FINITE ELEMENT ANALYSIS OF A ROTOR SUPPORTED WITH HYDRODYNAMIC JOURNAL BEARINGS

A

Project Report

Submitted in partial requirements for the degree of **Batchelor of Technology** in **Mechanical Engineering**

By

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Declaration

I hereby declare that this submission is my own work and that, to the best of my knowledge and belief, it contains no material previously published or written by another person nor material which to a substantial extent has been accepted for the award of any other degree or diploma of the university or other institute of higher learning, except where due acknowledgement has been made in the text.

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Certificate

This is to certify that the work in this project report entitled ***"Finite element analysis of a rotor supported with hydrodynamic journal bearings",*** has been performed out under my supervision in partial fulfilment of the requirements for the Degree of Batchelor of Technology in Mechanical Engineering during session 2014-2015 at Department of Mechanical Engineering, National Institute of Technology, Rourkela.

To the best of my knowledge, this work has not been submitted to any other University/Institute for the award of any degree or diploma.

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TABLE OF CONTENTS

Abstract

This work was aimed at studying and performing an analysis of a Rotor Bearing System with Journal Bearings by finite element analysis techniques. The Eigenvalue problems for equation of motions of Finite elements applied to rotors and bearings are stated. Design of a rotor bearing system is based upon calculation of various natural frequencies, critical speeds and stability analysis. The results for up to 80 element mesh were plotted. A computational procedure for finding natural frequencies of vibrating rotors with damping, gyroscopic and inertial effects is presented. The dynamic coefficients associated were computed by adopting perturbation scheme on governing equation for fluid film bearings. Rotating systems are widely studied under rotordynamics in industries for designing the systems below critical speeds and stability.

Keywords – Finite element analysis, rotor bearing system, perturbation scheme, critical speeds, damping, fluid film bearings

Nomenclature :-

D	Journal diameter, m
E	Journal center eccentricity, m
S	Sommerfeld number
M	Interial mass matrix
K	Stiffness coefficients matrix
C	Generalized damping matrix
C_1	Bearing damping coefficients
G	Gyroscopic Matrix
I	Identity Matrix
F_x, F_y	Components of reaction force in bearing, *N*
E	Young's Modulus, *pa*
s	Eigen value
x	X-direction
y	Y-direction
l	Shaft Length, *m*
θ, Φ	Angular shaft displacements, *radians*

α	Cross sectional shape factor for shear deformation of shaft
Ω	Angular speed of shaft, *rad/sec*
ρ	Density of shaft material, kg/m^3
ε	Logarithmic decrement of internal shaft damping, divided by Π
λ	Damping exponents of free vibrations of shaft, sec^{-1}

Chapter -1

1.1 Introduction:–

An important part of the standard design procedure for a rotor is the calculation of its critical speeds. In recent years methods have become available to investigate whether a rotor may experience instability because of the journal bearings, internal shaft damping, and aerodynamic excitation or from other sources.

The critical speeds of a rotor are frequently computed assuming the bearings to act as rigid supports. It is, however, well known that the bearings have flexibility which inherently lowers the critical speeds. A substantial bearing damping acts in series with the shaft flexibility, thereby contributing to a stiffening of the bearing. The effect, which depends on the ratio between the shaft and bearing stiffness's, is included here. A conventional critical speed calculation finds, by its very nature, the very nature, the undamped resonant frequencies of the rotor. In the general case, as considered, it is the damped natural frequencies which are to be determined. Thus denoting any rotor amplitude as x, a free vibration can be expressed as

$$x = |x|.e^{\lambda t} \cos(\omega t + \Psi) \qquad (1)$$

where $|x|$ is the amplitude, Ψ is an appropriate phase angle, ω is the damped natural frequency, λ is the corresponding damping exponent, and s is the complex frequency

$$s = \lambda + i\omega \qquad (2)$$

Usually λ is negative such that the vibration dies out exponentially with time. The critical speeds of the rotor and its threshold of instability are determined from a calculation of those values of s, also called the eigenvalues, at which the system can perform damped free vibrations.

Beam theory is used for establishment of shape functions. The shape functions are the basis for derivation for the matrices of finite element system governing equations. The shape functions have an inclusion of a shear a parameter which accounts the transverse shear deformation effects.

The effect of material internal damping and shear deformations combined was presented. Natural speed of whirl and response in unbalance were analysed.

Chapter – 2

2.1 Literature Review:-

The vibration analyses for different types of rotor bearing systems have been performed by various researchers in the past and recently. Below, some of the published journals are reviewed and discussed.

FEA of whirl speeds for rotor bearing systems with Internal damping. The study was done by M.Ku. In this publication, formulation of a finite element model of a C^0 Timoshenko beam is presented. Internal damping with shear deformations effected forward and backward whirl speeds. Speeds with instable threshold for system with linearized stiffness and damping by viscosity in bearings are also effected.

In the past Ruhl and Booker were among the first persons who utilised FEM to study stability and understand the unbalance behaviour of turbo rotor systems. But, they had considered only translational kinetic energy and bending energy due to elasticity. They had not included many other effects.

F. M. Dimentberg had published a book on flexural vibrations of rotors. He discusses the importance of consideration of many effects, such as the rotary inertial effects, shear deformations, gyroscopic moments, internal and external damping in finite element analyses.

Utilisation of Rayleigh beam model for devising formulation of finite element model with consideration of various other effects which were previously neglected was done by Nelson and McVaugh generalising the work done by Ruhl. Also a flexible rotor system was adopted in the work.

Guyan's work consisted of a reduction procedure to decrease the system matrices size. This saves computational time. Facilitation for computation of various natural whirl speeds had been done by transformation of element equations to rotating frame for undamped bearings.

The work with finite element formulation by Rayleigh beam model was extended by Zorzi and Nelson. The same model now consisted both hysteric and internal viscous damping inclusion.

2.1.1 Conclusion for Literature Review:

All the works conclude the use of finite element modelling to be useful with possibility for complicated rotor bearing system problems formulation and yielding successful data.

Chapter – 3

3.1 Theoretical Analysis

3.1.1 Rotor Bearing System:-

A typical **rotor bearing system** consists of a rotor shafts, discrete disks and discrete bearings. The model incorporated here is axis-symmetric physically. The rotor bearing systems can be categorised under continuous mass systems. The rotor is typically a shaft which can flexible in nature. The bearing can be of two types ball bearing or hydrodynamic journal bearing. The rotor model can simulate any practical shaft geometry.

3.1.2 Jeffcott Rotor Model:-

This model was proposed by Jeffcott in 1919 which overcame the limitations of the Rankine model. Figure 1 shows a typical Jeffcott rotor model. It is also called Föppl or Laval rotor model. It consists of a simply supported flexible massless shaft with a rigid thin disc mounted at the mid-span.

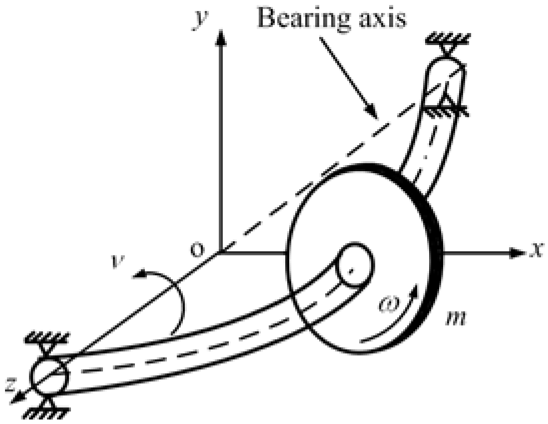

Fig1. Jeffcott Rotor Model

The transverse stiffness, *k*, of a simply supported shaft is expressed as

$$k = \frac{\text{Load}}{\text{Deflection}} = \frac{P}{PL^3/(48EI)} = \frac{48EI}{L^3} \tag{6}$$

E Young's modulus,

I Second moment of area of the shaft cross-section,

L Shaft length

3.1.3 Hydrodynamic Journal Bearings:-

The supportive part of shaft of a rotor by bearings is inside fluid. When shaft rotates the journal forms an eccentricity e to the centre with vertical and horizontal components. A radial clearance allows feasibility of the rotational operation. A ratio of radial clearance to radius of bearing is minimum at the vertically lowest point on circumference of internal housing for shaft and fluid.

3.1.4 Rotor Disk:

The kinetic energy of an axis asymmetrical rigid disk with mass centre coincident with its geometry centre is given as

$$T^d = \tfrac{1}{2} m^d \{ (Y' - \Omega Z)^2 + (Z' + \Omega Y)^2 \} + \tfrac{1}{2} (\rho I_x^d \omega_x^2 + \rho I_y^d \omega_y^2 + \rho I_z^d \omega_z^2) - \rho I_{yz} \omega_y \omega_z \qquad (3)$$

Where, m^d – mass, ρI_x^d, ρI_y^d , ρI_z^d and ρI_{yz}^d are the mass moments of inertia of the rigid disk, and ω_x, ω_y and ω_z the angular rates of the deformed cross-sectional relation to R: OXYZ, respectively. Superscript 'd' denotes the rigid disk. Application of Lagrange's Equations yield four simultaneous second order differential equation with respect to Y,Z,B and Γ.

The resulting linearized equations then become

$$M^d + q^{d''} + \Omega G^d q^{d'} - \Omega^2 N^d q^d = Q^d \qquad (4)$$

Where, the terms on the left are related to the relative acceleration, the Coriolis acceleration plus gyroscopic moment and the centripetal acceleration, respectively.

3.1.5 Shaft Modelling :-

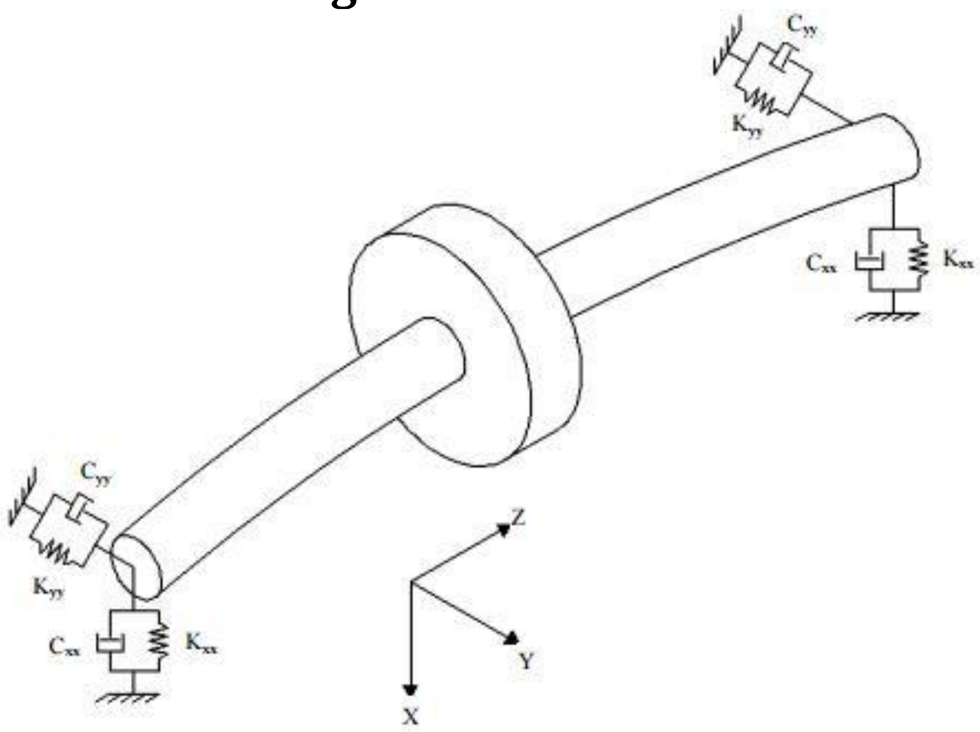

Fig2. A flexible rotor with bearings reduced to linearalised stiffness model.

The rotor base is sufficiently rigid compared to the flexible rotor–shaft. So the effect of base movement is conceived in terms of six motion parameters: three translational displacement components of any point attached to the base and three rotations about a set of three orthogonal axes. Due to rotation of the base, dynamics of the rotor–shaft system, where the rotor–shaft continuum is discretized using beam finite elements, is influenced by the Coriolis Effect and parametric excitations of different forms, in addition to the effects due to translational as well as rotary inertia, gyroscopic moment, flexural stiffness and internal damping. The equations of motion for individual disc and shaft elements are derived in a frame attached to the base using Lagrange's equation. Since the work aims at mitigation of bending vibration response of the flexible rotor–shaft system with respected to the rigid base, the effect of base-inertia does not appear in the equations of motion. Duchemin et al. developed similar mathematical model of a rotor–shaft–bearing system; however, used a lumped parameter approach to keep the analysis simple.

3.1.5.1 Finite Rotor Shaft Element:

A typical finite rotor element is shown here in Fig. 3. It is assumed that the nodal cross sectional displacements (V, W, θ, ζ) time dependent and function of position (s) throughout the element axis. The rotations (θ, ζ) are related with translations (V, W). V, W are account for degrees of freedom translationally and θ, ζ are for rotational degrees of freedom.

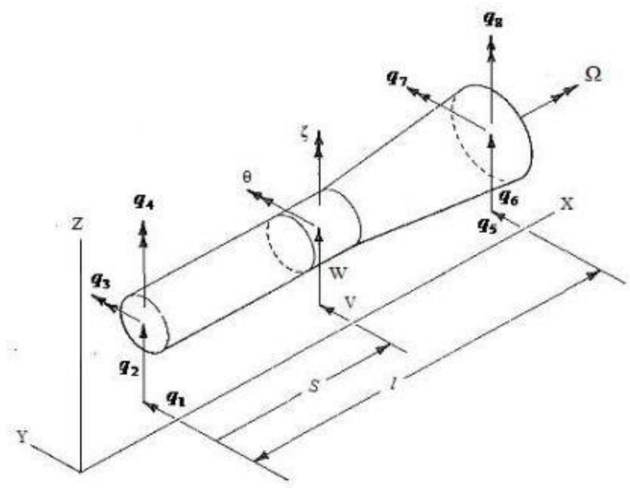

Fig3. Finite rotor shaft element and coordinates

3.1.6.2 Matrix Displacement Equations:-

In matrix displacement method stiffness matrix of an element is assembled by direct approach while in FEM through direct stiffness matrix may be treated as an approach for assembling element properties, it is energy approach which has revolutionized entire FEM.

The matrix displacement method is presented and solution techniques for simultaneous equations are discussed briefly.

The standard form of displacement equation is,

$$[K]\{\delta\} = [F] \quad (5)$$

Where, K is the stiffness matrix

$\{\delta\}$ is displacement vector and

$\{F\}$ is force vector in the coordinate directions

3.1.7 Bearing support modelling:-

The bearing support flexibility may be modelled by placing a support spring, mass and damper in series with the bearing's fluid film stiffness and damping properties. The equations for the resulting equivalent support stiffness and damping are derived and listed. Horizontal and vertical damping properties are included in the model at each bearing.

Although the proposed model assumes only a single level of support flexibility, multiple levels exist in real rotating machinery. These levels include the bearing pivot for tilting-pad bearings, the bearing support bracket or case, the machine casing, the support pedestals, the I-beam base and the concrete foundation. The first step in employing this model is to determine which part of the overall supporting structure is the most flexible. Often, but not always, the bearing case and bearing support case are more flexible than the other parts of the structure.

3.1.7.2 Reynold's Equation

The governing equation for pressure distribution in the fluid film from Lubrication theory is a partial differential equation. This equation is called Reynold's Equation.

$$\frac{\partial}{\partial X}\left(\frac{h^3}{12\mu}\frac{\partial P}{\partial X}\right)+\frac{\partial}{\partial Z}\left(\frac{h^3}{12\mu}\frac{\partial P}{\partial Z}\right)=\frac{\Omega R}{2}\frac{\mathrm{d}h}{\mathrm{d}X}+\frac{\partial h}{\partial t} \quad (6).$$

3.1.7.3 Damping and Stiffness Coefficients:

A particularly mathematically useful type of damping is linear damping. Linear damping occurs when a potentially oscillatory variable is damped by an influence that opposes changes in it, in direct proportion to the instantaneous rate of change, velocity or time derivative, of the variable itself. In engineering applications it is often desirable to linearize non-linear drag forces. This may be done by finding an equivalent work coefficient in the case of harmonic forcing. In non-harmonic cases, restrictions on the speed may lead to accurate linearization.

$$[K_m]=\begin{bmatrix} K_{XX} & K_{XY} \\ K_{YX} & K_{YY} \end{bmatrix};\ [C_m]=\begin{bmatrix} C_{XX} & C_{XY} \\ C_{YX} & C_{YY} \end{bmatrix} \quad (7)$$

3.1.8 Global Equation of Motion:-

Equations of motion for the finite element model of the rotor–shaft–bearing system may be obtained after assembling appropriately equations for individual discs, shaft elements as well as mass unbalance.

[M+N]{U’’}+[C]{U’}+[K]{U}={R} (8)

In the above equation, $[M]$ is the assembled inertia matrix, $[D]$ is the assembled matrix coefficient to global velocity vector including the gyroscopic matrix, Coriolis matrix, damping matrix, etc. and $[K]$ is the assembled matrix coefficient to global displacement vector including bending stiffness matrix, circulatory matrix, parametric stiffness matrices due to base motion and the bearing stiffness matrix. $\{f\}$ is global load vector containing the effects of mass unbalance and the inertia force due to the base motion. It is to be noted that both the matrices $[K]$ and $[D]$ have time-dependent terms originating due the base motion. Under fluctuating motion of the base, the time-dependent terms of the stiffness matrix cause parametric excitation and possible instability for a particular set of base motion parameters. To find out a method for avoiding the onset of such an instability phenomenon during the operation of the rotor–shaft system is one of the main objectives of the present work.

$$U_i = \begin{Bmatrix} y_i \\ x_i \\ \varphi_i \\ \theta_i \end{Bmatrix} \qquad (9)$$

Where, y,x,Φ, θ have same configuration as V, W, θ, ζ defined earlier for finite rotor shaft element.

3.1.8 Finite Element Equations:-

In engineering problems some unknowns are there which are basic. Finding them, prediction of whole structure gets possible. These basic unknowns can be called to be field variables. They are generally encountered in the engineering problems. Some of them are displacements, velocities, electric and magnetic potentials in solid mechanics, fluid mechanics, electrical engineering and temperature in heat flow problems.

In continuum, these unknowns are infinite. The finite element procedure reduces such unknowns to a finite number by dividing the solution region into small parts called elements and by expressing the unknown field variables in terms of assumed approximating functions (Interpolating functions/Shape functions) within each element. The approximating functions are defined in terms of field variables of the nodal points. Thus in the finite element analysis the unknowns are the field variables of the nodal points. Once these are found in the field variables at any point can be found by using interpolation functions.

After selecting elements and nodal unknowns next step in finite element analysis is to assemble element properties of each individual element. For example, in solid mechanics, we have to find force-displacement i.e. stiffness characteristics of each individual element. Mathematically this relationship is of the form

$$[K]_e\{\delta\}_e = \{F\}_e \qquad (10)$$

Where $[K]_e$ is the element stiffness matrix, $[\delta]_e$ is nodal displacement vector of the element and $\{F\}_e$ is the nodal force vector. The element of stiffness matrix k_{ij} represents the force in coordinate direction 'i' due to displacement in coordinate direction 'j'. Four methods are available for formulating these element properties viz. direct approach, variational approach, weighted residual approach and energy balance approach. Any of these methods can be used for assembling element properties. In solid mechanics variational approach is commonly employed to assemble stiffness matrix and nodal force vector (consistent loads).

3.1.8.1 Steps involved in finite element analysis:

1) Select suitable field variables and the elements.
2) Discretise the continua.
3) Select interpolation functions.
4) Find the elemental properties.
5) Assemble element properties to get global properties.
6) Impose the boundary conditions.
7) Solve the system equations to get the nodal unknowns.
8) Make the additional calculations to get the required values.

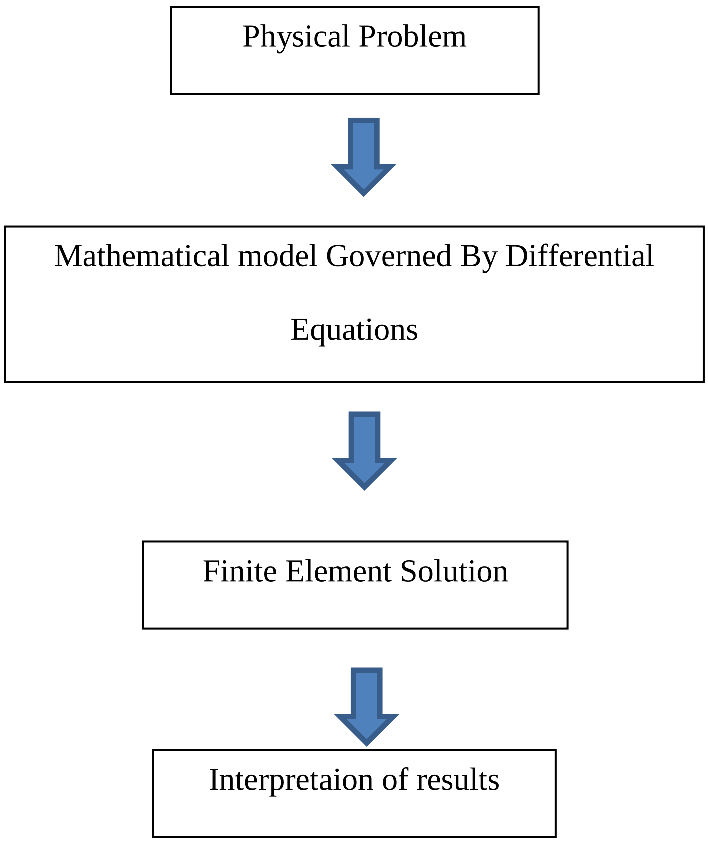

Fig4. A flow chart for the steps in process of Finite Element Analysis

3.1.8.2 Advantages with use of Finite Element Method:

One of the advantages of finite element model resides in its suitability for the automatic formation of the system equations with the separately developed component equations with the separately developed component equations.

3.1.9 Eigen Value Problems:-

Matrix eigenvalue problems occur in different situations. The eigenvalues of a matrix can describe behaviour independent of coordinates. Matrix eigenvalues are very useful in analyzing Markov chains and in the fundamental theorem of demography. Efficient computation of matrix powers can be done by theorems about diagonalization

An example of where a matrix eigenvalue problem arises is the determination of the main axes of a second order surface Q=xTAx=1 (defined by a symmetric matrix A). The task is to find the places where the normal is parallel to the vector x, i.e Ax=λx.

Methodology :-

For the purpose of computing damped natural frequencies (eigenvalues), a rotor bearing system can be presented mathematically by a stiffness matrix, damping matrix and an inertia matrix from which the eigenvalue problem can be formulated. The dimension of resulting matrix however, equals, 8 times the number of mass stations in system which, in a practical rotor calculation, may mean a matrix size of, for example, 240*240. Further complications arise from the matrix being symmetric, requiring special modification in the available standard methods for eigenvalues calculation. For this reason an alternative method is desirable and in following a computational procedure is developed.

The rotor is supported in fluid-film bearings whose dynamic properties are given through a set of stiffness and damping coefficients. The bearing reaction in x - direction can be expressed as reaction in x-direction =

$$\begin{aligned} &-K_{xx}x - K_{xy}y - K_{x\theta}\theta - K_{x\phi}\phi \\ &-B_{xx}\dot{x} - B_{xy}\dot{y} - B_{x\theta}\dot{\theta} - B_{x\phi}\dot{\phi} \end{aligned} \qquad (11)$$

And similarly for the reactions in the y,θ and Φ directions ("dot" indicates differentiation with respect to time). There are 32 coefficients in all which can be arranged in stiffness matrix K and a damping matrix B, respectively.

The coefficients are obtained from the lubrication equation (Reynold's equation) as the gradients of the hydrodynamic forces. For conventional journal bearings, only the radial coefficients are of importance. On the other hand, for very short rotors or at higher-order shaft modes the angular coefficients may also be of important significance.

Equation also describes certain aerodynamic forces in turbomachinery. In axial flow compressors and turbines, a radial displacement of the wheel in a stage sets up a transverse force, proportional to the displacement. With the notation of equation (12) the coefficient of proportionality becomes

$$K_{xy} = -K_{yx} = \beta * T/2rh \qquad (12)$$

Where T is the stage torque, r is the pitch radius, h is the vane height, and β is the dimensionless parameter. The remaining stiffness and damping coefficients are equal to zero.

The radial amplitudes at station number x_n and y_n , and the corresponding angular amplitudes are θ_n and Φ_n. For free vibrations of the form of equation (1), these quantities become complex and the equations of motion for station n are in figure 5.

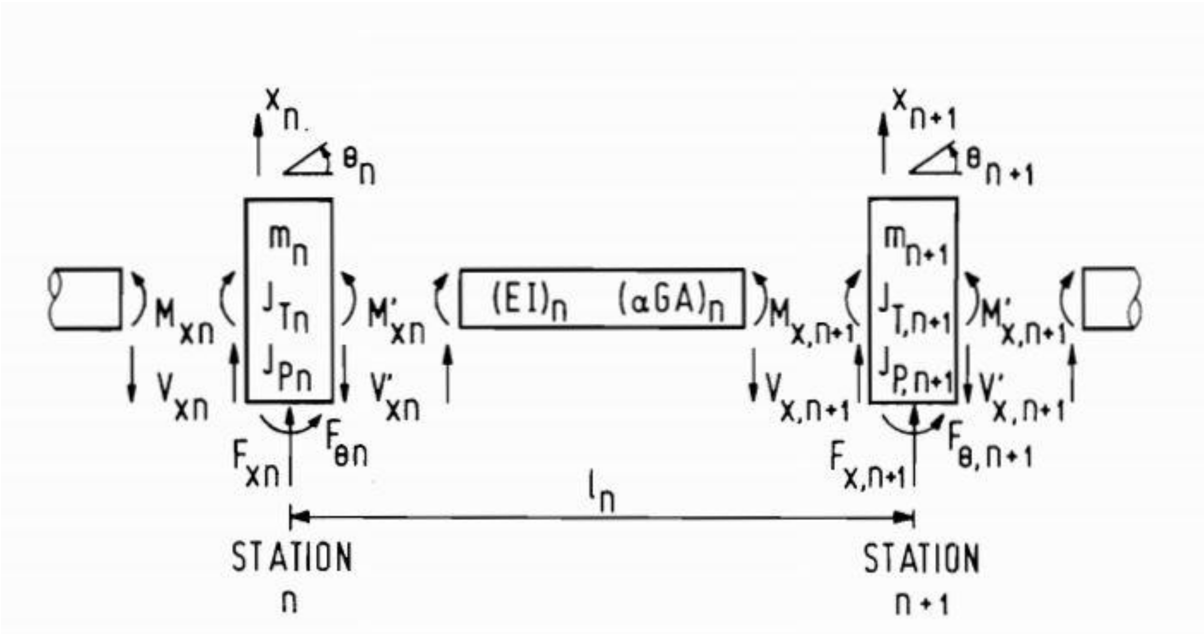

Figure 5 – Sign convention for radial displacement, angular displacement, bending moment, and shear force

$$\begin{Bmatrix} -V'_{xn} \\ -V'_{yn} \\ M'_{xn} \\ M'_{yn} \end{Bmatrix} = \begin{Bmatrix} -V_{xn} \\ -V_{yn} \\ M_{xn} \\ M_{yn} \end{Bmatrix} + \begin{Bmatrix} s^2 m_n x_n \\ s^2 m_n y_n \\ s^2 J_{Tn}\theta_n + s\Omega J_{Pn}\phi_n \\ s^2 J_{Tn}\phi_n - s\Omega J_{Pn}\theta_n \end{Bmatrix} + (\underline{\underline{K}} + s\underline{\underline{B}})_n \begin{Bmatrix} x_n \\ y_n \\ \theta_n \\ \phi_n \end{Bmatrix} \tag{13}$$

In further analysis, the derivatives of the variables with respect to s are also required.

$$\begin{Bmatrix} -\dfrac{dV'_{xn}}{ds} \\ -\dfrac{dV'_{yn}}{ds} \\ \dfrac{dM'_{xn}}{ds} \\ \dfrac{dM'_{yn}}{ds} \end{Bmatrix} = \begin{Bmatrix} -\dfrac{dV_{xn}}{ds} \\ -\dfrac{dV_{yn}}{ds} \\ \dfrac{dM_{xn}}{ds} \\ \dfrac{dM_{yn}}{ds} \end{Bmatrix} + \begin{Bmatrix} s^2 m_n \dfrac{dx_n}{ds} \\ s^2 m_n \dfrac{dy_n}{ds} \\ s^2 J_{Tn} \dfrac{d\theta_n}{ds} + s\Omega J_{Pn} \dfrac{d\phi_n}{ds} \\ s^2 J_{Tn} \dfrac{d\phi_n}{ds} - s\Omega J_{Pn} \dfrac{d\theta_n}{ds} \end{Bmatrix}$$

$$+ (\underline{\underline{K}} + s\underline{\underline{B}})_n \begin{Bmatrix} \dfrac{dx_n}{ds} \\ \dfrac{dy_n}{ds} \\ \dfrac{d\theta_n}{ds} \\ \dfrac{d\phi_n}{ds} \end{Bmatrix} + \begin{Bmatrix} 2sm_n x_n \\ 2sm_n y_n \\ 2sJ_{Tn}\theta_n + \Omega J_{Pn}\phi_n \\ 2sJ_{Tn}\phi_n - \Omega J_{Pn}\theta_n \end{Bmatrix} + \underline{\underline{B}} \cdot \begin{Bmatrix} x_n \\ y_n \\ \theta_n \\ \phi_n \end{Bmatrix} \tag{14}$$

Chapter -4

4.1 Results And Discussion :-

4.1.1 Problem Statement

Numerical results have been obtained for a uniform shaft with length of 50inch. a diameter 4inch., Young's modulus 207 $*10^9$ Pa. For simplicity the shaft is treated as a uniform beam. It is supported at the ends with identical fluid-film bearings and natural frequencies. The results are shown in figures.

4.1.1.1 Table -1: Input values taken for numerical analysis

Serial No.	Parameter	Description	Value	Unit
1	L	Shaft span	1.190	m
2	D	Shaft Diameter	0.1036	m
3	E	Modulus of Elasticity	$211*10^9$	Pa
4	P	Density	8081	Kg/m^3
5	k_b	Direct coefficient of stiffness for bearings	$10.18*10^6$	N/m
6	c_b	Direct coefficient of damping for bearings	16542	N.s/m

4.1.2Results:

4.1.2.1 Table -2 Results of Natural Frequencies in different modes

FEM results								
ω/n	5	10	20	40	60	80	100	120
1st	5117	5296	5373	5342	5329	5323	5319	5316
2nd	6278	6062	5983	5985	5994	6001	6003	6008
3rd	16543	16834	16938	16864	16829	16809	16796	16788
4th	46428	46480	46447	46188	46118	46054	45992	45966
5th	93100	92934	92113	91826	91613	91508	91456	91418

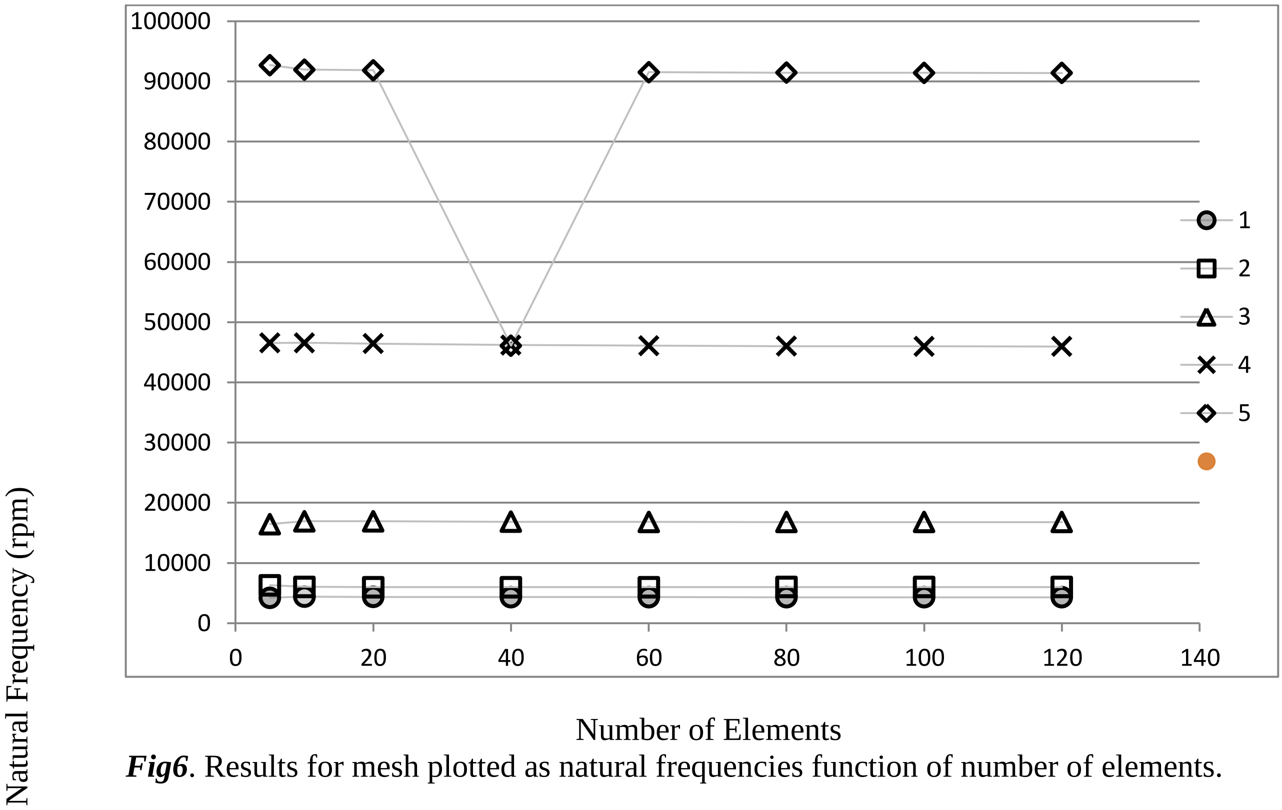

Fig6. Results for mesh plotted as natural frequencies function of number of elements.

4.1.2.2 Table – 3: Calculated Natural Frequencies relative error in percentage values.

FEM results								
ω/n	5	10	20	40	60	80	100	120
1st	-0.3%	2.9%	2.5%	2.1%	2.0%	1.8%	1.6%	1.5%
2nd	4.70%	0.9%	-0.3%	-0.2%	-0.1%	0.0%	0.1%	0.1%
3rd	-0.2%	2.5%	2.6%	2.1%	2.0%	1.9%	1.8%	1.7%
4th	3.5%	3.5%	3.2%	2.7%	2.4%	2.3%	2.2%	2.1%
5th	3.0%	2.2%	2.1%	1.8%	1.7%	1.6%	1.6%	1.5%

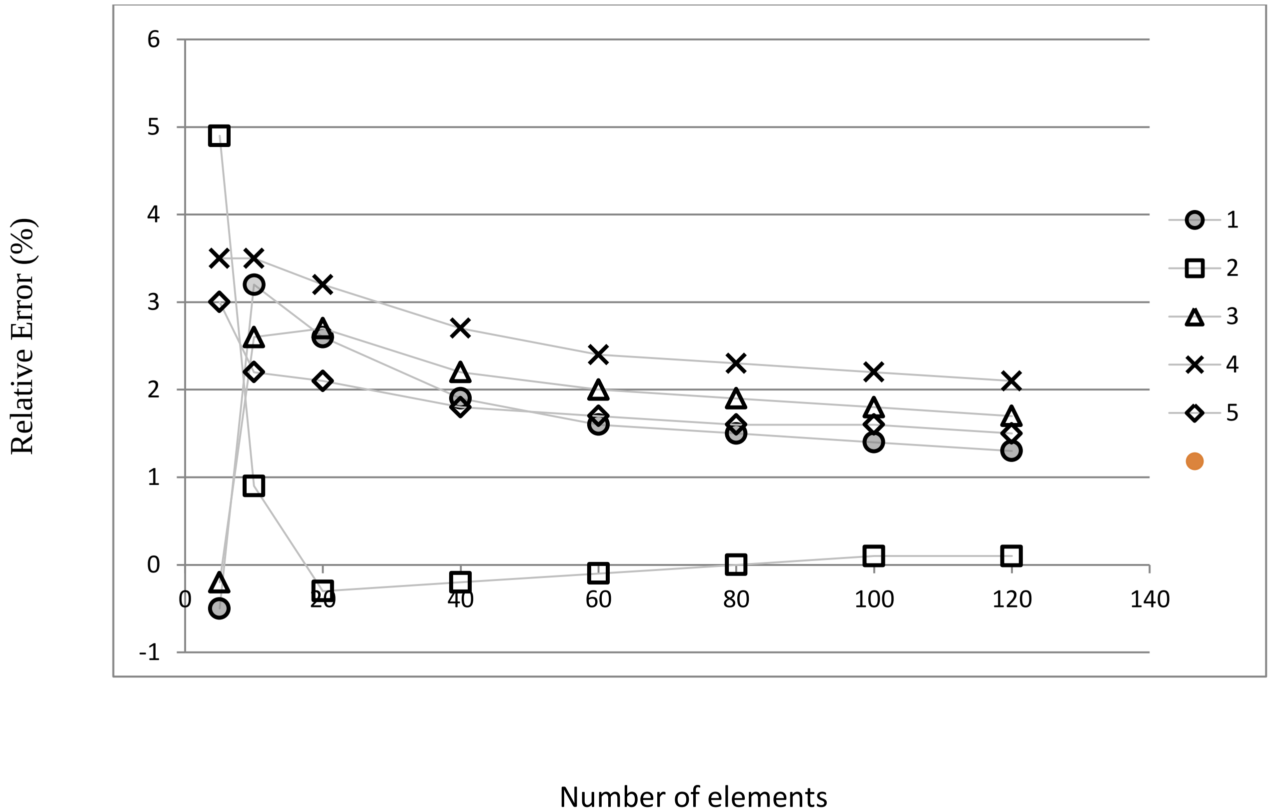

Fig7 Results for mesh plotted as relative error as function of number of elements.

4.1.3 Discussion:

A analysis for finite element model had its basis on natural frequencies of the rotordynamic system. “NF” represents natural frequency while “n” represents number of elements.

A n = 80 element mesh resulted for an error lesser than 2% at calculation of 1st, 2nd, 3rd and 5th natural frequencies and at the 4th natural frequency , the error is lesser than 2.4%.

The lowest two modes of natural frequency are critically damped. The third shows up at a frequency slightly below the first critical speed. The fourth mode comes in just above the second bearing critical and similarly for the higher modes. The mode number differs with the critical speed number.

In an actual rotor, the bearings usually have different stiffness and damping properties, in vertical and horizontal directions. Each mode will split up into two, one mode corresponding to the minimum bearing stiffness.

In the absence of any damping and with no interference with other modes, it is readily seen that the rotor will be in a state of backward precession between the two modal frequencies and although the presence of damping and overlapping modes complicate the picture, it is normally found that one mode has predominantly forward precession while the other mode is backward precession. This is further amplified by the gyroscopic moments in the rotor.

Stiffening up the bearing results in second and third mode being critically damped instead of first and second. In this case, the first critical speed corresponds with the first mode and the frequency is just below the value computed.

In the analysis it is shown that maximum two modes can be critically damped for simple shaft bearing system. It is true for systems with reasonable symmetry.

When the value of logarithmic decrement exceeds 1, the particular mode is well damped. Each rotor actually consists of two modes, one with forward precession, identified by the letters F and B respectively. In addition, E identifies even modes where the amplitudes are in phase and O are for the odd modes where the end amplitudes are out of phase. The first rotor mode is even, the second odd, and so on. The backward precession modes are critically damped.

Mass unbalance excites the rotor. The intersection between synchronous frequency and the modal curves give damped critical speeds of the rotor. It is seen that the first and second modes are never excited while the third mode is synchronous.

The whirl frequency at oneset of instability is close to one half of the rotational frequency. The rotor whirls in its first mode (forward precession) but this mode is not the first critical speed. The threshold of instability is not twice the first critical speed, as it is otherwise accepted as the rule of thumb, as seen, it equals twice the frequency of the first mode. Even though a bearing calculation may be successful in predicting unbalanced response peaks, it cannot be used for whip encounter speed.

4.2 Conclusions and Future Scope:-

The damped natural frequencies (eigenvalues) of a general flexible rotor supported in fluid film journal bearings. The rotor model also incorporates internal hysteric shaft damping and destabilizing aerodynamic forces.

The method is basically an extension of the previous method for calculating critical speeds, which utilizes the computational procedure of the well-known Holzer method. As such the method can be readily applied to a wide variety of rotor and support configurations and is easily programmed for numerical computation. The program is simple with fast execution.

The calculated eigenvalues establish the stability margin of the rotor system, conveniently expressed in terms of logarithmic decrement of the eigenvalue closest to the threshold of instability. If the margin is insufficient, or the rotor is even found to become unstable, the program can be used to explore possible means of improvement, either by reducing or eliminating the sources causing the instability, by the design modifications of shaft or bearings, or by providing stabilizing external damping through damper bearings. In performing such investigations, and to optimize corrective measures, the program can be a valuable design tool.

The obtained damped natural frequencies also establish the actual critical speeds of the rotor, including the stiffening effect of the damping in the bearings. These results give a more realistic base than conventional critical speed calculation for assessing any potential trouble from passing through or operating close to a critical speed. In addition, knowing the logarithmic decrement at critical speeds and thereby, the response amplification factor, means are provided for evaluating the rotors sensitivity to mass unbalance.

However, in the present studies the bearing data are calculated on the assumption that the portion of the shaft in the bearing is rigid, so iterative method between flexible rotor analysis and bearing analysis will give more realistic results for a real rotor bearing system.

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