

**FINITE ELEMENT MODELLING AND
ANALYSIS OF CARBON NANOTUBE BASED
NANO COMPOSITE STRUCTURES**

A thesis submitted in partial fulfillment of the requirements for the degree

of

Bachelor of Technology in Mechanical Engineering by

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CERTIFICATE

This is to certify that the thesis entitled, “**Finite Element Modelling and Analysis of Carbon Nanotube based Nano Composite Structures**” submitted by **ABHISHEK NANDA** in partial fulfilment of the requirement for the award of Bachelor of Technology degree in Mechanical Engineering at National Institute of Technology, Rourkela is an authentic work carried out by him under my supervision and guidance. To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University/Institute for the award of any Degree or Diploma.

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Abstract:

The objective of the present work is to finite element (FE) modelling and analysis of different nanocomposite structures such as airfoil and parabolic antenna for free vibration analysis (to find the different modal natural frequencies and mode shapes) as well as forced vibration analysis (to obtain the equivalent von Mises stresses and maximum transverse displacements). The analysis is done for Carbon Nanotube reinforced Composites with varying percentage of carbon nanotube content. First the mathematical model on Carbon Nanotube reinforced Composite was used to find out the different properties; i.e. Young's Modulus, Bulk modulus and so on with different volume fraction of Carbon Nanotube content present. Next step is to model the structures; i.e. airfoil and parabolic antenna by solid modelling and then convert it into Finite Element model by giving the proper meshing, conditions and input data. The FE model was then analysed with the help of ANSYS and the different mode shapes and fundamental vibration frequencies were found. The some important results were then presented into a table for comparison and discussion.

Table of contents:

Serial number	Contents	Page number
	<i>Certificate</i>	ii
	<i>Acknowledgement</i>	iii
	<i>Abstract</i>	iv
	<i>Table of contents</i>	v
	<i>List of figures</i>	vii
	<i>List of tables</i>	viii
CHAPTER 1	Introduction	1
1.1	Introduction to Nano-composite	1
1.2	Applications of Nano-composites	2
	<i>Motivation and Objective</i>	4
CHAPTER 2	Literature review	5
2.1	Classical Modelling	5
	<i>Beam</i>	5
	<i>Shell/Plate</i>	6
2.2	Finite Element Modelling	7
2.3	Literature Review	10
CHAPTER 3	Materials and Methods	13
3.1	Formulation of nano composite properties	13
3.2	3D Solid Modelling of Structures	14
	<i>Airfoil</i>	14

	<i>Parabolic antenna</i>	17
3.3	Finite Element Modelling	20
3.3.1	ANSYS	20
3.3.2	Element Type	20
3.3.3	Boundary Condition	23
3.3.4	No of Elements and Nodes	24
CHAPTER 4	Results and Discussion	26
	<i>Problem Definition</i>	
	<i>ANSYS Analysis</i>	
4.1	Free Vibration Analysis	26
4.1.1	Airfoil	27
4.1.2	Parabolic Antenna	28
4.2	Forced Vibration Analysis	29
4.2.1	Airfoil	30
4.2.2	Parabolic Antenna	31
4.3	Analysis of different variations of airfoil	32
4.3.1	Free Vibration Analysis	32
4.3.2	Forced Vibration Analysis	34
4.4	Effect of CNT on Various Response of Airfoil and Antenna	34
	<i>Conclusion</i>	37
	<i>Scope of future work</i>	
	REFERENCES	38

List of figures:

Figure number	Description	Page number
1	The different airfoil components	15
2	Airfoil 3D model	17
3	Parabola and its definitions	18
4	Parabolic antenna 3D model	19
5(a), 5(b)	Finite Element model of airfoil	21
6(a), 6(b)	Finite Element model of parabolic antenna	22
7(a), 7(b), 7(c), 7(d), 7(e), 7(f):	Different mode shapes of an airfoil under free vibration. The figures are the mode shapes from mode, $m=1$ to $m=6$ respectively.	27
8(a), 8(b), 8(c), 8(d), 8(e), 8(f):	Different mode shapes of a parabolic antenna under free vibration. The figures are the mode shapes from mode, $m=1$ to $m=6$ respectively.	28
9(a)	Stress profile of an airfoil under forced vibration.	30
9(b)	Displacement profile of an airfoil under forced vibration.	31
10(a)	Stress profile of a parabolic antenna under forced vibration.	31
10(b)	Displacement profile of a parabolic antenna under forced vibration.	32
11(a), 11(b)	Variation of frequency with respect to modes and percentage of CNT of airfoil and parabolic antenna respectively in free vibration.	35
12(a)	Variation of max. deformation of airfoil with respect to percentage of CNT.	35

12(b)	Variation of equivalent stress of airfoil with respect to percentage of CNT.	36
13(a), 13(b)	Variation of max. deformation and equivalent stress of parabolic antenna with respect to percentage of CNT.	36

List of tables:

Figure number	Description	Page number
1	Convergence analysis of airfoil	24
2	Convergence analysis of parabolic antenna	25
3	Element statistics for variations of airfoil	25
4	Modal frequencies of structures at different percentage of Carbon Nanotube content	26
5	Stresses and deformation of airfoil at different conditions	29
6	Stresses and deformation of parabolic antenna at different conditions	30
7	Modal frequencies of varied airfoil structures at different percentage of Carbon Nanotube content	33
8	Stresses and deformation of airfoil shell at different conditions	34
9	Stresses and deformation of airfoil shell with ribs at different conditions	34

1. Introduction

Despite the fact that many theories and experiments are helpful in securing the nano-composite stress–strain conducts, their utilization under real auxiliary usages is a definitive reason needed to improve this materials, best in their class. Therefore, there is a need to watch the worldwide reaction of CNTRCs in a genuine basic component.

1.1 Introduction to Nano-composite:

Nanocomposites are a multiphase strong material where one of the stages has one, a few measurements of fewer than 100 nanometres, or nano scale structures. In the best way, permeable membranes, copolymers , gels and colloidal solutions are included in this definition. Nano-composites have entirely different electrical, mechanical and thermal properties from components. Proposal for of confinement for these impacts are made, <5 nanometer for reactant action, <20 nanometer for flexing of a rigid material, <50 nanometer for changes in refractive files, and <100 nanometer for accomplishing super-paramagnetism, fortifying and confining network disengagement development.

Large surface to volume proportions make nano-composites different from regular composites in fortifying stage and the astoundingly large viewpoint proportion. The connection of the network fortification phase is ordinarily of greatness. The properties of network are altogether influenced in fortification region .

Carbon Nanotubes:

Carbon Nanotubes (CNTs) are one type of Carbon allotrope with a nano structure of tubular shape. CNTs can have aspect ratio $1:10^8$, altogether larger than others. The carbon atoms, which are hollow and round, have abnormal properties, especially nanotechnology application. Thanks to their phenomenal electrical, mechanical and thermodynamic properties, CNTs discover applications as additive to various auxiliary composites.

CNTs are individuals from the auxiliary fullerene crew. Sheets of carbon with one-molecule thickness make up the CNTs; hence deriving its name from long tubular hollow structures. There is movement of sheets at discrete and specific points; the blend of the span and the point applies the properties; e.g. if the individual CNT is a semiconductor or metal. Types of nanotubes are^[1]:

- 1) SWCNT: Single walled Carbon Nanotube;
- 2) MWCNT: Multi walled Carbon Nanotube.

1.2 Application:

- ❖ *More prominent force yielding batteries.* Analysts have made strategies to build electrodes for Li-ion batteries from a nano-composite shaped with nano circles of silicon and nano particles of carbon. The silicon-carbon nano composite electrode reaches the Li electrolyte permitting quicker charging.
- ❖ *Broken bone mending is faster.* The building of alternative bone is faster when a platform made up of polymer nano composites with CNT which manages the building up of alternative bone.

- ❖ *Basic parts production having high quality to-weight proportion.* An Carbon nanotube containing nano composite with epoxy matrix is used to create the edges of the windmills. This outcomes in a light and solid cutting edge. These razor sharp edges expand the power created by each windmill.
- ❖ *Composites containing graphene with high quality to-weight proportions.* Epoxy composites are made stronger/stiffer by adding graphene than CNT based epoxy nano-composites. The bonding of graphene is better with the epoxy polymers, resulting in successful coupling.
- ❖ *Making lightweight nano-composite sensors.* A CNT based nano composite behaviours power depending on the division of CNTs. This property permits patches of polymer-nanotube nano composite to go about as anxiety sensors on windmill edges. When a strong flow of wind twists the razor-edges of the windmill, nano composite also twists. The electrical conductance of the nano composite sensor is changed by any twisting, thus sounding an alarm.
- ❖ *Adaptable battery production.* A conductive paper is made from CNT based nano composite. When this conductive paper stores up the electrolyte, it forms an adaptive battery.
- ❖ *Tumour examination and evacuation.* An joint venture of fluorescent nano particles and attractive nano particles is up for production. Fluorescence helps in examination of the tumour while attractiveness helps easy detection of tumours in MRI.

Motivation and Objective:

Nano composites containing Carbon Nanotubes have novel electronic, mechanical, other physical and chemical properties. Carbon Nanotube based nano-composites are very stiff compared to other composites; hence have application in nano-electronics, medical treatments and composite made structures. Their mechanical behaviour is of topmost requirement and many current researchers focus on that too.. Hence the objective of present study includes:

- i. Formulation of nano-composite properties;
- ii. Finite Element modelling of airfoil and parabolic antenna;
- iii. Free vibration analysis of structure and system;
- iv. Forced vibration analysis of structures;
- v. Comparison of results for different percentages of CNT in nano-composite.

2. Literature Review

2.1 Classical Modelling:

2.1.1 Beam:

A structure with axial dimension, l , predominant than the other orthogonal components is known as a Beam. Cross-section of a beam, A , is defined by the intersection of its axis and the orthogonal plane of the structure. An ortho-normal reference coordinate system (Cartesian system) contains the geometry of the beam, stresses, strains and deformations.

The Euler–Bernoulli model:

These assumptions are used to define the EBBT model:

- (I) There is the fixed cross-section on the plane; no deformation in plane
- (II) A neutral plane has all the cross-sections rotate around it; axial displacement is linear against the planar components.
- (III) The neutral plane has the cross-sections orthogonal to it. $\gamma_{yz} = \gamma_{yx} = 0$.

The Timoshenko model:

The third assumption of EBBT is not taken in TBT. The rigidity of the cross-section in-plane is maintained, rotation around neutral plane is allowed, but the orthogonality constraint is removed. Now σ_{xy} and σ_{yz} , the shear deformations are considered.

2.1.2 Plate/Shell:

Two-dimensional methodologies:

A 3D problem is converted into a 2D one to decrease the simulation time and various analytical problems. The following diagrams show the 3D to 2D transformations.

A conceivable order of 2D methodologies, despite the fact that there is a sure level of contention in the writing, can be made as takes after^[3]:

1. Stress resultant-based models;
2. Asymptotic- type approaches;
3. Axiomatic- type approaches.

Classical Models:

Traditional hypotheses were initially produced for structures with single layer and isotropic properties. These hypotheses are separated into 2 principle gatherings^[3]:

- 1) LFAT Love first-approximation theory and
- 2) LSAT : Love second-Approximation theory.

Classical Lamination Theory (CLT):

CLT is formulated from the Kirchhoff speculations: the transverse normals remain same before and after deformation; they are inextensible; they pivot to stay opposite of mid-surface after deformity.

The initial 2 suspicions infer that transverse uprooting is autonomous of the thickness, so σ_{zz} is zero. The transverse shear strains are considered zero by the third suspicion: $\gamma_{xz} = \gamma_{yz} = 0$.

First-order Shear Deformation Theory (FSDT):

The 3rd assumption of Kirchhoff speculations is uprooted; accordingly the mid-surface doesn't have the transverse normal opposite to them after distortion. The inextensibility of the transverse typical remains, The thickness z remains consistent by w removal in this way. But above hypothesis contains definite transverse shear strains γ_{xz} and γ_{yz} .

The FSDT displacement field has 5 variables (CLT has 3 variables); u_0 , v_0 and w_0 : the displacement of mid-surface and ϕ_x and ϕ_y : the torsion of normal around x-axis and y-axis. The torsions and the derivatives $\phi_x = -\partial w_0 / \partial x$ and $\phi_y = -\partial w_0 / \partial y$ coincide in CLT.

2.2 Finite Element Modelling:

The examinations needed to discover the arrangement of non-exclusive investigative or specialized issues all in all make utilization of numerical models which are either discrete or nonstop. Discrete issues include a limited number of segments with a predetermined number of degrees of flexibility (DOFs).

A discretization is regularly presented and the issue is communicated regarding a limited number of discrete variables by including a limited number of DOFs. Along these lines, the arrangement of the persistent issue is approximated by taking care of the discrete issue. The slip because of discretization can be decreased by expanding the quantity of discrete variables. The discretization methodology of constant issues have been widely institutionalized and summed up. A bound together treatment of "standard discrete issues" is exhibited by characterizing the limited component handle as a strategy for estimate to constant issues so that; the continuum is partitioned into a limited number of parts (components), the conduct of which is determined by a limited number of parameters; and the

arrangement of the complete framework, as a gathering of its components, takes after definitely the same guidelines as those relevant to standard discrete issues. The above close estimation strategy is regularly known as the finite element system (FEM). Let us examine the different ventures in FE modelling by utilizing the illustration of a bar.

The arrangement speaks to one of the fruitful purposes of the whole detailing of FE model.

The accompanying fundamental issues will be examined:

1. Pre-processing actualities;
2. Finite Element analysis of structures;
3. Post transforming of acquired results.

Pre-processing and information:

Pre-processing shows every operation which is obliged for situating information essential Finite Element examination. Consideration accounts for information required testing of the Finite Element model; it is isolated into 2 gatherings: average inputs and particular input. Each one of that information that is regularly utilized as a part of most FE codes.

Geometry:

Bar length, L , characterizes structural geometry along pillar hub. When L is altered, discretization of the bar must be done, i.e. one cross section must be made. This cross section is characterized by: kind of pillar component; the aggregate number of bar components, NBE . Those 2 parameters are adequate if the lattices utilized are homogenous; which implies steady shaft length components received. Component parameter type demonstrates quantity of every component. What FE code gets as information is for the most part a situated of two records: the hub area record and the integration document. The first contains the compass directions of

every hub, the second one characterizes the arrangement of hubs component. A merging investigation is typically performed to characterize the cross section size of the issue.

Application of loads and the boundary conditions:

A mixed bag of stacking conditions are utilized as a part of Finite Element investigation, i.e. inertial, thermal and mechanical burdens. A point load is characterised by only a few parameters:

- Point co-ordinates of application, $[x_p, y_p, z_p]$;
- Load size;
- Load bearing.

Nodal level discretisation is done for the boundary conditions; which implies after the compilation of hub is done, all the cross-segment indicates corresponding that hub are obliged. The dislodging segments are secured a clasped point; turns are just permitted in a pivoted point. In the event that fundamental, it is likewise conceivable to compel the particular degree of freedom at that point, like common in Finite Element examination.

Material properties:

The material properties depend, as it is, on the use of a kind of material, like, orthotropic, isotropic or cross-utilize covers, and so forth. These Finite Element examinations done on isotropic material structures; this simplifies material attributes, because of requirement of only E and ν : Young's modulus and Poisson ratio.

Analysis type:

This information characterizes the investigation that must be led. The meaning of the examination will naturally focus the FE lattices that must be processed and the yield documents. On account of straight static investigation the solidness lattice and the stacking power will be processed and thus we acquire the dislodging, strain, and anxiety fields. For the natural frequency examination, mass and stiffness networks are processed to investigate the Eigen vectors. Characteristic frequencies and modular shapes will be given as yield information.

Post processing:

The outcome from post handling speaks to the last venture of a FE investigation. In an uprooting based model, the post preparing information is vector sum of nodal relocation values, for static testing, or Eigen vectors, for modular investigation. The extension of these vectors id required over all of the focuses if post preparing information is required. The development methodology includes shape capacities, $N_i(y)$, along the pillar pivot and extension capacities, $F\tau$, over the cross-segment where $[x_p, y_p, z_p]$ is a non specific purpose of the structure in which the uprooting parts must be calculated.

2.3 Literature review:

- Vibrations of Carbon Nanotubes are of significant significance in various nano-mechanical gadgets^[4], for example, oscillators, charge indicators, tickers, field emanation gadgets and sensors. Likewise, Carbon Nanotube vibrations happen amid certain assembling methods and some non-damaging assessment forms.

- The MWCNT has the modes of vibration autonomous on axial stress^[5]; however the resonance frequency is expanded by axial tensile load and diminished by hub compressive load. The initial axial stress has no effect on intertube resonant frequencies while affecting the natural frequency heavily.
- Vibration of Carbon Nanotubes show highlights of diminishing frequencies with increment in aspect ratio^[6]. Common frequencies of zigzag Carbon Nanotubes are higher contrasted with arm chair Carbon Nanotubes. Be that as it may, contrasts in frequencies decreases with increment in aspect ratio.
- The aspect ratio and number-of-layers assume an imperative part, impacting the fundamental methods of vibration of the Nanotubes^[7]. The impact of van der Waals collaborations is vital in the vibration investigation of Multi Walled Carbon Nanotubes.
- The internal non-coaxial vibration is energized at higher frequency values; accordingly MWCNT can't keep their concentricity at very high frequencies^[8]. So non-coaxial vibrations bend the geometry of MWCNTs, critically modifying some imperative physical properties.
- Results anticipated by Parabolic Shear Deformation Theory are marginally higher for all aspect ratios than Timoshenko Beam Theory and are similar for higher aspect ratios with Euler Beam Theory^[9]. There is no compelling reason to utilize a correction variable for the shear term.

- A fuzzy fiber reinforced composite (FFRC) is proposed which is made out of zigzag SWCNT, carbon strands and a polyimide network. For this, the demonstrating and different perspectives are examined^[10].
 - The load twisting moments of a plate are altogether expanded as the consequence of a practically computed strength^[11]. It likewise affirms nonlinear deformation qualities are essentially impacted by rise of temperature, the in-plane limit condition characters, the displacement thanks to transverse, the plate aspect ratio and also presence of nanotube.
-

3. Materials and Methods

3.1 Formulation of Nano-composite properties:

As a mechanical model of micro scale, the compelling physical properties may be assessed by the mixture principle or principle of Mori-Tanaka. Mori-Tanaka plan correlates the nano-particles and the various properties and reactions of different structures are found by standard of mixture. As from the mixture principle, Shear modulus and Young's modulus can be computed as^[10]:

$$\begin{aligned}
 E_{11} &= \eta_1 V_{CN} E_{11}^{CN} + V_m E^m \\
 \frac{\eta_2}{E_{22}} &= \frac{V_{CN}}{E_{22}^{CN}} + \frac{V_m}{E^m} \\
 \frac{\eta_3}{G_{12}} &= \frac{V_{CN}}{G_{12}^{CN}} + \frac{V_m}{G^m}
 \end{aligned}
 \tag{3.1}$$

where E_{11}^{CN} ; E_{22}^{CN} and G_{12}^{CN} are the Young's modulus of carbon nanotubes and the shear modulus of carbon nanotube respectively, E_m and G_m are the respective properties of the polymer matrix. To account for the different material properties dependent on scaling, the CNT efficiency parameter is included in Eq. (3.1). The volume fractions of CNT and polymer matrix are related by^[10]:

$$V_{CNT} + V_m = 1 \tag{3.2}$$

where V_{CNT} and V_m are the volume fractions of CNT and polymer matrix respectively.

3.2 3D Solid Modelling of structures:

For the practical application of nano-composites, various structures were sought out and finally two shapes were used for the purpose, i.e. a) Airfoil

b) Parabolic Antenna

3.2.1 Airfoil:

For airfoil modelling, first the cross sectional shape is obtained. The cross sectional shape is done by means of a spline whose coordinates were obtained from NACA (National Advisory Committee for Aeronautics) airfoil tool: - NACA 4 digit airfoil generator.

The NACA airfoil series is designated by 4 digits, which gives the following respectively: the maximum camber, the maximum camber position and the foil thickness. For example:

NACA ABCC e.g. *NACA 5623*

- A is ratio of maximum camber to 100. Example: M=5 so the maximum camber is 0.05 or 5% of principle chord.
- B is the ratio of maximum camber position to 10. E.g. P=6 so the maximum camber position is at 0.6 or 60% of the principle chord.
- CC is ratio of the foil thickness to 100. E.g. XX=23; the foil thickness is 0.23 or 23% of the principle chord.

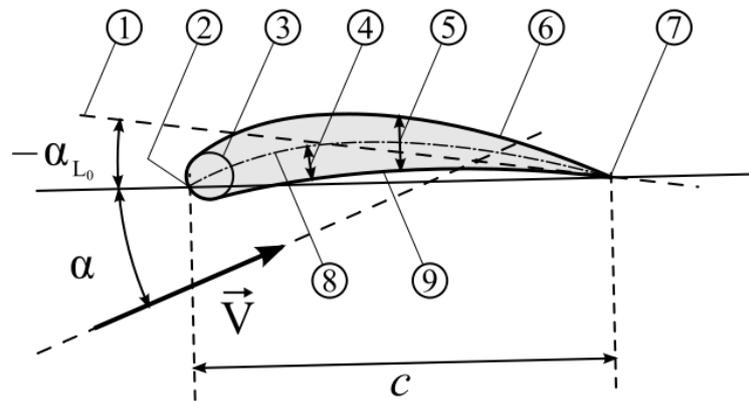


Figure 1: The different airfoil components^[12] – 1: Lift line (zero); 2: Foremost edge; 3: Nose curve (circle); 4: General Camber; 5: Maximum thickness; 6: Up surface; 7: Ending edge; 8: Mean camber line; 9: Down surface

Governing equations:

An airfoil section is plotted from a mean camber line and a thickness function orthogonal to the camber mean-line. The camber line equation is given for both sides of the position of maximum camber (P). The camber gradient line is essential for airfoil plotting. The thickness function is satiated by^[12]:

$$y_t = \frac{T}{0.2} (c_0 x^{0.5} + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4) \quad \dots (3.3)$$

in which: $c_0=0.2969$ $c_1=-0.126$ $c_2=-0.3516$ $c_3=0.2843$ $c_4=-0.1015$ or -0.1036

- The terms c_0 to c_4 are for airfoil with 20% thickness. The term $T/0.2$ normalises the constants.
- At the ending edge (20% airfoil), a chord of thickness $t=0.0021$. Requirement of a closed ending edge is satiated by adjusting the value of c_4 .
- The half thickness value is y_t which is applied on either side of camber mean line.

Using the above equations, it is possible to calculate the camber line position y_c for a given value of x , also the thickness and the camber line gradient. Then lower and upper surface positions can be calculated orthogonal to the camber line^[12]:

$$\theta = a \tan\left(\frac{dy_c}{dx}\right) \quad \dots\dots\dots (3.4)$$

Upper surface: $x_u = x_c - y_t \sin(\theta)$ $y_u = y_c + y_t \cos(\theta)$

Lower surface: $x_l = x_c + y_t \sin(\theta)$ $y_l = y_c - y_t \cos(\theta)$

The airfoil is plotted by taking differently spaced, but controlled x values and computing the y coordinates from the above equation. The points are spaced more about the foremost edge, because of the greater curvature; hence plots contain flat sections around there.

Modelling:

The generated points were then joined with the help of a spline. The generated curve is swept translationally with solid along the z-axis on both sides with equal gradients, so as to form the aerofoil 3-D shape.

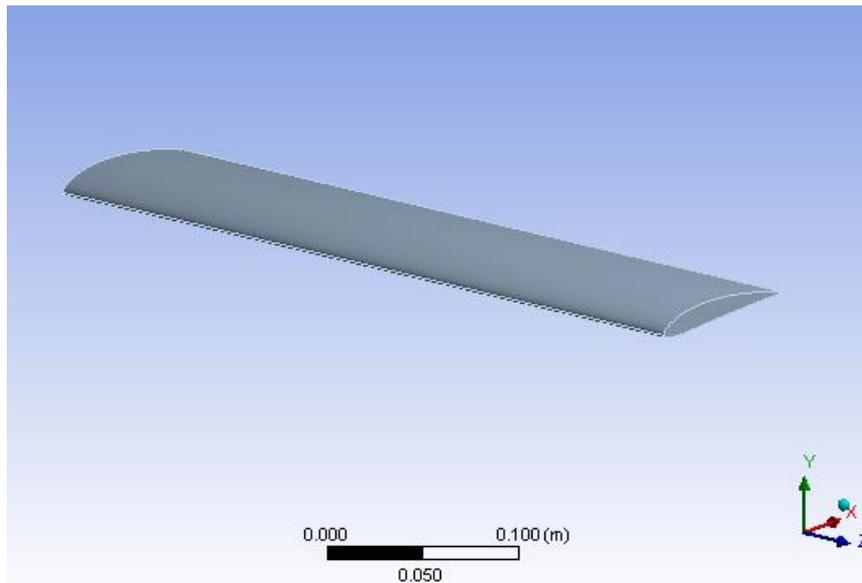


Figure 2: Airfoil 3D model

3.2.2 Parabolic Antenna:

As the name suggests, the antenna is a paraboloid in shape. Hence the first thing to model is the shape of the parabola curve governing the paraboloid. The coordinates of the parabola at some points generated were calculated by using the equations of parabola and giving the required dimensions of the parabola, i.e. depth and diameter.

Parabola is a planar curve whose points are at an equal distance from the focus and the directrix of the curve. It is essentially a conic section. It is created by the intersection of a plane parallel to the tangential of a cone and the cone itself. All the conic sections involve a focus and a directrix, both of which are independent entities.

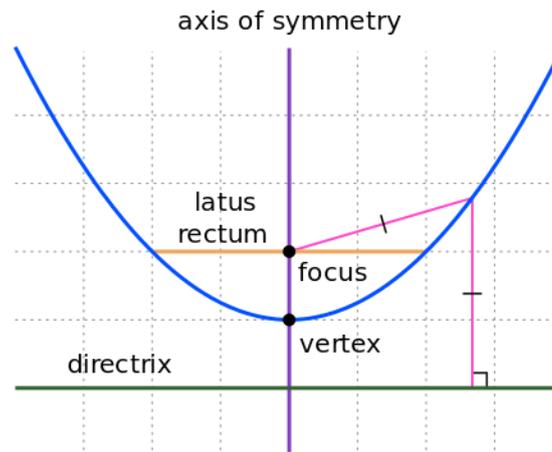


Figure 3: Parabola and its definitions.

Governing equations:

The general equation of a parabola is $y = ax^2 + \beta x + \gamma$; but if parabola is horizontal, the equation is $x = \alpha y^2 + \beta y + \gamma$. The significance is the quadratic variable: horizontal parabolas have the y coordinate squared while the vertical parabolas have the x coordinate squared.

The general equation of a parabola is given by:

$$(ax + by)^2 + cx + dy + e = 0 \quad \dots(3.5)$$

The result is obtained from the general conic equation stated as:

$$Px^2 + Qxy + Ry^2 + Ax + By + C = 0 \quad \dots(3.6)$$

The parabola relation is,

$$Q^2=4PR$$

Modelling:

The points hence generated were joined by means of a spline to form the parabola. Then the parabola was offsetted and filled to create a closed curve. The new curve was now made to be swept rotationally around the Y-axis with solid filling. Hence the antenna model was generated.

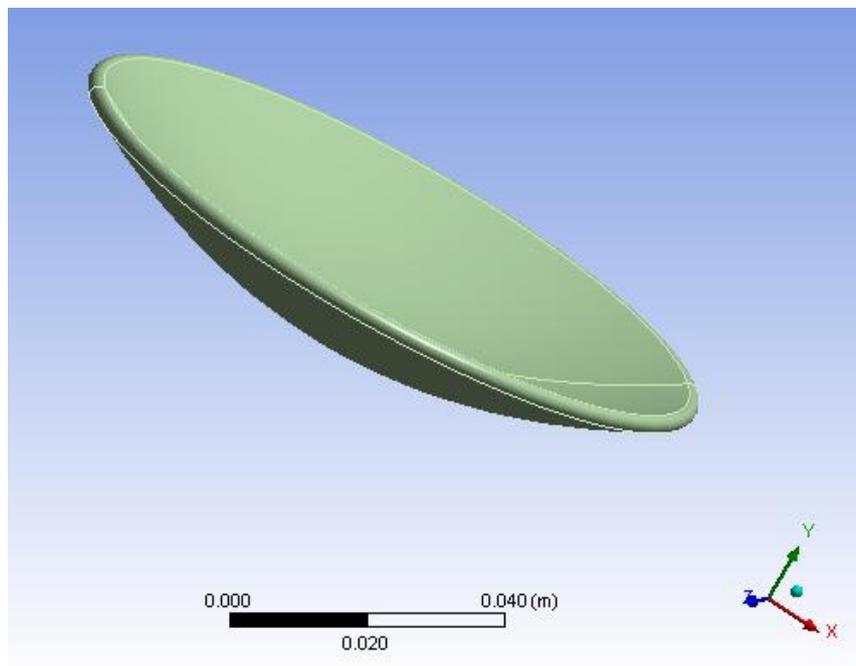


Figure 4: Parabolic antenna 3D model

3.3 Finite element modelling:

3.3.1 ANSYS:

ANSYS is a general software for various analysis. It simulates interactions of many types; including structural, modal, fluid dynamics, thermodynamics and physics. ANSYS helps in testing of prototypes in virtual environments before production by simulating working conditions and various tests. It imports CAD data from various softwares and builds up geometry using pre-processing. The pre-processor also generates finite element models/meshing for analysis. After setting up boundary and working conditions, analysis is done and the post-processing data is obtained as graphs, contours and vectors.

ANSYS Workbench is used for automation and performance. It integrates the simulation techniques and parametric CAD systems. ANSYS Workbench also verifies and improves product and its simulation in a virtual environment. It is more than just an interface with quite high compatibility with PC.

3.3.2 Element type:

After 3D solid modelling of the structure, they are passed through Finite element modelling in Ansys. ANSYS is used to import the 3D solid geometric models from the modelling software for further processing.

Then the meshing on both the structures; i.e. Airfoil and parabolic antenna are done by means of automatic method, which was retained from the default meshing style. Hence it produces hexahedral meshes for the airfoil geometry, while producing tetrahedral meshing for the antenna geometry. Different edge sizings are given as according to the requirement of no of divisions of each edge. The sizings are required to keep the mesh sizes as uniform

as possible, so that it may not be too scarce for convergence nor it should be too much for processing. Thus the 3D models obtained are given below.

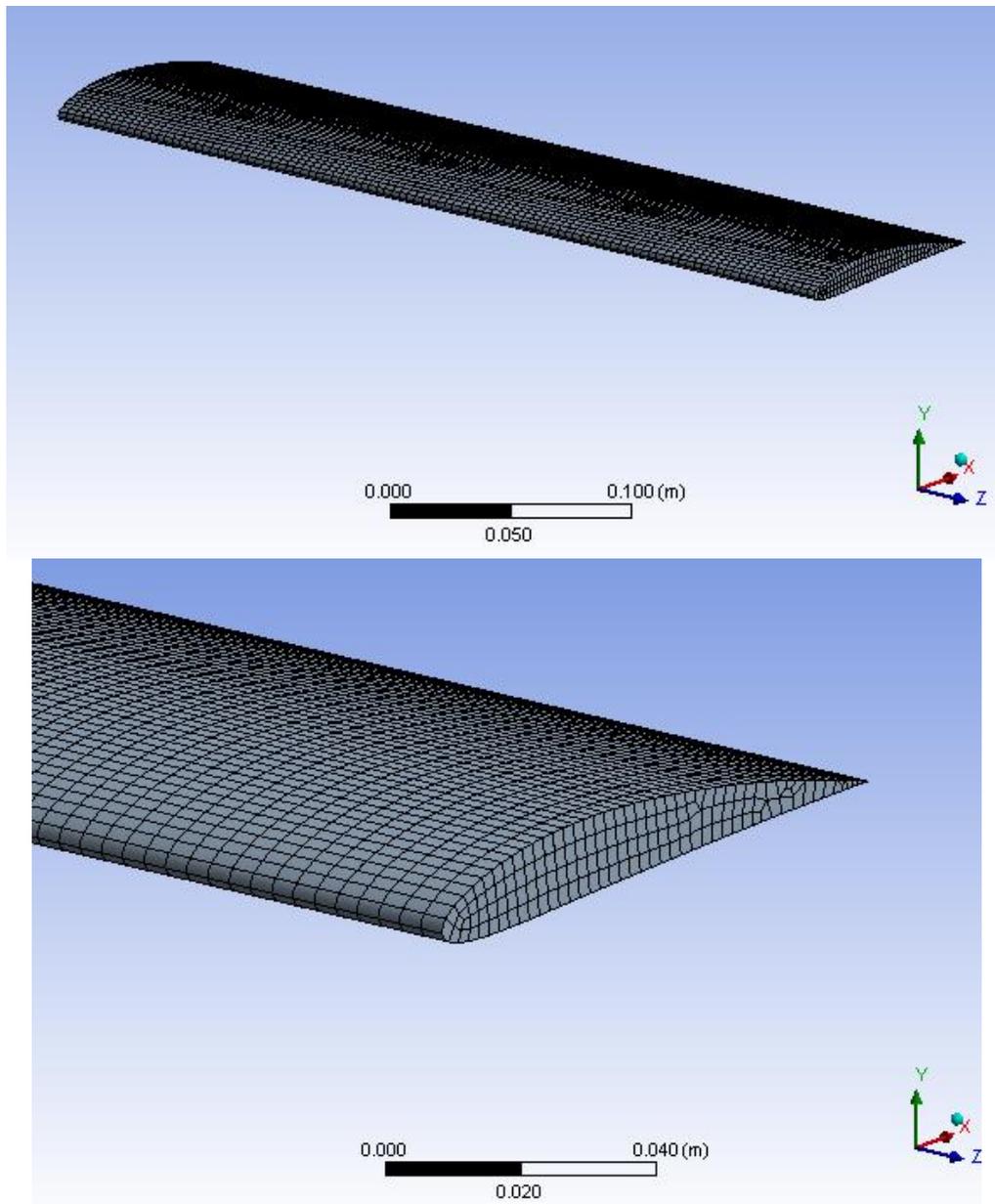


Figure 5(a) and (b): Finite Element model of airfoil

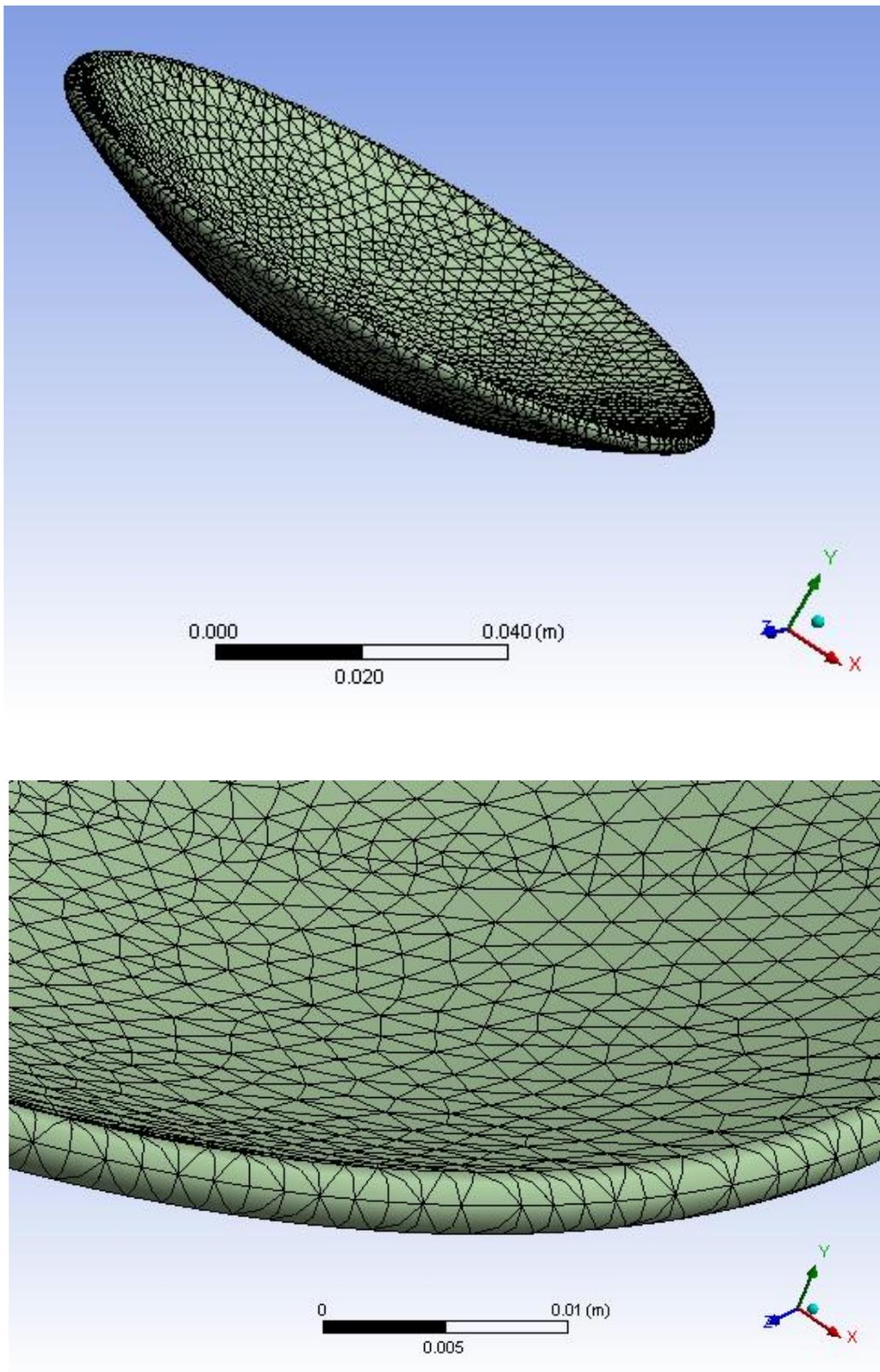


Figure 6 (a) and (b): Finite Element model of parabolic antenna

3.3.3 Boundary conditions:

Before running analysis, first we have to consider the different vibration conditions, namely:

1) Free vibration analysis

2) Forced vibration analysis

The different modelled structures had different boundary conditions set for them. The boundary conditions are discussed below in different categories separately.

1) Free vibration analysis:

In free vibration analysis, there is no external force on the structures. Hence we obtain the natural frequencies in this case. For the airfoil model, one side surface was used as a fixed surface, like it is used in an airplane.

For antenna, the upper surface joining the inner and outer surface was used as a fixed surface as seen in normal antenna mountings supported by various structures.

2) Forced vibration analysis:

In forced vibration analysis, there is an external force on the structures. Here we have taken consideration for uniformly distributed load instead of point loads for simple and uniform analysis. The different air pressures at nominal working heights are used. Also the respective surfaces are taken as fixed supports as in the free vibration analysis.

While the airfoil have an uniformly distributed force on the upper surface resembling the force applied by air pressure while being used as an airplane wing, the antenna has the inner parabolic surface applied with an uniformly distributed force which comprises of the air pressure and the nominal load applied by various equipment.

3.3.4 No of elements and nodes:

The finite element modelled structures of airfoil and parabolic antenna are first statistically checked for the mesh properties. The different number of nodes and elements are enlisted in the following tables.

Table 1: Convergence analysis of airfoil

Structure	Number of elements	Number of nodes	1st mode Frequency (Hz)	Element type
Airfoil	13800	69103	40.179	Hexahedral
	1750	10398	40.179	
	340	2072	40.195	
	130	836	40.359	
	40	241	40.417	

Table 2: Convergence analysis of parabolic antenna

Structure	Number of elements	Number of nodes	1st mode Frequency (Hz)	Element type
Parabolic Antenna	15189	28269	7714.4	Tetrahedral
	10823	16781	7714.9	
	2672	5551	7733.1	
	1038	471	8311.2	

Table 3: Element statistics for variations of airfoil

Structure	Number of elements	Number of nodes	Element type
Airfoil shell	11400	70706	Tetrahedral
Airfoil shell with ribs	12854	24933	Tetrahedral

4. Results and Discussion

Problem Definition:

Our objectives for this report include Finite Element modelling different structures under free vibration analysis of structure to find the different modal natural frequencies and under forced vibration analysis of structures to find the Equivalent stress and maximum displacement. The analysis is done for Carbon Nanotube reinforced Composites with varying percentage of carbon nanotube content.

ANSYS Analysis:

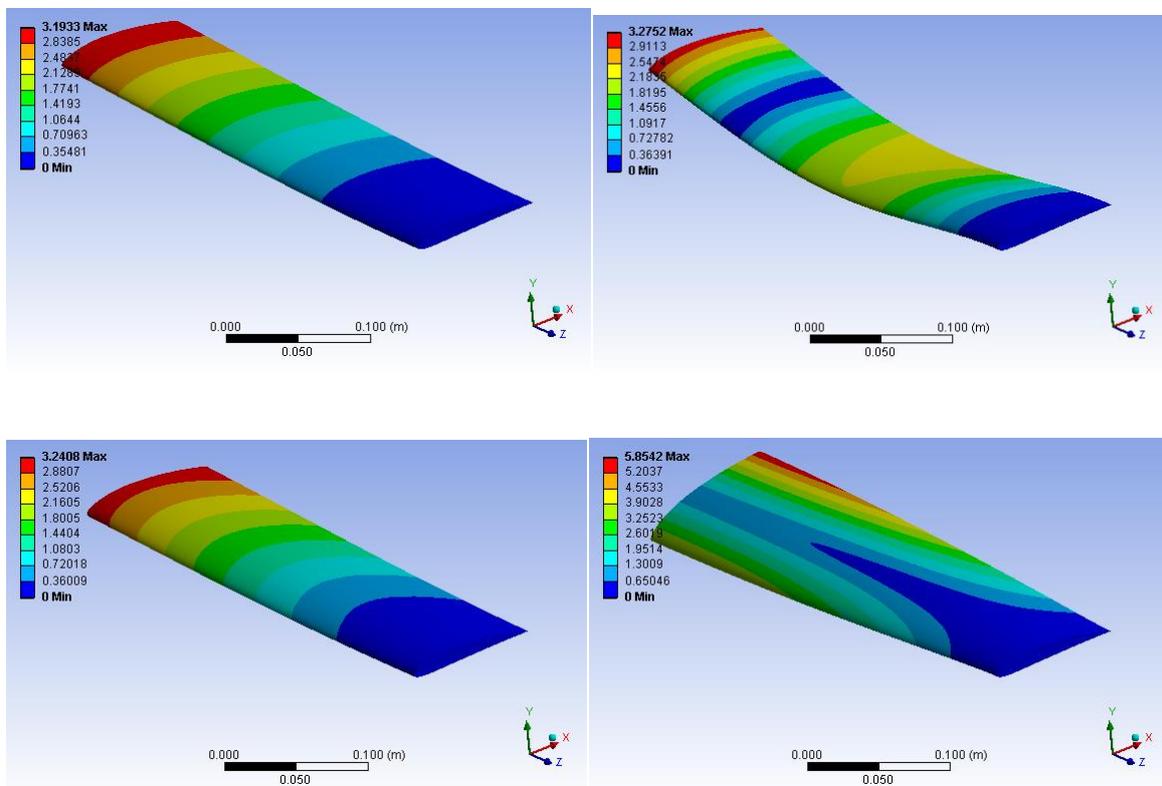
4.1 Free vibration analysis:

Table 4: Modal frequencies of structures at different percentage of Carbon Nanotube content

Percentage of CNT (V_{CNT})	5% ($V_{CNT}=0.05$)		10% ($V_{CNT}=0.10$)		15% ($V_{CNT}=0.15$)	
	Mode	Frequency (Hz)	Mode	Frequency (Hz)	Mode	Frequency (Hz)
Airfoil Structure	1	40.179	1	47.339	1	53.461
	2	249.74	2	294.23	2	332.28
	3	304.15	3	358.33	3	404.66
	4	401.74	4	472.82	4	533.7
	5	690.17	5	813.1	5	918.2
	6	1198	6	1410	6	1591.6

Parabolic Antenna Structure	Mode	Frequency (Hz)	Mode	Frequency (Hz)	Mode	Frequency (Hz)
	1	7714.4	1	9088.9	1	10264
	2	7715.2	2	9090	2	10265
	3	8467.6	3	9980.5	3	11273
	4	8714.8	4	10268	4	11596
	5	8716.7	5	10271	5	11599
	6	9951.2	6	11730	6	13250

4.1.1 Airfoil:



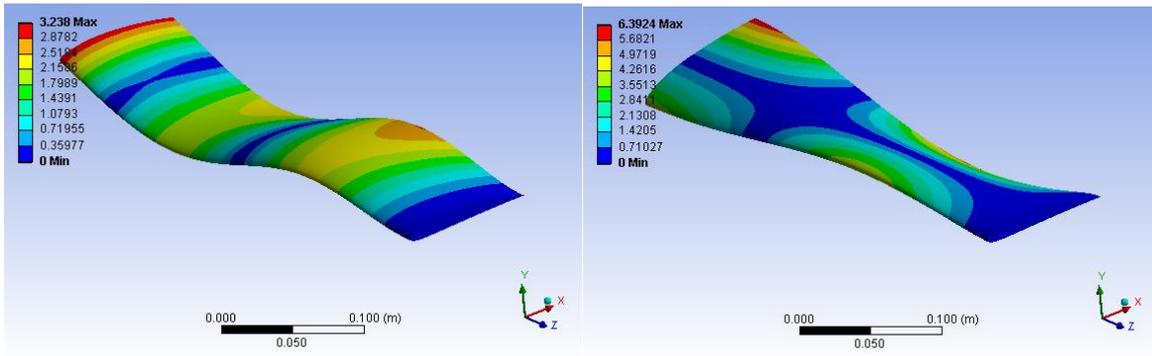
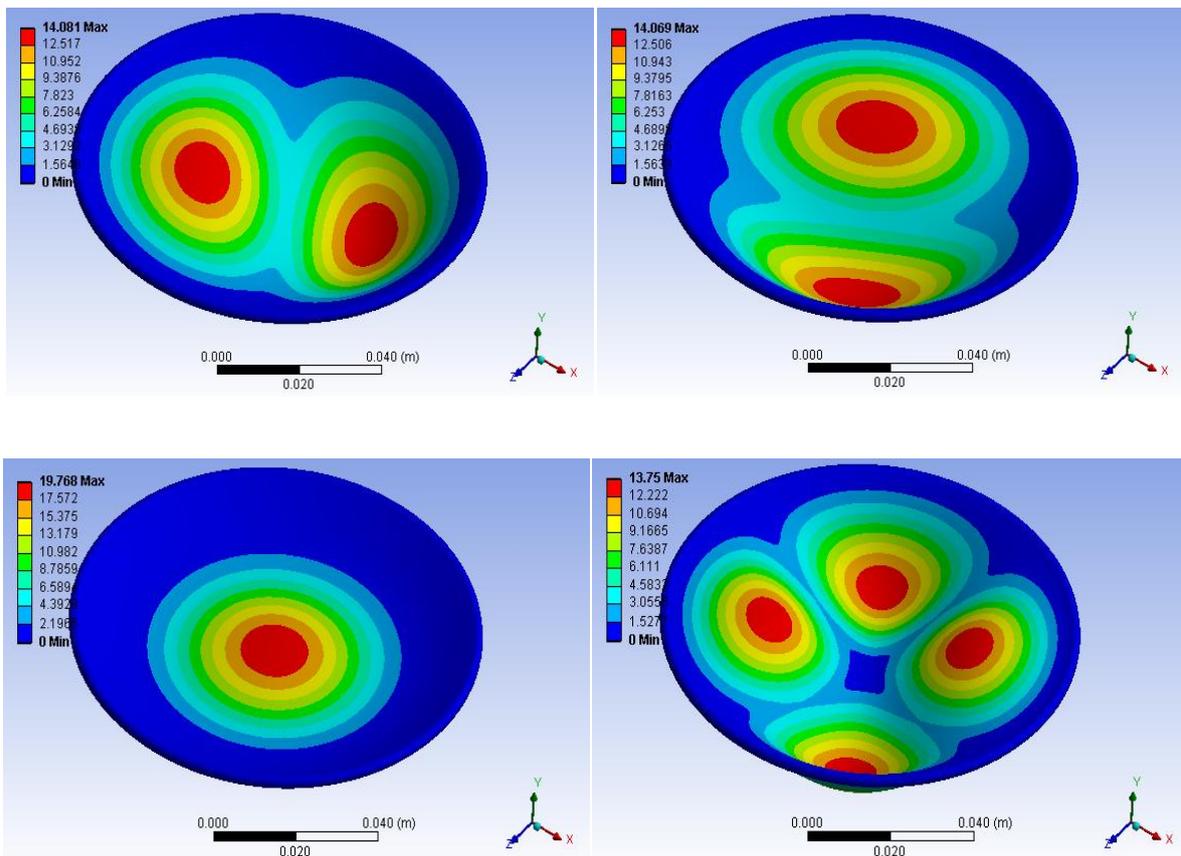


Figure 7 (a), (b), (c), (d), (e) and (f): Different mode shapes of an airfoil under free vibration.

The figures are the mode shapes from mode, $m=1$ to $m=6$ respectively.

4.1.2 Parabolic Antenna:



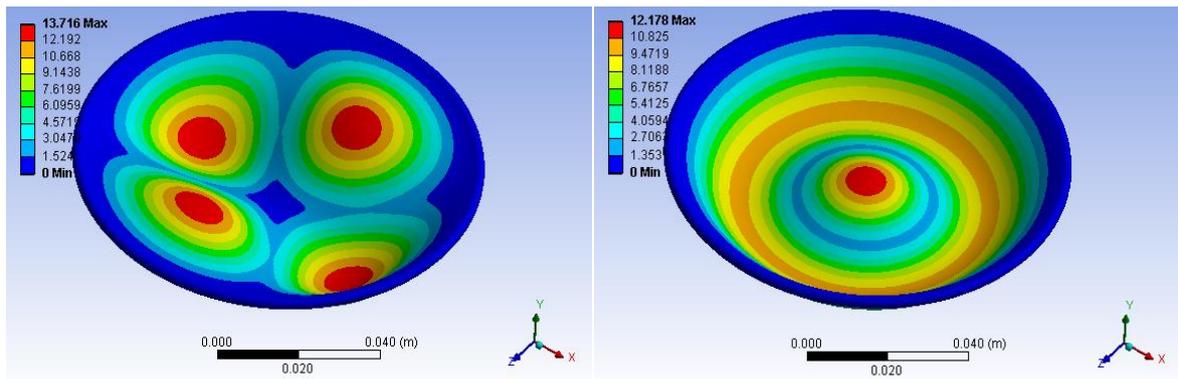


Figure 8 (a), (b), (c), (d), (e) and (f): Different mode shapes of an parabolic antenna under free vibration. The figures are the mode shapes from mode, $m=1$ to $m=6$ respectively.

4.2 Forced vibration analysis:

The maximum deformation and equivalent stress was obtained for different percentages of CNT in nano-composite from ANSYS analysis.

Table 5: Stresses and deformation of airfoil at different conditions

Percentage of CNT (V_{CNT})	Max. Deformation (m)	Equivalent stress (Pa)
5 ($V_{CNT}=0.05$)	0.05016	1.4307×10^8
10 ($V_{CNT}=0.10$)	0.034768	1.43×10^8
15 ($V_{CNT}=0.15$)	0.02335	1.4297×10^8

Table 6: Stresses and deformation of parabolic antenna at different conditions

Percentage of CNT (V_{CNT})	Max. Deformation (m)	Equivalent stress (Pa)
5 ($V_{CNT}=0.05$)	1.10×10^{-5}	5.25×10^6
10 ($V_{CNT}=0.10$)	7.64×10^{-6}	5.24×10^6
15 ($V_{CNT}=0.15$)	5.77×10^{-6}	5.23×10^6

4.2.1 Airfoil:

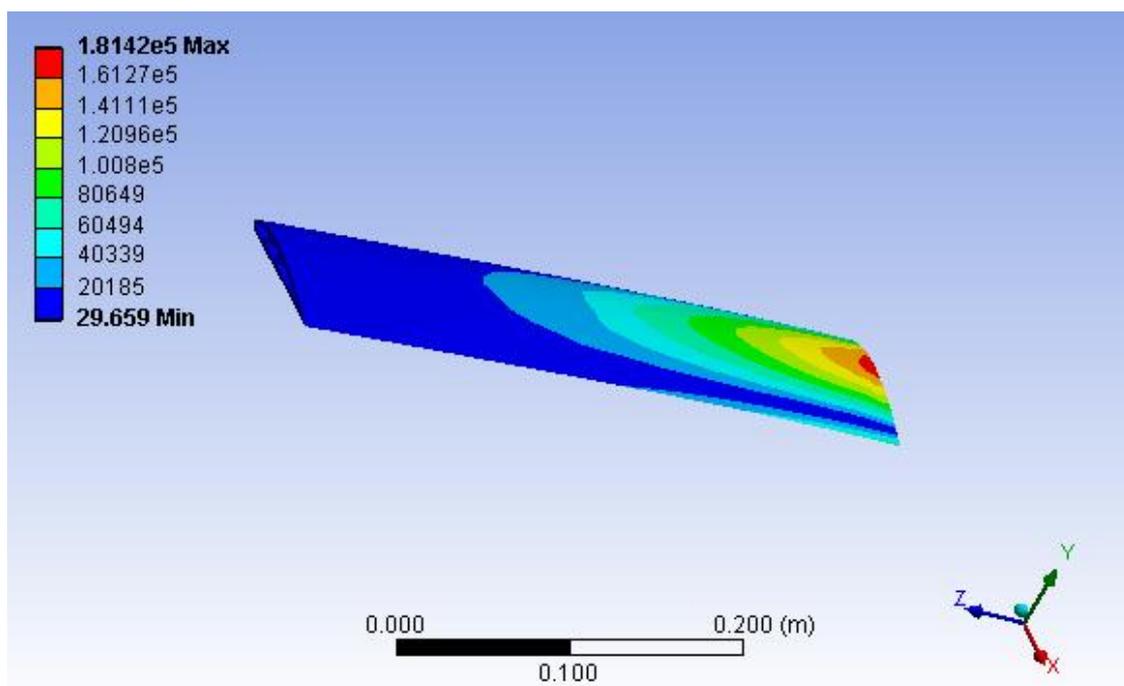


Figure 9 (a): Stress profile of an airfoil under forced vibration.

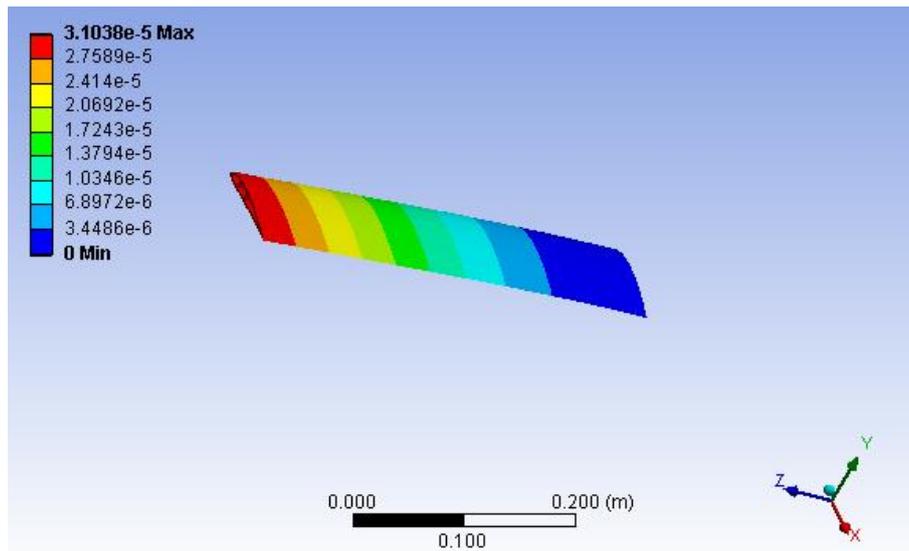


Figure 9 (b): Displacement profile of an airfoil under forced vibration.

4.2.2 Parabolic Antenna:

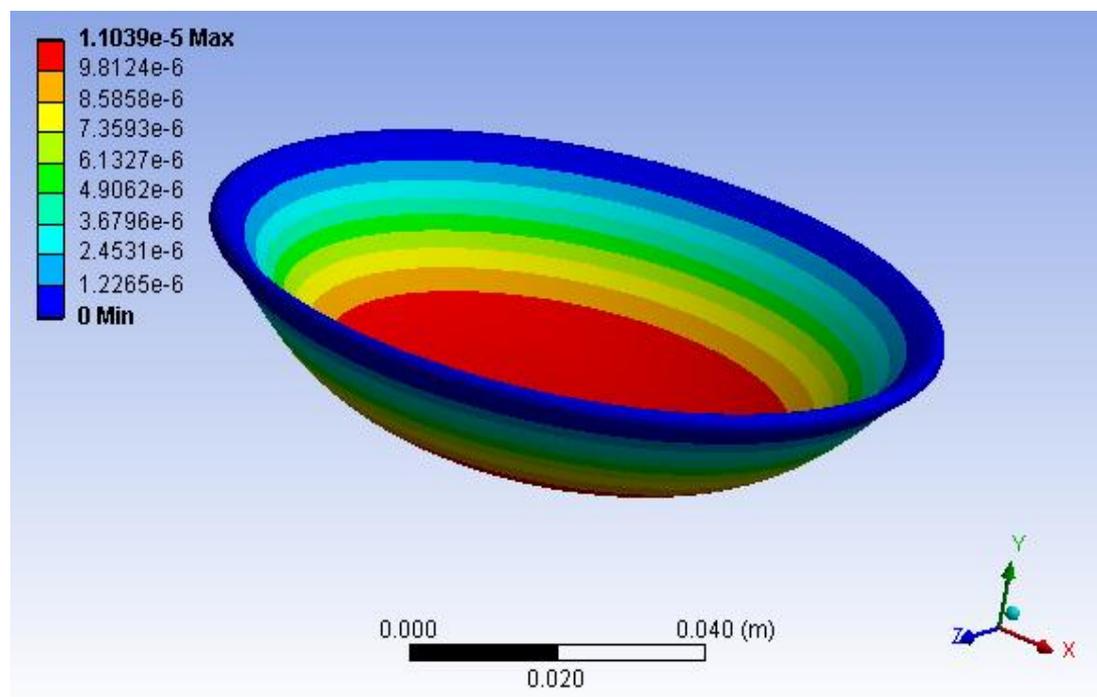


Figure 10 (a): Stress profile of parabolic antenna under forced vibration.

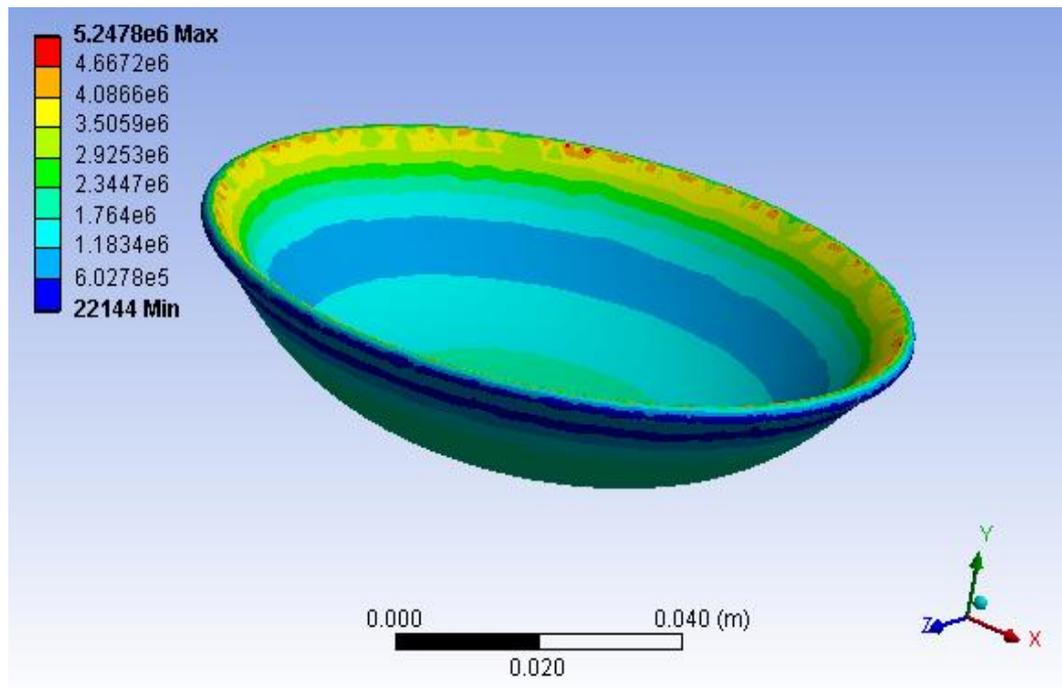


Figure 10 (b): Displacement profile of parabolic antenna under forced vibration.

4.3 Analysis of different variations of airfoil:

4.3.1 Free vibration analysis:

The natural frequencies of different variations of airfoil structures is enlisted in tabular form.

The variations include a model with shell structure of an airfoil and another one with the shell structure of airfoil containing ribs.

Table 7: Modal frequencies of varied airfoil structures at different percentage of CNT content

Percentage of CNT (V_{CNT})	5% ($V_{CNT}=0.05$)		10% ($V_{CNT}=0.10$)		15% ($V_{CNT}=0.15$)	
	Mode	Frequency (Hz)	Mode	Frequency (Hz)	Mode	Frequency (Hz)
Airfoil Shell	1	48.305	1	56.911	1	64.27
	2	272.23	2	320.7	2	362.15
	3	340.45	3	401.08	3	452.93
	4	408.73	4	481.08	4	543.05
	5	647.82	5	763.16	5	861.8
	6	1018.2	6	1199.5	6	1354.6
Airfoil shell with ribs	1	47.705	1	56.206	1	63.474
	2	286.2	2	337.18	2	380.76
	3	333.25	3	392.6	3	443.35
	4	415.54	4	489.07	4	552.05
	5	771.12	5	908.36	5	1025.7
	6	1199.2	6	1411.5	6	1593.4

4.3.2 Forced vibration analysis:

Table 8: Stresses and deformation of airfoil shell at different conditions

Percentage of CNT (V_{CNT})	Max. Deformation (m)	Equivalent stress (Pa)
5 ($V_{CNT}=0.05$)	0.062201	1.99×10^8
10 ($V_{CNT}=0.10$)	0.04312	1.99×10^8
15 ($V_{CNT}=0.15$)	0.032581	1.99×10^8

Table 9: Stresses and deformation of airfoil shell with ribs at different conditions

Percentage of CNT (V_{CNT})	Max. Deformation (m)	Equivalent stress (Pa)
5 ($V_{CNT}=0.05$)	5.93×10^{-5}	1.82×10^5
10 ($V_{CNT}=0.10$)	4.11×10^{-5}	1.81×10^5
15 ($V_{CNT}=0.15$)	3.10×10^{-5}	1.81×10^5

4.4 Effect of CNT on various response of airfoil and antenna:

Various nano-composites were used (having different percentage of Carbon Nanotubes) for the material for the structure under two conditions; i.e. free vibration and forced vibration. The different mode shapes were plotted under the different conditions and the natural frequencies and vibration frequencies are found out.

Tables were made to compare the frequencies of the different nano composites to draw conclusion regarding the percentage of CNT present in the nano-composite.

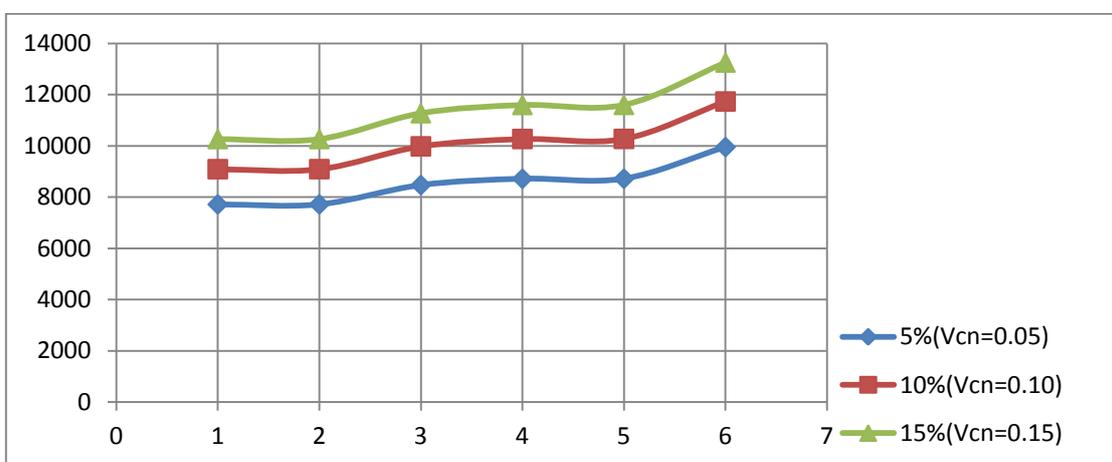
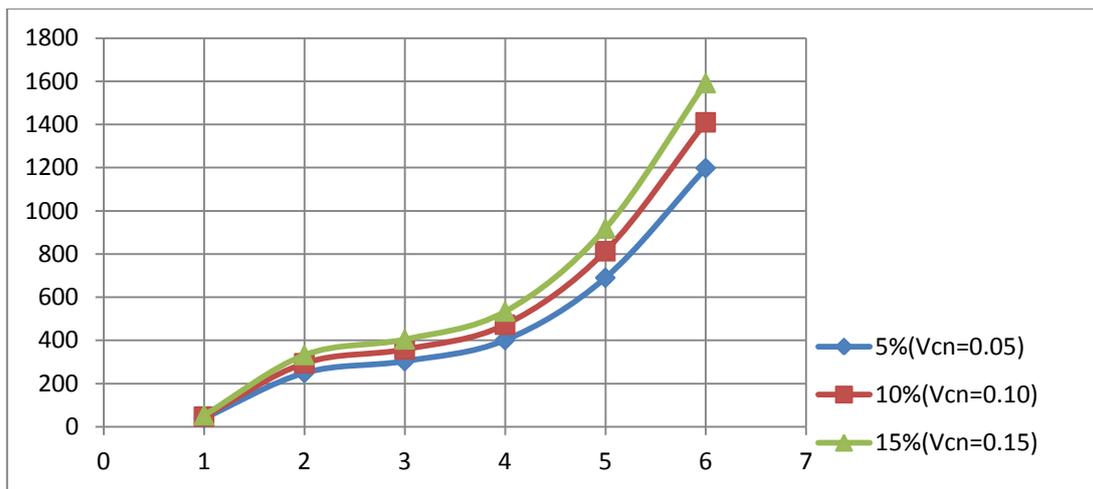


Figure 11 (a) and (b): Variation of frequency with respect to modes and percentage of CNT of airfoil and parabolic antenna respectively in free vibration (Table 4).

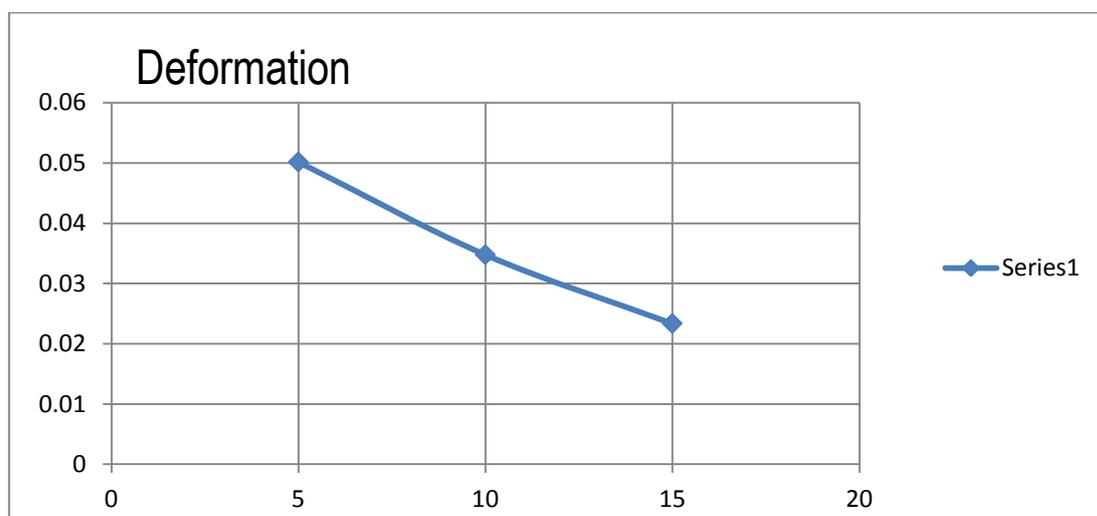


Figure 12 (a): Variation of max. deformation of airfoil with percentage of CNT (Table 5).

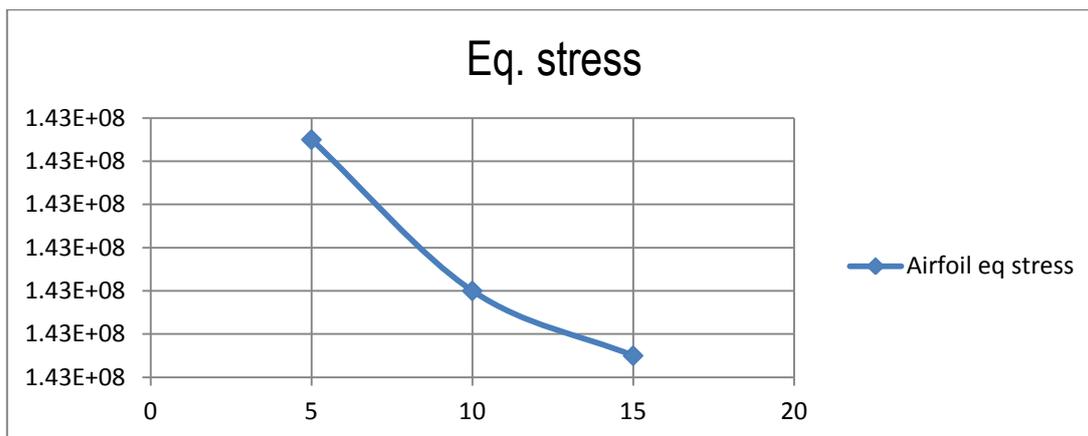


Figure 12 (b): Variation of equivalent stress of airfoil with percentage of CNT (Table 5).

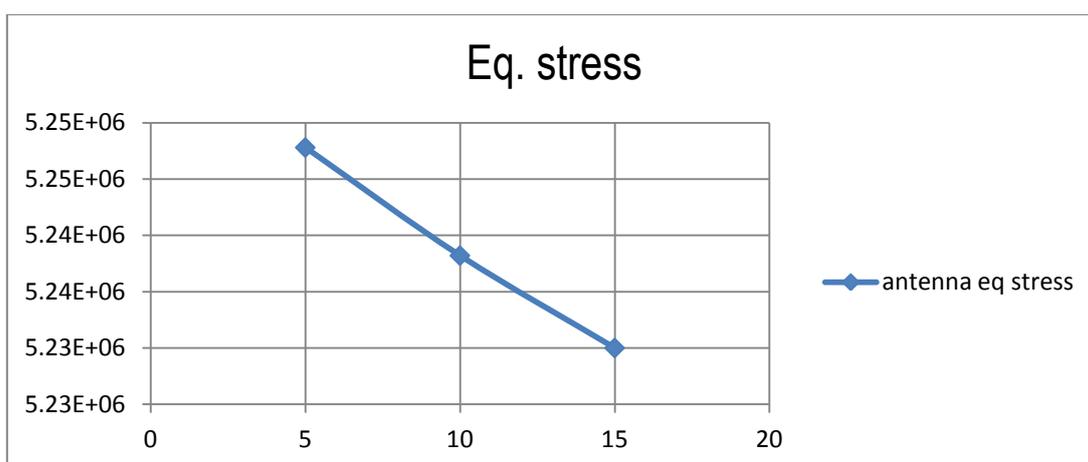
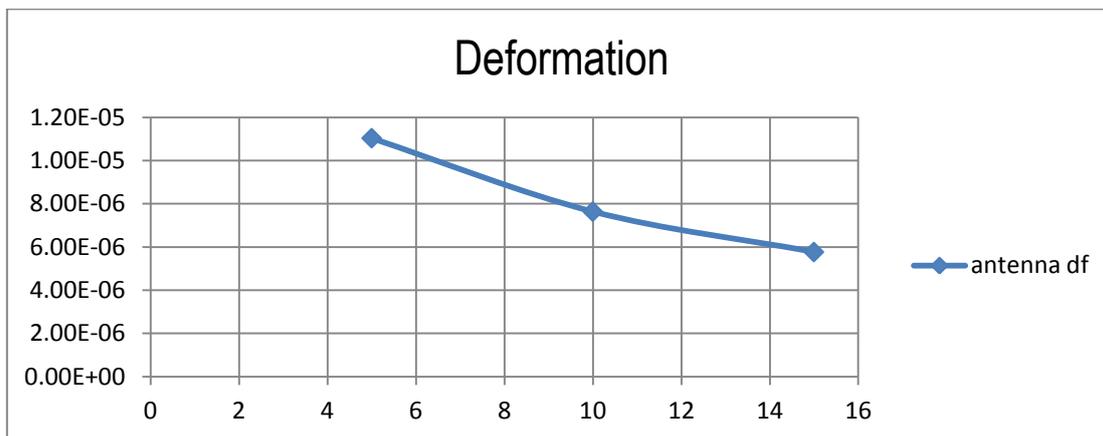


Figure 13 (a) and (b): Variation of max. deformation and equivalent stress with respect to percentage of CNT (Table 6).

Conclusion:

From the free vibration analysis of airfoil and parabolic antenna structure, we observe natural vibration frequencies increase appreciably with the increase in percent Carbon Nanotube content present in the nano-composite structures. Hence the stiffness of the structures increases. From the forced vibration analysis of the same structures, the max equivalent stress and the maximum displacement is found to decrease with the increase in percentage Carbon Nanotube content. The free and forced vibration analysis of the airfoil shell and airfoil shell with ribs structures also give similar results.

Scope of future work:

- i. While in this work, the percentage of carbon Nanotubes is varies in the structure, there was no study on the orientation of the Nanotubes in the composites. There can be further study on the orientation on the different orientations of Nanotubes; i.e. aligned, random and so on the various structure.
- ii. In this work, the study was based on free vibration and a single type of distributed load. In further works, complex loading patterns and their effects can be studied
- iii. Furthermore, any more complex models or any assembled products can be put under similar observations.

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