Free vibration of isotropic rectangular beam based on orthogonal finite element method

A Thesis
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in

Mathematics

by

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Declaration

I, the undersigned, declare that the work contained in this thesis entitled **Free vibration of isotropic rectangular beam based on orthogonal finite element method** , in partial fulfillment of the requirement for the award of the degree of Master of Science, submitted in the Department of Mathematics, National Institute of Technology Rourkela, is entirely my own work and has not previously in its entirety or part been submitted at any university for a degree, and that all the sources I have used or quoted have been indicated and appropriately acknowledged by complete references.

Saurav Naik

May 11, 2015

This is to certify that the above statement made by the candidate is correct to the best of my knowledge.

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Abstract

Present work deals with free vibration of isotropic rectangular beam subject to various sets of boundary conditions. Governing differential equation has been solved by finite element method where the shape function has been taken as orthogonal polynomials generated from simple algebraic polynomials. Orthogonal polynomials are obtained first by means of Gram-Schmidt orthogonalization procedure. The generalized eigenvalue problem for free vibration is obtained after finding the concerned stiffness and mass matrices from orthogonal finite element method. We have also considered various types of discretizations in the beam element for the simulation. Results for eigenfrequencies are incorporated after checking the test of convergence and comparison of present results with available literature in special cases.

Keywords: Isotropic beam, vibration, finite element method, generalized eigenvalue problem .

Contents

Chapter 1

Introduction

A beam is defined as a structural member designed primarily to support forces acting perpendicular to the axis of the member. In the analysis of continuous beams basically axial deformation is negligible (small deflection theory); hence the transverse deflections and the rotations about the axis are considered at the nodes of the member.

1.1 Isotropic beam

Beam may be classified based on the type of support

1. Simply Supported Beams (support at both end)

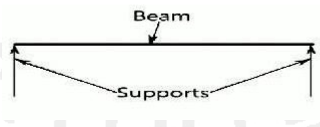

Figure 1.1: Simply supported beams

2. Cantilever Beam (Clamped at one end and other end is free)

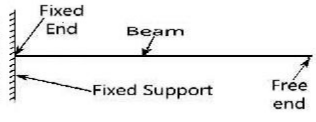

Figure 1.2: Cantilever beam

3. Continuous Beam (supported at more than two points)

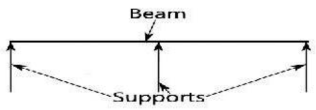

Figure 1.3: Continuous beam

Also classification of beam based on observation propose may be mention as

1. Euler–Bernoulli beam: Only translation mass and bending stiffness are considered.

2. Rayleigh Beam: Here the effect of rotary inertia is taken in to consideration .

3. Timoshenko beam: In this case both the rotary inertia and transverse shear deformation are assumed.

In this study we have considered the Euler–Bernoulli beam.

Assumptions of Euler–Bernoulli beam may be written as :

- The cross-sectional plane perpendicular to the axis of the beam remains plane after deformation (assumption of a rigid cross–sectional plane).

- The deformed cross-sectional plane is still perpendicular to the axis after deformation.

- The classical theory of beam neglects transverse shearing deformation where the transverse shear stress is determined by the equations of equilibrium.

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1.2 Finite element method

Three most well–known computational techniques are Finite Difference Method (FDM), Finite Element Method (FEM) and Boundary Element Method (BEM). The FEM is a numerical procedure for solving a differential or integral equation by discretizing the specified domain into finite elements (Bhavikatti, 2005; Reddy, 2005). By this technique, the domain of the beam is to be divided into specific number of elements and compute stiffness and mass matrices for each element. Consequently, these matrices are to be assembled to generate the global stiffness and inertia matrices. Here shape function has been taken as orthogonal polynomials derived from simple algebraic polynomials. The orthogonal polynomials are acquired by the Gram–Schmidt orthogonalization process. It may be noted that both potential and kinetic energy of the beam are dependent on the shape function.

1.3 Literature review

The vibration characteristics of isotropic beam have been studied earlier by using analytical as well as computational methods. As such, Bhat (1986) has predicted vibration frequencies of rotating cantilever beam using characteristic orthogonal polynomials in Rayleigh–Ritz method. Natural frequencies of an Euler–Bernoulli beam with a mass are calculated by Öz (2000). A new beam element has bee developed by Chakraborty et al. (2003) to study the thermoelastic behavior of functionally graded beam structures using first order shear deformation theory. Vibration of plates using orthogonal polynomials in Rayleigh-Ritz Method can be briefly found in (Chakraverty, 2009). Alshorbagy et al. (2011) have used finite element method to the dynamic characteristics of functionally graded beam. Recently, the method of differential quadrature is employed by Nassar et al. (2013) to free vibration of a cracked cantilever beam resting on Winkler–Pasternak foundations.

1.4 Gaps

To the best of the authors' knowledge, free vibration of isotropic beam using finite element method with orthogonal sets of polynomials has not been done yet. As such, we have considered the orthogonal set of polynomials rather than taking only simple algebraic polynomials by means of Gram–Schmidt orthogonalization process. The natural frequencies are computed based on two different types of discretizations viz. homogeneous and non–homogeneous. The natural frequencies of isotropic beam have been evaluated for various sets of boundary conditions.

1.5 Objectives

The major objectives of the present investigation may be given as below:

1. Finite element formulation

2. Gram–Schmidt orthogonalization process

3. Discretizations

4. Implementation of boundary conditions

5. Computation of results

Chapter 2

Numerical modeling

2.1 Finite element formulation

In FEM, the transverse displacement component for a beam may be assumed as

$$w(x,t) = a_1 + a_2 x + a_3 x^2 + a_4 x^3 \tag{2.1}$$

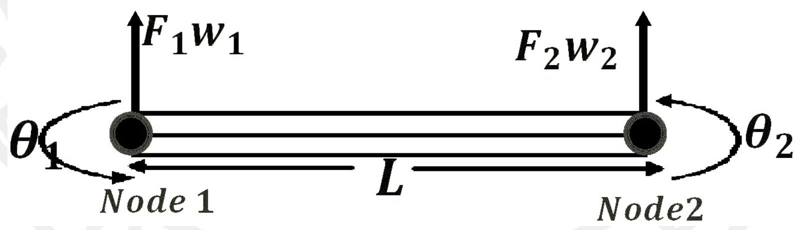


Figure 2.1: Beam element with two nodes

Accordingly, the rotational deflection at each node of the beam may be written as

$$\theta(x,t) = \frac{dw}{dx} = a_2 + 2a_3 x + 3a_4 x^2 \tag{2.2}$$

subject to $w(0) = w_1$ and $\theta_1 = \frac{dw}{dx}]_{x=0}$; $w(L) = w_2$ and $\theta_2 = \frac{dw}{dx}]_{x=L}$

In matrix form, we have

$$\begin{bmatrix} w_1 \\ \theta_1 \\ w_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & L & L^2 & L^3 \\ 0 & 1 & 2L & 3L^2 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} \tag{2.3}$$

From Eq.(2.3), one may find

$$\begin{aligned} a_1 &= w_1 \\ a_2 &= \theta_2 \\ a_3 &= \frac{1}{L^2}(-3w_1 - 2L\theta_1 + 3w_2 - L\theta_2) \\ a_4 &= \frac{1}{L^3}(2w_1 + L\theta_1 - 2w_2 - L\theta_2) \end{aligned}$$

Substituting the values of a_1, a_2, a_3 and a_4 in Eq. (2.1), we have

$$w(x) = \left(1 - \frac{3x^2}{L^2} + \frac{2x^3}{L^3}\right) w_1 + \left(x - \frac{2x^2}{L} + \frac{x^3}{L^2}\right) \theta_1 + \left(\frac{3x^2}{L^2} - \frac{2x^3}{L^3}\right) v_2 + \left(-\frac{x^2}{L} + \frac{x^3}{L^2}\right) \theta_2 \tag{2.4}$$

It can also be written as

$$w(x) = [N] \begin{Bmatrix} w_1 \\ \theta_1 \\ w_2 \\ \theta_2 \end{Bmatrix} \tag{2.5}$$

with

$$[N] = [N_1 \;\; N_2 \;\; N_3 \;\; N_4] \tag{2.6}$$

where N_1, N_2, N_3 and N_4 are the components of the shape functions and may be found as

$$N_1 = 1 - 3\xi^2 + 2\xi^3,$$
$$N_2 = L(\xi - 2\xi^2 + \xi^3),$$
$$N_3 = 3\xi^2 - 2\xi^3,$$
$$N_4 = L(\xi^3 - \xi^2)\ ;\ \text{where } \xi = \frac{x}{L}$$

It can be easily checked that N_1 is equal to 1 when ξ is equal to 0 and N_3 is equal to 1 at ξ is equal to 1. If we plot these shape functions by normalizing N_2 and N_4 with respect to L; that is, the equation corresponding to N_2 is divided on either side with L. Then, it becomes a function of ξ alone. So, we can plot how these shape functions look like, when ξ goes from 0 to 1.

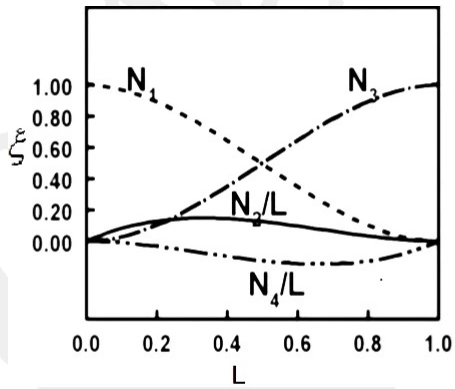


Figure 2.2: plot of shape functions (Bhavikatti, 2005)

Now we can find connectivity matrix ($[B]$) by taking the second derivatives for components of $[N]$ matrix as

$$[B] = [B_1\ \ B_2\ \ B_3\ \ B_4] \tag{2.7}$$

where

$$B_1 = 12\xi - 6,$$
$$B_2 = L(6\xi - 4),$$
$$B_3 = 6 - 12\xi,$$
$$B_4 = L(6\xi - 2);\ \text{where } \xi = \frac{x}{L}$$

Stiffness Matrix

The potential energy V of a mechanical system during small displacements from an equilibrium position may be written as

$$V = \frac{1}{2}q^T[K]q$$

where $[K]$ is stiffness matrix and q is the vector whose components are the generalized components of the system with respect to time and q^T is the transpose of q.
We may have the element stiffness matrix as.

$$\begin{aligned}[K_e] &= \int_w EI[B]^T[B]dw \\ &= \int_0^L EI[B]^T[B]dx \\ \text{Taking } \xi &= \frac{x}{L} \Longrightarrow dx = Ld\xi \\ \Longrightarrow [K_e] &= EIL\int_0^1 [B]^T[B]d\xi\end{aligned}$$

Then the element stiffness matrix becomes

$$[K_e] = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^4 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \tag{2.8}$$

Mass Matrix

In analytical mechanics, the mass matrix is a symmetric matrix $[M]$ that expresses the connection between the time derivative $\dot{q}$ of the generalized coordinate vector q of a system and the kinetic energy T , by the equation

$$T = \frac{1}{2}\dot{q}^T[M]\dot{q}$$

where $\dot{q}^T$ denote the transpose of the vector $\dot{q}$ The element mass matrix may be written as

$$\begin{aligned}[M_e] &= \int_w \rho A[N]^T[N]dw \\ &= \int_0^L \rho A[N]^T[N]dx \\ \text{Taking } \xi &= \frac{x}{L} \implies dx = ad\xi \\ \implies [M_e] &= \rho AL \int_0^1 [N]^T[N]d\xi\end{aligned}$$

Then the element mass matrix becomes

$$[M_e] = \frac{\rho AL}{420}\begin{bmatrix} 156 & 22L & 54 & -13L \\ 22L & 4L^2 & 13L & -3L^2 \\ 54 & -13L & 156 & -22L \\ -13L & -3L^2 & -22L & 4L^2 \end{bmatrix} \tag{2.9}$$

In present study, the length L plays major role in finding $[K_e]$ and $[M_e]$. Consequently, the discretizations of beam element has been performed in two different ways viz. **homogeneous** and **non–homogeneous**. The **Gauss–Chebyshev–Lobatto** points $x_i = \frac{1}{2}\left[1 - cos\left(\frac{i-1}{n-1}\right)\pi\right]; \quad i = 1, 2, 3, ..., n$ are assumed here for **non–homogeneous** discretization.

2.2 Gram–Schmidt orthogonalization process

If $P(x)$ and $Q(x)$ are two functions in $\mathbb{R}^2$, then polynomials P and Q are said to be orthogonal if the inner product $\langle P, Q\rangle = 0$, where $\langle P, Q\rangle = \int_0^1 P(x)Q(x)\mathrm{d}x \; ; \; x\epsilon[0, 1]$. By using **Gram–Schmidt procedure**, we may now find the orthogonal basis for $f_1 = 1$, $f_2 = x$, $f_3 = x^2$, $f_4 = x^3$. If ϕ_i's are the orthogonal polynomials, then we may get

$$\phi_1 = 1,$$
$$\phi_2 = x - \frac{1}{2},$$
$$\phi_3 = x^2 - x + \frac{1}{6},$$
$$\phi_4 = x^3 - \frac{3}{2}x^2 + \frac{3}{5}x - \frac{1}{20}.$$

Now taking the shape functions N_1, N_2, N_3 and N_4 as ϕ_1, ϕ_2, ϕ_3 and ϕ_4 respectively, then Eq. (2.6) become

$$[N] = \left[1 \quad \left(x - \frac{1}{2}\right) \quad \left(x^2 - x + \frac{1}{6}\right) \quad \left(x^3 - \frac{3}{2}x^2 + \frac{3}{5}x - \frac{1}{20}\right) \right] \tag{2.10}$$

By assuming this shape function (Eq. (2.10)), the same stiffness and mass matrices may be obtained. So the expressions of these matrices may be similar as yielded for simple algebraic polynomials.

2.3 Implementation of boundary conditions

After finding global stiffness and mass matrices using finite element method, the boundary conditions may be implemented to find the reduced stiffness and mass matrices. In present investigation, five different sets of boundary conditions involving clamped (C), simply supported (S) and free (F) have been assumed in case of isotropic beam. The criteria to define such boundary supports can be given as below.

2.3.1 Boundary conditions

Case 1: Clamped–clamped supports

$$w_i = 0, \quad \theta_i = 0, \quad w_f = 0, \quad \theta_f = 0 \tag{2.11}$$

Case 2: Clamped–simply supported

$$w_i = 0, \quad \theta_i = 0, \quad w_f = 0 \tag{2.12}$$

Case 3: Clamped–free supported

$$w_i = 0, \quad \theta_i = 0 \tag{2.13}$$

Case 4: Simply supported–simply supported

$$w_i = 0, \quad w_f = 0 \tag{2.14}$$

Case 5: Simply supported–free

$$w_i = 0 \tag{2.15}$$

where the subscripted notations $(\ )_i$ and $(\ )_f$ denote initial and final nodes of the beam. After implementing the boundary conditions, we can obtain the generalized eigenvalue problem of the following from.

2.3.2 Generalized eigenvalue problem

For the total length of the beam , the element mass and stiffness matrices are combined and global mass and stiffness matrices are obtained. Those are known as global mass and global stiffness matrices. The equation of motion can be obtained for free vibration by applying finite element method.

$$M\{\ddot{w}\} + K\{w\} = \{0\} \tag{2.16}$$

Here, M and K are global mass and stiffness matrices respectively, w is the displacement vector. Let us assume a solution in the form of

$$w(x,t) = W(x)e^{jw_n t} \tag{2.17}$$

where j and w_n denote $\sqrt{1}$ and natural frequencies; $W(x)$ is displacement amplitude vector. Substituting Eq. (2.17) into Eq. (2.16), we obtain the above generalized eigenvalue problem which may be used to get the vibration characteristics .

$$\left[K - w_n^2 M\right]\{W\} = \{0\} \tag{2.18}$$

Chapter 3

Numerical results and discussions

This chapter deals with finding natural frequencies of isotropic beam subjected to five different sets of boundary conditions involving clamped (C), simply supported (S) and free (F). The validation of these results has been carried out with available literature along with the test of convergence. First six natural frequencies have been evaluated and incorporated in subsequent sections. As such, homogeneous grids are being considered in Tables 3.1 to 3.5, whereas non–homogeneous grids (Gauss–Chebyshev–Lobatto points in the domain) are assumed in Tables 3.6 to 3.10. Tables 3.1 and 3.6 are meant for the first six natural frequencies of isotropic beam with C–C edge conditions. In the similar fashion, Tables 3.2 and 3.7 are for C–S beam; Tables 3.3 and 3.8 are for cantilever beam; Tables 3.4 and 3.9 are for S–S beam and Tables 3.5 and 3.10 are for S–F beam. Following facts may be summarized with the obtained natural frequencies:

- There occurs convergence of natural frequencies with increase in number of discretizations irrespective of griding criteria, which may play major role in such computations.

- In case of homogeneous grids, the convergence of natural frequencies is slow with respect to number of grids (n), whereas such convergence is faster in case of non–homogeneous grids. As regards, it is better to assumed non–homogeneous grids rather than homogeneous ones.

- From the validation made performed in last two rows of Tables 3.1 to Tables 3.5 and

Tables 3.6 to Tables 3.10 it can be easily said that the present results are in excellent agreement with the available literature for non–homogeneous discretizations which are also demonstrated in Fig. 3.1.

- In our formulations, we have also attempted the use of orthogonal polynomials in finite element method rather than using only simple algebraic polynomials. But such evaluations are not so worthy and yields same stiffness and mass matrices. As such, the natural frequencies in case of orthogonal FEM will definitely be same as found for simple algebraic polynomials.

3.1 Homogeneous discretizations

In this head, various results for C–C, C–S, C–F, S–S and S–F beams are incorporated in Tables 3.1 to Tables 3.5 for homogeneous discretizations.

Table 3.1: Convergence and validation of first six natural frequencies of C–C beam with homogeneous discretizations

n	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
5	35.0047	97.2551	192.9462	365.0377	603.7180	972.7442
8	29.2265	80.6385	158.5431	263.7223	397.8721	556.5600
10	27.6228	76.1694	149.4865	247.7202	371.7275	522.8376
15	25.6839	70.8027	138.8283	229.5921	343.2611	480.1127
20	24.7904	68.3369	133.9753	221.4974	330.9634	462.4553
30	23.9429	65.9997	129.3872	213.8888	319.5283	446.3198
Öz (2000)	22.3733	61.6729	120.9039	199.8616	298.5627	–
Exact (Özakaya and Pakdemirli, 1997)	22.3733	61.6728	120.9032	199.8604	298.5569	–

Table 3.2: Convergence and validation of first six natural frequencies of C–S beam with homogeneous discretizations

n	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
5	24.1062	78.5571	166.5577	312.7748	509.9369	814.1832
8	20.1394	65.3062	136.5654	234.7775	361.6766	518.1849
10	19.0353	61.7011	128.8441	220.7847	338.2401	482.4767
15	17.6995	57.3602	119.6953	204.7612	312.6831	443.7024
20	17.0839	55.3635	115.5166	197.5618	301.5359	427.5104
30	16.4999	53.4703	111.5623	190.7821	291.1362	412.6377
Öz (2000)	15.4182	49.9649	104.2482	178.2713	272.0364	–
Exact (Özakaya et al., 1997)	15.4182	49.9648	104.2482	178.2706	272.0322	–

Table 3.3: Convergence and validation of first six natural frequencies of cantilever beam with homogeneous discretizations

n	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
5	5.4940	34.4690	97.1483	191.6526	356.4647	572.4838
8	4.5924	28.7836	80.6662	158.4957	263.4617	396.8536
10	4.3408	27.2044	76.1987	149.4751	247.6651	371.5042
15	4.0362	25.2949	70.8307	138.8260	229.5886	343.2463
20	3.8959	24.4150	68.3639	133.9736	221.4969	330.9611
30	3.7627	23.5803	66.0258	129.3856	213.8889	319.5281

Table 3.4: Convergence and validation of first six natural frequencies of S–S beam with homogeneous discretizations

n	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
5	15.4253	61.9285	141.3274	273.8613	435.3013	688.3728
8	12.8913	51.5864	116.2699	207.6146	327.1967	477.6069
10	12.1848	48.7467	109.7511	195.4424	306.4208	443.8340
15	11.3299	45.3209	101.9835	181.3583	283.5485	408.7613
20	10.9359	43.7438	98.4268	174.9966	273.4832	393.9482
30	10.5620	42.2481	95.0588	168.9962	264.0657	380.2787
Öz (2000)	9.8696	39.4784	88.8267	157.9147	246.7442	–
Exact Özakaya et al. (1997)	9.8695	39.4784	88.8264	157.9144	246.7413	–

Table 3.5: Convergence and validation of first six natural frequencies of S–F beam with homogeneous discretizations

n	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
5	24.1050	78.4866	165.6954	307.9450	489.3738	763.3250
8	20.1394	65.3041	136.5383	234.6020	360.9267	515.9042
10	19.0353	61.7006	128.8384	220.7476	338.0784	481.9368
15	17.6995	57.3602	119.6949	204.7587	312.6724	443.6662
20	17.0839	55.3635	115.5165	197.5614	301.5342	427.5048
30	16.4999	53.4703	111.5623	190.7821	291.1360	412.6373

3.2 Non–homogeneous discretizations

In this head various results for C–C, C–S, C–F, S–S and S–F beams are incorporated in Tables 3.6 to Tables 3.10 for Non–homogeneous discretizations.

Table 3.6: Convergence and validation of first six natural frequencies of C–C beam with non–homogeneous discretizations

n	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
5	22.4908	64.8679	125.0958	230.9118	487.4263	585.6315
8	22.3892	61.9058	122.7981	201.4761	330.3318	466.7648
10	22.3792	61.7640	121.5043	201.8458	310.4589	421.6543
15	22.3743	61.6890	121.0110	200.3150	299.9984	420.8040
20	22.3736	61.6776	120.9358	199.9981	299.0020	418.1718
30	22.3733	61.6737	120.9094	199.8856	298.6404	417.2180
Öz (2000)	22.3733	61.6729	120.9039	199.8616	298.5627	–
Exact (Özakaya and Pakdemirli, 1997)	22.3733	61.6728	120.9032	199.8604	298.5569	–

Table 3.7: Convergence and validation of first six natural frequencies of C–S beam with non–homogeneous discretizations

n	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
5	15.4570	51.3339	107.5990	207.0716	383.2959	549.0196
8	15.4231	50.0853	105.3144	180.5077	294.2674	420.5870
10	15.4200	50.0110	104.6211	179.8376	279.4785	391.8562
15	15.4185	49.9730	104.3149	178.5895	273.1206	388.5222
20	15.4183	49.9673	104.2679	178.3668	272.3667	386.4633
30	15.4182	49.9653	104.2515	178.2880	272.0947	385.7102
Öz (2000)	15.4182	49.9649	104.2482	178.2713	272.0364	–
Exact (Özakaya et al., 1997)	15.4182	49.9648	104.2482	178.2706	272.0322	–

Table 3.8: Convergence and validation of first six natural frequencies of cantilever beam with non–homogeneous discretizations

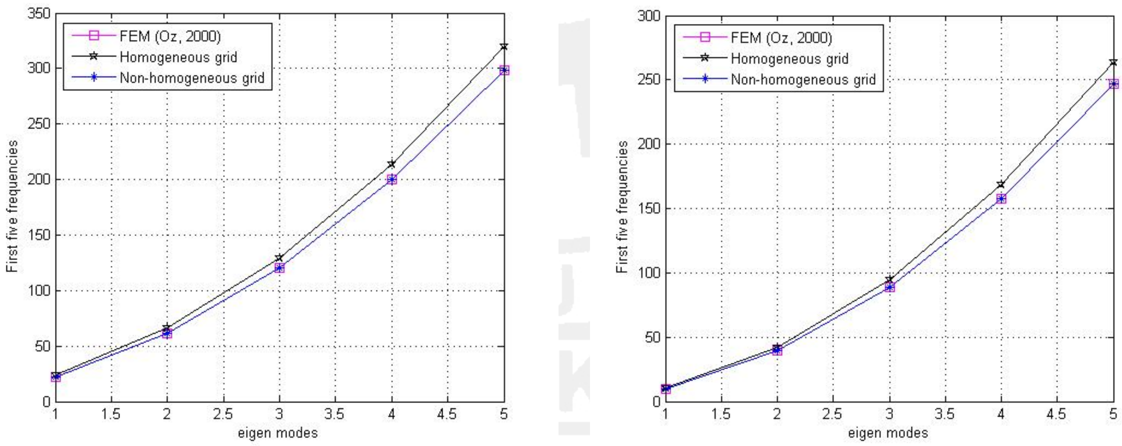
n	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
5	3.5163	22.1091	64.4807	124.5820	231.5564	482.6280
8	3.5160	22.0459	61.8978	122.6933	201.2669	329.6807
10	3.5160	22.0388	61.7763	121.4640	201.7579	310.2725
15	3.5160	22.0352	61.7113	121.0028	200.2997	299.9689
20	3.5160	22.0347	61.7014	120.9323	199.9936	298.9933
30	3.5160	22.0345	61.6980	120.9076	199.8848	298.6388

Table 3.9: Convergence and validation of first six natural frequencies of S–S beam with non–homogeneous discretizations

n	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
5	9.8798	40.1263	89.5532	184.0552	344.0300	443.6823
8	9.8709	39.5323	89.4581	159.2565	266.6035	375.8417
10	9.8701	39.4990	89.0481	159.0056	252.4690	357.7353
15	9.8697	39.4820	88.8663	158.1318	247.5481	357.6446
20	9.8696	39.4795	88.8384	157.9798	246.9882	356.0326
30	9.8696	39.4786	88.8287	157.9261	246.7871	355.4448
Öz (2000)	9.8696	39.4784	88.8267	157.9147	246.7442	–
Exact (Özakaya et al., 1997)	9.8695	39.4784	88.8264	157.9144	246.7413	–

Table 3.10: Convergence and validation of first six natural frequencies of S–F beam with non–homogeneous discretizations

n	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
5	15.4423	51.0729	106.9410	207.5145	383.3795	535.7269
8	15.4215	50.0630	105.2347	180.3178	293.8640	419.1964
10	15.4194	50.0028	104.5910	179.7637	279.3334	391.5729
15	15.4184	49.9716	104.3097	178.5767	273.0952	388.4775
20	15.4183	49.9669	104.2663	178.3630	272.3591	386.4502
30	15.4182	49.9652	104.2512	178.2873	272.0933	385.7078



(a) C–C beam (b) S–S beam

Figure 3.1: Validation of first five natural frequencies having $n = 30$ with (Öz, 2000)

Chapter 4

Concluding remarks

Following facts may be summarized with the obtained natural frequencies:

- There occurs convergence of natural frequencies with increase in number of discretizations irrespective of griding criteria, which may play major role in such computations.
- In case of homogeneous grids, the convergence of natural frequencies is slow with respect to number of grids (n), whereas such convergence is faster in case of non–homogeneous grids. As regards, it is better to assumed non–homogeneous grids rather than homogeneous ones.
- From the validation made performed in last two rows of Tables 3.1 to Tables 3.5 and Tables 3.6 to Tables 3.10 it can be easily said that the present results are in excellent agreement with the available literature for non–homogeneous discretizations which are also demonstrated in Fig. 3.1.
- In our formulations, we have also attempted the use of orthogonal polynomials in finite element method rather than using only simple algebraic polynomials. But such evaluations are not so worthy and yields same stiffness and mass matrices. As such, the natural frequencies in case of orthogonal FEM will definitely be same as found for simple algebraic polynomials.

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