a particular mass ratio and again it is increasing on further increment of mass ratio. Here optimum reduction is occurring corresponding to a mass ratio of 0.15. Table-3 illustrates the percentage reduction in peak displacement as a function of mass ratio and frequency ratio.
Table 3 Maximum structural displacements with and without TMD and percentage reduction

<table>
<thead>
<tr>
<th>Frequency ratio</th>
<th>Maximum displacement without TMD in mm</th>
<th>Maximum displacement with TMD in mm</th>
<th>Percentage reduction in the displacement</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MR=0.25</td>
<td>MR=0.20</td>
<td>MR=0.15</td>
</tr>
<tr>
<td>0.4</td>
<td>11.2</td>
<td>8.6</td>
<td>6.2</td>
</tr>
<tr>
<td>0.6</td>
<td>15.4</td>
<td>12.3</td>
<td>10.7</td>
</tr>
<tr>
<td>0.8</td>
<td>27.0</td>
<td>5.2</td>
<td>10.1</td>
</tr>
<tr>
<td>1.0</td>
<td>70.1</td>
<td>10.2</td>
<td>12.3</td>
</tr>
<tr>
<td>1.2</td>
<td>21.4</td>
<td>14.0</td>
<td>20.2</td>
</tr>
<tr>
<td>1.4</td>
<td>10.0</td>
<td>5.2</td>
<td>7.1</td>
</tr>
<tr>
<td>1.6</td>
<td>6.2</td>
<td>5.5</td>
<td>5.2</td>
</tr>
</tbody>
</table>

4.4.3. Effect of tuning ratio on structural response for single storey frame:

The tuning ratio of TMD is defined as the ratio of the damper frequency to the fundamental frequency of the primary structure. The damper is mistuned to different frequency by varying the length of the damper at a mass ratio of 0.15 and the responses are experimentally compared to observe the effect of tuning ratio as shown in figure 4.17.
Figure 4.17 Displacement response of structure for varying tuning ratio at a mass ratio of 0.15

Table 4 Damper parameters at different tuning ratio

<table>
<thead>
<tr>
<th>case</th>
<th>Tuning ratio</th>
<th>Frequency of damper (Hz)</th>
<th>Length of damper (L) in cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0</td>
<td>1.75</td>
<td>52.44</td>
</tr>
<tr>
<td>2</td>
<td>1.3</td>
<td>2.26</td>
<td>44.23</td>
</tr>
<tr>
<td>3</td>
<td>0.8</td>
<td>1.43</td>
<td>60.00</td>
</tr>
</tbody>
</table>

From the graph shown in figure 4.17, it can be observed that, there is optimum reduction in response when the tuning ratio is closed to unity. With increase or decrease in tuning ratio, the response is going on increasing.
4.5. Analysis of double storey frame:

The single storey frame model developed earlier is further extended to a double storey frame model by attaching another iron plate of same mass as previous one. Now the two storey frame as shown in figure 4.18 (a) is subjected to harmonic excitation defined by the expression, \( x = x_0 \omega^2 \sin (2\pi ft) \) where, \( x_0 \) and \( f \) are the amplitude and frequency of excitation respectively which are the two varying parameters.

Weight of each plate=15.44 Kg

Height of column in each floor (l)= 50 cm

Diameter of column (D)= 0.77 cm

Fundamental frequency of frame(f) = 1.8 Hz (obtained from free vibration analysis)

Displacement Amplitude =0.5 cm

![Figure 4.18 Frame model-2](image)
4.6. Frequency domain analysis for double storey frame:

The frame is excited under sinusoidal excitation containing various exciting frequency ranging from 0.18 Hz to 2.97 Hz and the signals obtained are studied both in time and frequency domain which gives two different outlooks to examine the nature of vibration. The maximum amplitude response for each excitation frequency is obtained for each floor from corresponding time domain plots and presented graphically against the frequency ratio as shown in figure 4.19.

![Figure 4.19 Displacement response of two storey frame structure for varying frequency ratios without TMD](image)

The nature of the frequency domain curve obtained for the two storey frame model in figure 4.19 resembles the curve in figure 4.11 obtained for single storey model. It can also be observed that the 1st floor response is always lower than the 2nd floor response.

4.6.1 Effect of frequency ratio on displacement response:

A mass damper is attached at top floor of the primary frame as shown in figure 4.18(b) to observe the reduction in response taking different mass ratio ranging from 0.025 to 0.126. In this
study, various frequency ratios starting from 0.1 to 1.7 and mass ratio ranging from 0.025 to 0.126 are considered and the corresponding displacement responses are observed. At each mass ratio the damper is not tuned to the fundamental frequency of the primary structure.

At mass ratio of 12.5%, the damper is tuned to the fundamental frequency of the frame (1.8 Hz). Stiffness of damper \(k_d = m_d w^2\) (in N/m) = 486.08 N/m

\[
\text{Damper length } (L_d) = \left( \frac{3EI}{k_d} \right)^\frac{1}{3} \quad \text{(in cm)} = 43.42 \text{ cm}
\]

With decreasing the mass ratio, the length of the damper is increasing after tuning the damper to the fundamental frequency of the primary model which is not feasible for the experiment. Therefore the damper is tuned at only 12.5% mass ratio. Table 5 shows the different modal frequencies at different mass ratio.

Table 5 Different parameters of single storey frame model

<table>
<thead>
<tr>
<th>Mass ratio ((m_d/m))</th>
<th>Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1(^{st}) mode</td>
</tr>
<tr>
<td>0.126 (tuned)</td>
<td>1.25</td>
</tr>
<tr>
<td>0.100</td>
<td>1.45</td>
</tr>
<tr>
<td>0.075</td>
<td>1.6</td>
</tr>
<tr>
<td>0.050</td>
<td>1.65</td>
</tr>
<tr>
<td>0.025</td>
<td>1.7</td>
</tr>
</tbody>
</table>
Figure 4.20 Displacement response of two storey frame structure for varying frequency ratios with TMD at a mass ratio of 0.125

The amplitude response of the structure for different forcing frequency ratio with a mass ratio of 0.126 is plotted in figure 4.20 which shows two peaks corresponding to two modal frequency of the secondary structure as observed in previous case.

4.6.2. Effect of mass ratio on structural response of a double storey frame model:

Figure 4.21 First floor displacement response of a two storey frame structure for varying frequency ratios with different mass ratio
In figure 4.21 and 4.22, at each mass ratio the damper is not tuned to the fundamental frequency of the primary structure. At a mass ratio of 0.125, the damper is tuned by fixing the length 43.42 cm. It can be observed that maximum amplitude reduction is obtained when the damper is tuned to the natural frequency of the structure.
5. NUMERICAL STUDY

5.1 Problem statement-1

The structural response of the one storey frame analyzed experimentally with and without TMD in the previous chapter is validated numerically using present FEM and STAAD Pro by subjecting the frame model under sinusoidal acceleration. Figure 5.2 and 5.3 show the top floor displacement of the single storey frame obtained in STAAD Pro without and with TMD respectively.

Figure 5.1 Single storey model

Figure 5.2 Response of single storey frame in STAAD Pro. Without TMD
Figure 5.3 Response of single storey frame in STAAD Pro. With TMD

The displacement time history response of the single storey frame subjected to sinusoidal base excitation, obtained experimentally in the previous chapter is also determined using the numerical finite element method. Figure 5.4 to 5.12 shows the comparison of experimental and numerical time history response at different frequency ratios 0.8, 1 and 1.2 with and without TMD. It shows that both the results are in a very close agreement.

Figure 5.4 Comparison of numerical and measured structure displacement time history response without TMD for frequency ratio=0.8
Figure 5.5 Comparison of numerical and measured structural acceleration time history response without TMD for frequency ratio=0.8

Figure 5.6 Comparison of numerical and measured structural displacement time history response without TMD for frequency ratio=1.0

Figure 5.7 Comparison of numerical and measured structural acceleration time history response without TMD for frequency ratio=1.0
Figure 5.8 Comparison of numerical and measured structural displacement time history response without TMD for frequency ratio=1.2

Figure 5.9 Comparison of numerical and measured structural acceleration time history response without TMD for frequency ratio=1.2

Figure 5.10 Comparison of numerical and measured structural displacement time history response with TMD for frequency ratio=0.8
Figure 5.11 Comparison of numerical and measured structural displacement time history response with TMD for frequency ratio=1.0

Figure 5.12 Comparison of numerical and measured structural displacement time history response with TMD for frequency ratio=1.2

Table 6 shows the comparison of top floor displacement of the frame obtained in present FEM, experimental analysis and STAAD Pro which are in close agreement with each other.
Table 6 Comparison Study on the maximum displacement of the single storey frame without and with TMD

<table>
<thead>
<tr>
<th>Source</th>
<th>Displacement in mm</th>
<th>% Reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Without TMD</td>
<td>With TMD</td>
</tr>
<tr>
<td>Present FEM</td>
<td>68</td>
<td>9</td>
</tr>
<tr>
<td>Experiment</td>
<td>70</td>
<td>10</td>
</tr>
<tr>
<td>STAAD Pro.</td>
<td>71.2</td>
<td>14.9</td>
</tr>
</tbody>
</table>

5.2 Problem statement-2

The structural response of the two storey frame analyzed experimentally with and without TMD in the previous chapter is validated numerically using present FEM and STAAD Pro by subjecting the frame model under sinusoidal acceleration. Figure 5.14 and 5.15 show the top floor displacement of the single storey frame obtained in STAAD Pro without and with TMD respectively.

Figure 5.13 Two-storey Model
Figure 5.14 Response of double storey frame in STAAD Pro. Without TMD

Figure 5.15 Response of double storey frame in STAAD Pro. Without TMD

Table 7 shows the comparison of top floor displacement of the frame obtained in present FEM, experimental analysis and STAAD Pro which are in close agreement with each other.

Table 7 Comparison Study on the maximum displacement of the double storey building frame without and with TMD

<table>
<thead>
<tr>
<th>Source</th>
<th>Displacement in mm</th>
<th>% Reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Without TMD</td>
<td>With TMD</td>
</tr>
<tr>
<td>Present FEM</td>
<td>50.8</td>
<td>20.5</td>
</tr>
<tr>
<td>Experiment</td>
<td>58</td>
<td>55</td>
</tr>
<tr>
<td>STAAD Pro.</td>
<td>52.5</td>
<td>18.3</td>
</tr>
</tbody>
</table>
5.3. Problem statement-3

The dynamic amplitude response of a 4 and a 10 storey reinforced concrete frame model subjected to sinusoidal ground motion is analyzed with and without tuned mass damper.

A sinusoidal ground acceleration $x'' = x_0 \omega^2 \sin(\omega t)$ is considered for time history analysis on the 4-storey frame model with and without mass damper to show the dynamic structural response. Where, $x_0$ and $\omega$ are the maximum amplitude of acceleration and frequency of the sinusoidal acceleration respectively.

Following data are taken for the analysis.

Total height of the building=14 m
Height of each floor=3.5 m
Each bay width =5m
No of bays =2
Size of beam= (0.25 x 0.35) m
Size of column = (0.25 x 0.45) m
Grade of concrete =M_{20}
Modulus of elasticity = $2.236 \times 10^6$ kN/m$^2$

Member load at roof level=23.75 kN/m
floor level=41.25 kN/m

Joint load at roof level=42.11 kN
floor level=73.28 kN

Fundamental frequency of structure =1.373 Hz
Damping factor =0.05
Time step =0.01
Maximum acceleration amplitude =0.2 m/s$^2$
5.3.1. Comparison of predicted frequency (Hz) obtained in FEM and STAAD Pro:

Table 8 shows the value of four modal frequencies of the 4-storey frame calculated in present FEM and STAAD Pro from free vibration analysis. It is observed from Table 8 that both the results are in good agreement with each other.

<table>
<thead>
<tr>
<th>Mode</th>
<th>FEM</th>
<th>STAAD Pro.</th>
<th>% Variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.439</td>
<td>1.373</td>
<td>4.8</td>
</tr>
<tr>
<td>2</td>
<td>4.0824</td>
<td>3.913</td>
<td>4.6</td>
</tr>
<tr>
<td>3</td>
<td>6.112</td>
<td>5.941</td>
<td>3.4</td>
</tr>
<tr>
<td>4</td>
<td>7.232</td>
<td>7.046</td>
<td>2.6</td>
</tr>
</tbody>
</table>

5.3.2. Comparison Study on the top storey displacement of the 4-storey frame without and with TMD:

Figure 5.17 and 5.20 show the top floor displacement of the 4-storey frame obtained from finite element method and STAAD Pro without and with TMD respectively. Table 9 shows the comparison of top floor displacement of the frame obtained in present FEM and STAAD Pro which are in close agreement with each other.
Figure 5.17 Response of 4-storey RC frame in STAAD Pro. Without TMD

Figure 5.18 Response of 4 storey RC frame in Present FEM. Without TMD

Figure 5.19 Response of 4-storey RC frame in STAAD Pro. With TMD

Figure 5.20 Response of 4-storey RC frame in Present FEM. With TMD
Table 9 Comparison Study on the maximum displacement of the 4-storey frame without and with TMD

<table>
<thead>
<tr>
<th>Source</th>
<th>Displacement in mm</th>
<th>% Reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Without TMD</td>
<td>With TMD</td>
</tr>
<tr>
<td>FEM</td>
<td>30.9</td>
<td>22.8</td>
</tr>
<tr>
<td>STAAD Pro.</td>
<td>34.2</td>
<td>27.8</td>
</tr>
</tbody>
</table>

5.4. Problem Statement-4

The frame model analyzed in problem statement-3 is extended to a 10 storey frame and again analysis is done.

5.4.1. Comparison of predicted frequency (Hz) obtained in FEM and STAAD Pro.: 

Table 10 shows the value of four modal frequencies of the 10-storey frame calculated in present FEM and STAAD Pro from free vibration analysis. It is observed from Table 10 that both the results are in good agreement with each other.
Table 10 Free vibration frequency of 10-storey frame without TMD

<table>
<thead>
<tr>
<th>Mode</th>
<th>FEM</th>
<th>STAAD Pro.</th>
<th>% Variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.588</td>
<td>0.513</td>
<td>14.6</td>
</tr>
<tr>
<td>2</td>
<td>1.743</td>
<td>1.548</td>
<td>12.6</td>
</tr>
<tr>
<td>3</td>
<td>2.840</td>
<td>2.707</td>
<td>4.9</td>
</tr>
<tr>
<td>4</td>
<td>3.912</td>
<td>3.714</td>
<td>5.3</td>
</tr>
</tbody>
</table>

5.4.2. Comparison Study on the top storey displacement of the 10-storey frame without and with TMD:

![Figure 5.22 Response of 10-storey RC frame in Present FEM. without TMD](image)

![Figure 5.23 Response of 10-storey RC frame in STAAD Pro without TMD](image)
Figure 5.24 Response of 10-storey RC frame in Present FEM with TMD

Figure 5.25 Response of 10-storey RC frame in STAAD Pro. with TMD

Figure 5.22 and 5.25 show the top floor displacement of the 10-storey frame obtained from finite element method and STAAD Pro without and with TMD respectively. Table 11 shows the comparison of top floor displacement of the frame obtained in present FEM and STAAD Pro which are in close agreement with each other.

Table 11 Comparison Study on the maximum displacement of the 2D 10-storey frame building without and with TMD

<table>
<thead>
<tr>
<th>Source</th>
<th>Displacement in mm</th>
<th>% Reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Without TMD</td>
<td>With TMD</td>
</tr>
<tr>
<td>FEM</td>
<td>185.5</td>
<td>121.6</td>
</tr>
<tr>
<td>STAAD Pro.</td>
<td>259</td>
<td>168</td>
</tr>
</tbody>
</table>
5.4.3 Tuning of damper:

To know the effect of tuning in reducing the structural response, the mass damper attached to the 10-storey frame is tuned to different modal frequencies obtained from analysis. Table 12 shows the storey displacement at various mass ratios when the damper is tuned to different modal frequencies.

Table 12 Storey displacement when damper is tuned to different modal frequency

<table>
<thead>
<tr>
<th>Mass ratio (%)</th>
<th>Storey displacement(mm)</th>
<th>f_d=f_1</th>
<th>f_d=f_2</th>
<th>f_d=f_3</th>
<th>f_d=f_4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>fd=f1</td>
<td>fd=f2</td>
<td>fd=f3</td>
<td>fd=f4</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>121.6</td>
<td>188</td>
<td>187</td>
<td>185</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>125.6</td>
<td>192</td>
<td>189</td>
<td>192</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>142.4</td>
<td>201</td>
<td>201</td>
<td>201</td>
</tr>
</tbody>
</table>

f_d = frequency of damper in Hz.

f_1, f_2, f_3 & f_4 = structural frequency at different mode in Hz.

= 0.588, 1.734, 2.84 & 83.91 Hz respectively.

From the table 12 shown above it can be concluded that frequency of the damper has to be tuned to the fundamental frequency of the structure to obtain maximum response reduction.
5.4.4 Effect of TMD on displacement response with variation of mass ratio:

A study has been carried out to analyze the effect of variation of mass ratio by keeping the damping of TMD and structure constant 5%. The 10-storey frame with TMD is subjected to the harmonic motion of same amplitude and frequency as in case of structure without TMD. The reduced dynamic response of the structure is found numerically at various mass ratio and shown in figure 5.26.

Figure 5.26 Displacement time histories response of the 10 storey frame with and without TMD at various mass ratios
Table 13 Response of the structure with TMD at various mass ratios

<table>
<thead>
<tr>
<th>Mass ratio (%)</th>
<th>0.1</th>
<th>0.2</th>
<th>0.5</th>
<th>0.6</th>
<th>0.8</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Displacement (mm)</td>
<td>156.2</td>
<td>141.8</td>
<td>128.0</td>
<td>125.8</td>
<td>125.5</td>
<td>121.6</td>
<td>126.6</td>
<td>141.5</td>
<td>162.2</td>
</tr>
</tbody>
</table>

Figure 5.27 Variation of displacement response with mass ratio

It can be concluded from figure 5.27 that increasing the mass ratio of the TMD decreases the displacement response of the structure up to a certain point.

5.4.5 Effect of TMD on Displacement response with variation of damping ratio:

The effect of variation of damping ratio of TMD is studied through the amplitude response of the frame at a constant mass ratio of 0.01 under sinusoidal acceleration. From the figure 5.28 it is found that the response of the frame decreases with increase in damping ratio and then it will remains constant after a certain point.
Figure 5.28 Variation of displacement response with damping ratio
6. CONCLUSION

6.1. Conclusion:

The present study focuses on the capability of TMD in reducing the structural vibration induced due to earthquake. A single and a double storey frame model are examined experimentally with and without TMD to determine the structural response and presented in graphical and tabular forms. Effect of various parameters such as frequency ratio, mass ratio, damping ratio on the amplitude response has been studied with TMD. The results obtained are validated numerically using finite method. Further a four storey and a ten storey RC frame models are studied using Finite element method and STAAD Pro considering various parameters. The experimental and numerical investigation of various frame models under sinusoidal ground motion confirms that the structural response can be considerably reduced to a large extent by a properly designed TMD.

The following conclusions are made from the study.

- When the frame is subjected to sinusoidal ground motion without TMD, amplitude becomes maximum at the point of resonance.

- From the experimental study, it is observed that, after using damper optimum reduction is occurring at a frequency ratio nearer to the point of resonance. That is when the frequency ratio becomes nearer to unity.

- With increase in mass ratio, the peak displacement is going on decreasing up to a particular mass ratio and again it is increasing on further increment of mass ratio.
• It is more effective in reducing the displacement responses of structures when tuned to fundamental (1st mode) frequency of the structure.

• It is more effective to use high damping ratio.

• At a higher beat frequency, beating effect is prominent which diminishes as the forcing frequency approaches to the fundamental frequency and no beating effect is observed at the state of resonance.

• From this study, it can be concluded that properly designed TMD with efficient design parameters such as tuning ratio, frequency ratio and mass ratio is considered to be a very effective device to reduce the structural response.

6.2. Future scope for study:

1. A linear model is considered in the experimental study. This can be studied by considering a non-linear model.

2. In current study the damper mass is placed at the top of the frame model and the reduction in response is observed. A further study can be carried out by placing the damper mass at different floors of the frame structure.

3. Effect of mass damper on response reduction can be studied by designing the damper mass in a pendulum shape.

4. The study can be further extended by using multiple tuned mass damper (MTMD) tuned with various modal frequencies at different levels.

5. A future study can be done with active multiple tuned mass damp.
REFERENCES


