

**FREE VIBRATION STUDY OF ANNULAR
LAMINATED COMPOSITE CIRCULAR PLATES
WITH HOLES**

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR
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In

Civil Engineering

By

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UNDER THE ABLE GUIDANCE OF

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Technology, Rourkela May, 2015**



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CERTIFICATE

This is to certify that the thesis entitled “**FREE VIBRATION STUDY OF ANNULAR LAMINATED COMPOSITE CIRCULAR PLATES WITH HOLES**” submitted by **ARPITA MOHAPATRA** bearing roll no. **111CE0608** of **Civil Engineering Department**, National Institute of Technology Rourkela is a genuine work carried out by her under my supervision and support.

To the best of my knowledge, the matter embodied in this thesis has not been presented to any other University / Institute for the award of any Degree or Diploma.

Date: May, 2015

Place: Rourkela

Prof. A.V. Asha

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Finally, my grateful regards to my parents **Deba Sundar Mohapatra** and **Rajashree Mohapatra** who have been constantly supporting and encouraging me at the time of failure and they are always at my back. They are my constant source of inspiration for these four years. I want to dedicate this piece of work of mine to my parents.

Thanks to the Almighty for the blessing that he has bestowed upon me in all my endeavours.

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ABSTRACT

A composite laminate is layers of fibres of composite materials which are assembled to provide desired engineering properties. These have a wide range of application, especially in weight sensitive structures like aircraft and spacecraft. Laminated plates used in these structures are subjected to dynamic loads. These plates with numerous circular cut outs are either used to lessen the mass of the whole structure or to expand the inspection. The presence of cut outs in the plates changes the stress distribution of the plates and makes the analysis complex. When an external system vibrates and has exactly the same natural frequency as the internal system, resonance follows producing considerable deflections. Specific plans can minimise these unnecessary failures. Hence the analysis of the modal characteristics of these plates is significant.

First an isotropic circular plate's behaviour is studied and then a circular orthotropic plate has been considered for this study. The material properties have been fixed. The natural frequencies have been computed for different boundary condition, hole-radius ratio, and thickness to radius ratio, and fibre orientation, asymmetric and symmetric holes. The effects of these variables on the nature of vibration have been analysed and discussed. ANSYS 13.0 is used for the computation of natural frequencies.

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Chapter 1

INTRODUCTION

1.1 LAMINATED COMPOSITES

Laminated composites are layers of fibres of composite materials that are assembled to provide desired engineering properties. The individual layers are made up of high modulus, and high strength fibres in a ceramic matrix, metallic or polymeric material. Lamination is done to achieve the best properties of the combined layers or a new property in order to make the material more useful. With the requirement of economical and lightweight structures composite materials have become popular due to the following reasons:

- 1) They are rigid with high strength to weight ratio
- 2) Good Electrical resistance,
- 3) Resistance to chemicals and weather
- 4) Good directional properties



FIG. 1.1 LAMINATED COMPOSITES

The individual layers are usually orthotropic or transversely isotropic with the laminate then exhibiting anisotropic, orthotropic, or quasi-isotropic properties [7].

1.2 CIRCULAR PLATES WITH HOLES

Circular plates with holes are widely used in mechanical parts. Circular plates with multiple circular holes have applications in many engineering structures. They are used either to lessen the mass of the structure or to upturn the range of inspection or to satisfy engineering applications. Circular geometries are used in extensive variety of applications because of the ease of their manufacture [8].



FIG.1.2 CIRCULAR PLATE WITH HOLES

1.3 SIGNIFICANCE OF HOLES

A composite structure sometimes may be provided with various types of holes for assembling the parts or units in the structure, for passage of the cables and controlling mechanisms, for inspection, maintenance and attachment to other units [8]. These holes in the structure usually lead to the variation of natural frequency and also in the decrease of the respective potential. If the hole is eccentric, the natural frequency of structures is expected to differ considerably from that of a plate with central holes [8].

Chapter 2

REVIEW OF LITERATURE

2.1 REVIEW OF LITERATURE

Lin (1982) presented an analytical method for calculating the free transverse vibrations of uniform circular plates and membranes with eccentric holes. The wave equations governing the small transverse motion of the plates and membranes were solved to satisfy the boundary conditions at both inner and outer edges, and the associated frequency equations for the plates and membranes were derived. The analysis included the vibrations of uniform circular plates and membranes with concentric holes as special cases.

Srinivasamurthy et al. (1987) presented a non-linear shear deformable model to study the dynamic behaviour of circular plates composed of orthotropic layers. The basic equations were originated by Hamilton's principle and expression of the changing calculus in terms of the transverse displacement that yielded a non-linear ordinary differential equation for the time function. This was then resolved by using the methods of harmonic balance.

Bicos et al. (1989) used a finite element method to calculate the natural frequencies, mode shapes, and damping factors of rectangular plates, cylinders, and cylindrical panels with free, clamped, or simply supported edges, and with or without circular cut outs.

Chai Gin Boay (1996) used finite element method to compare results with experimental data on the free vibration of symmetric laminated composite plates containing a central hole. The effect of parameters such as the hole sizes, the boundary conditions along the edges of the plate and the aspect ratio of the plate on frequencies were studied. The stacking sequence and lamination materials were not changed. There was close correlation between numerical studies and experimental data.

Lee (2007) used semi analytical methodologies to solve vibrational characteristics and mode shapes for circular plates with various circular holes. The direct searching methodology was adopted to establish the vibrational characteristics through SVD. For the cases of circular plates with various eccentric and centric circular cut outs, the results were also matched with those of FEM using ABAQUS.

Cheng et al. (2009) studied the effects of the boundary conditions, eccentric holes, and size of the cut outs on vibration modes using FEM analysis code NASTRAN and examined it through both universal and local analysis.

Malekzadeh et al. (2010) operated a 3 dimensional layer wise FEM in the direction of the thickness to study the natural frequencies of a thick laminated circular annular plate supported on a flexible foundation.

Hosseini-Hashemi et al. (2012) presented the natural frequency of laminated transversely isotropic circular plates. The plate had an axisymmetric rigid core attached at the centre. The governing equation of motion was obtained using Mindlin's first order shear deformation (FSDT) plate theory.

Pushparaj (2012) used the ANSYS software for analysing the plates with various boundary conditions and different orientation of laminates. Eight-noded Shell99 was used throughout the analysis. Two different boundary conditions were considered - CFFF-(clamped free free free) and CFCE-(clamped free clamped free) conditions.

Thakare (2013) determined experimentally the natural frequencies of a plate with various holes and compared the results obtained from ANSYS. The relationship among parameter variations and frequencies was analysed.

Hussain et al. (2014) investigated the natural frequency of four layered angle ply symmetrically laminated plates with various lamination angles of laminas with different hole locations. The ANSYS 13.0 software was used for modelling and analysis of the laminated plates under clamped boundary conditions. The eight noded shell 99 was used throughout the analysis. Length to height ratio considered is 40 and 200. The ratio of hole area to the plate area was maintained constant throughout the analysis as 0.05. From the analysis it was inferred that the fundamental frequency of laminated composite plates decreases with increase in L/h ratio.

Rahimi et al. (2014) presented a three-dimensional elasticity theory by means of a state-space centred differential quadrature technique for natural frequency investigation of fibre metal laminate plate with cut outs. A semi analytical approach which uses state-space in the thickness and differential quadrature in the radial direction was implemented for evaluating the vibration characteristics and frequencies of the annular circular plates.

Chapter 3

OVERVIEW

3.1 OBJECTIVE OF PRESENT STUDY

“the free vibrations of circular plates have been of practical and academic interest for at least a century and a half” [11]

Resonance can occur when an external system's natural frequency matches with the internal system, causing considerable deflection and sudden collapse in inappropriate constructions. Precise designs have potential to minimise these unnecessary vibrations. Although many results of vibration analysis for solid or annular plates are already presented in the literature work and the analysis of these circular plates have been done by various methods like differential quadrature method, finite element analysis, SVD, ABAQUS, ANSYS, Chebyshev–Ritz method, moving Least Square Method and several other semi analytical methods, few studies have been done on the problem of laminated circular plate with holes and I believe it to be a newly considered study that could be used as a means of comparison of frequencies with those of solid circular plates. Plates with holes are frequently used in various structures. This would also enhance my knowledge on the topic.

3.2 SCOPE OF STUDY

Laminated composites are materials made by bonding layers to bring out the superior and desirable qualities of the individual materials. Due to its qualities like light weight and high strength it has been applied in various engineering applications. Hence after a thorough review of literature, it was decided to study the free vibration of orthotropic laminated composite plates with circular holes that can be centric, eccentric, symmetric and asymmetric for various changes in parameters like aspect ratio, fibre orientation, number of layers, thickness to radius ratio, etc. The present study deals with the natural frequency of orthotropic laminated composite plates having single cut-out as well as various eccentric, symmetric and asymmetric cut outs. The stress distribution is complicated in such type of structures. It can fail due to resonance occurring due to vibration. Hence this study is important to know the behaviour of such type of plates.

3.2 OUTLINE OF PRESENT WORK

An attempt has been made to study the vibration characteristics of circular laminated orthotropic composite plates with holes. For generalisation of the analysis, first an isotropic plate with a hole is assumed with inner radial edge clamped and outer radial edge free and analysis is done by varying the outer to inner radius ratio and the element thickness for mesh size. The effect of boundary conditions, hole-size, and lamination scheme and fibre orientation on natural frequencies of vibration is studied, and based on these results; the analysis of an annular orthotropic plate for various boundary conditions is carried out.

First, a single circular hole at the centre is considered and then the analysis is further extended to holes that are eccentric. All the calculations have been done with the finite element package ANSYS 13.0.

This thesis comprises of seven chapters. The first chapter introduces the importance of the laminated composite plates in various fields, explains the complexity of circular plates with holes and significance of holes in a structure.

In the second chapter, the literature review is presented. All the research papers relevant to this study were reviewed in detail and have been discussed briefly

The third chapter states the objective and scope of the study based on the detailed literature review.

The fourth chapter presents the theoretical formulation. All the computation was carried out using ANSYS. The details of modelling in ANSYS are also presented.

In chapter 5, the results obtained in the present investigation are tabulated. The effects of various parameters like plate thickness, boundary conditions, number of layers, size of hole, aspect ratio, fibre orientation and hole shape is discussed.

Chapter 6 states all the conclusions that are inferred from the results obtained.

Chapter 7 gives all sources of the research papers using which studies were carried out.

Chapter 4

THEORY AND FORMULATION

4.1 EQUATIONS OF EQUILIBRIUM

The equations of motion for the free vibration of a symmetric cross-ply laminated circular plate can be expressed as [6]

$$K_S A_{55} \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial \phi_x}{\partial x} \right) + K_S A_{44} \left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial \phi_y}{\partial y} \right) = I_0 \frac{\partial^2 w}{\partial t^2}$$

$$D_{11} \frac{\partial^2 w}{\partial x^2} + D_{12} \frac{\partial^2 \phi_y}{\partial y \partial x} + D_{66} \left(\frac{\partial^2 \phi_x}{\partial y^2} + \frac{\partial^2 \phi_y}{\partial y \partial x} \right) - K_S A_{55} \left(\frac{\partial w}{\partial x} + \phi_x \right) = I_2 \frac{\partial^2 \phi_x}{\partial t^2}$$

$$D_{66} \left(\frac{\partial^2 \phi_y}{\partial x^2} + \frac{\partial^2 \phi_x}{\partial y \partial x} \right) + D_{12} \frac{\partial^2 \phi_x}{\partial y \partial x} + D_{22} \frac{\partial^2 \phi_y}{\partial y^2} - K_S A_{44} \left(\frac{\partial w}{\partial y} + \phi_y \right) = I_2 \frac{\partial^2 \phi_y}{\partial t^2}$$

Where, I_0 and I_2 are the mass inertia tensor components defined as,

$$I_0 = \rho h$$

$$I_2 = \frac{\rho h^3}{12}$$

ρ and h denote the density and total thickness.

As per the First order Shear Deformation Theory (FSDT) [11], after deformation the transverse normals to the mid surface do not remain perpendicular to the mid surface due to the effects of transverse shear strains. The displacement field at time t is given by

$$U(x, y, z, t) = u_0(x, y, t) + z\phi_x(x, y, t),$$

$$V(x, y, z, t) = v_0(x, y, t) + z\phi_y(x, y, t),$$

$$W(x, y, z, t) = w_0(x, y, t),$$

Where (u_0, v_0, w_0) are the displacements of a point on the plane $z = 0$, and ϕ_x and ϕ_y are, respectively, the rotations of a transverse normal about the y and x axes.

4.2 STRAIN DISPLACEMENT RELATIONS

Strain displacement relationships in cylindrical co-ordinates [11],

$$\begin{Bmatrix} \varepsilon_r \\ \varepsilon_\theta \\ \gamma_{\theta z} \\ \gamma_{rz} \\ \gamma_{r\theta} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u_o}{\partial r} \\ \frac{1}{r} \frac{\partial u_o}{\partial \theta} \\ \frac{1}{r} \frac{\partial w_o}{\partial \theta} + \phi_r \\ \frac{\partial w_o}{\partial r} + \phi_r \\ \frac{1}{r} \frac{\partial u_o}{\partial \theta} + \frac{\partial v_o}{\partial r} \end{Bmatrix} + z \begin{Bmatrix} \frac{\partial \phi_r}{\partial r} \\ \frac{1}{r} \frac{\partial \phi_\theta}{\partial \theta} \\ 0 \\ 0 \\ \frac{1}{r} \frac{\partial \phi_r}{\partial \theta} + \frac{\partial \phi_\theta}{\partial r} \end{Bmatrix}$$

Where u_o , v_o , w_o are mid-plane displacements in the r , θ and z direction and ϕ_r , ϕ_θ are rotations of the transverse normal about r and θ axes.

4.3 CONSTITUTIVE RELATIONS

The relationship of the force and moment resultants to the mid-plane strains and curvatures has been expressed by the laminate constitutive equations. The force resultants and moment resultants are given by

$$\begin{Bmatrix} N_r \\ N_\theta \\ N_{r\theta} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \frac{\partial u_o}{\partial r} \\ \frac{1}{r} \frac{\partial u_o}{\partial \theta} \\ \frac{1}{r} \frac{\partial u_o}{\partial \theta} + \frac{\partial v_o}{\partial r} \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \frac{\partial \phi_r}{\partial r} \\ \frac{1}{r} \frac{\partial \phi_\theta}{\partial \theta} \\ \frac{1}{r} \frac{\partial \phi_r}{\partial \theta} + \frac{\partial \phi_\theta}{\partial r} \end{Bmatrix}$$

$$\begin{Bmatrix} M_r \\ M_\theta \\ M_{r\theta} \end{Bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \frac{\partial u_o}{\partial r} \\ \frac{1}{r} \frac{\partial u_o}{\partial \theta} \\ \frac{1}{r} \frac{\partial u_o}{\partial \theta} + \frac{\partial v_o}{\partial r} \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \frac{\partial \phi_r}{\partial r} \\ \frac{1}{r} \frac{\partial \phi_\theta}{\partial \theta} \\ \frac{1}{r} \frac{\partial \phi_r}{\partial \theta} + \frac{\partial \phi_\theta}{\partial r} \end{Bmatrix}$$

$$A_{ij} = \sum_{k=1}^N Q_{ij}^{(k)} (h_{k+1} - h_k),$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^N Q_{ij}^{(k)} (h_{k+1}^2 - h_k^2)$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^N Q_{ij}^{(k)} (h_{k+1}^3 - h_k^3)$$

Where A_{ij} denotes the extensional stiffnesses, D_{ij} denotes the bending stiffnesses, and B_{ij} denotes the bending–extensional coupling stiffnesses of a laminate.

4.4 METHODOLOGY

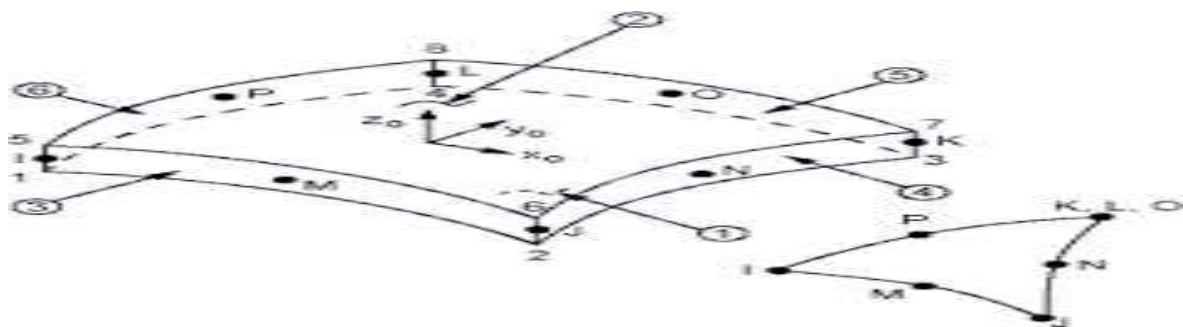
ANSYS 13.0 version is used for modelling and vibrational analysis of circular plates with holes. ANSYS Mechanical software is a finite element method software for structural analysis, including linear, nonlinear and dynamic studies.



The element used for modelling the circular plate is the SHELL 281 element.

SHELL281 is suitable for analysing structures. The element has eight nodes with six degrees of freedom at each node: translations in the x, y, and z axes, and rotations about the x, y, and z-axes. SHELL281 is well-suited for linear, large rotation, and large strain nonlinear applications. Change in shell thickness is accounted for in nonlinear analyses. For layered applications for modelling composite shells or sandwich construction, SHELL281 is applied for the modelling of an accurate composite shell. This is governed by the FSDT theory (Mindlin-Reissner shell theory). Logarithmic strain and true stress measures is the base for element formulation. Finite membrane strains (stretching) are allowed by the element kinematics. However, the curvature variations are assumed to be insignificant. The shell section information and eight nodes (I, J, K, L, M, N, O and P) defines the element [1].

FIG 4.1 SHELL281 ELEMENT

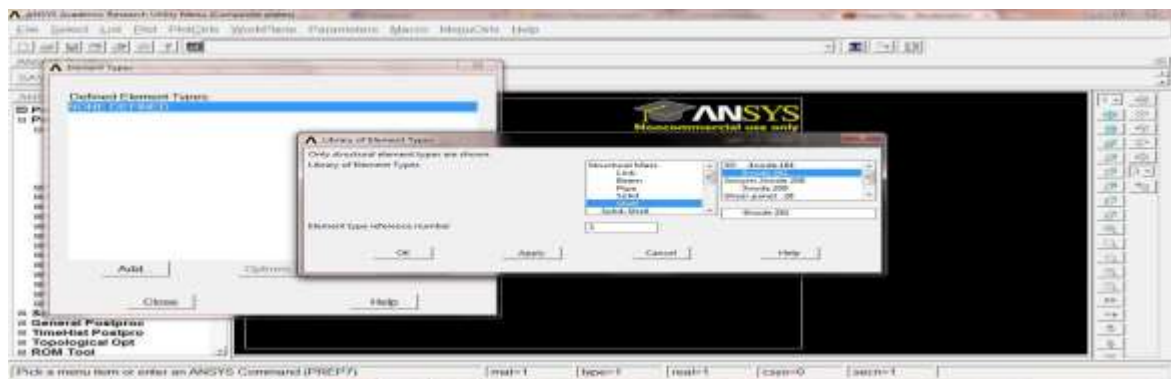


4.5 PROCEDURAL STEPS FOR MODELING IN ANSYS

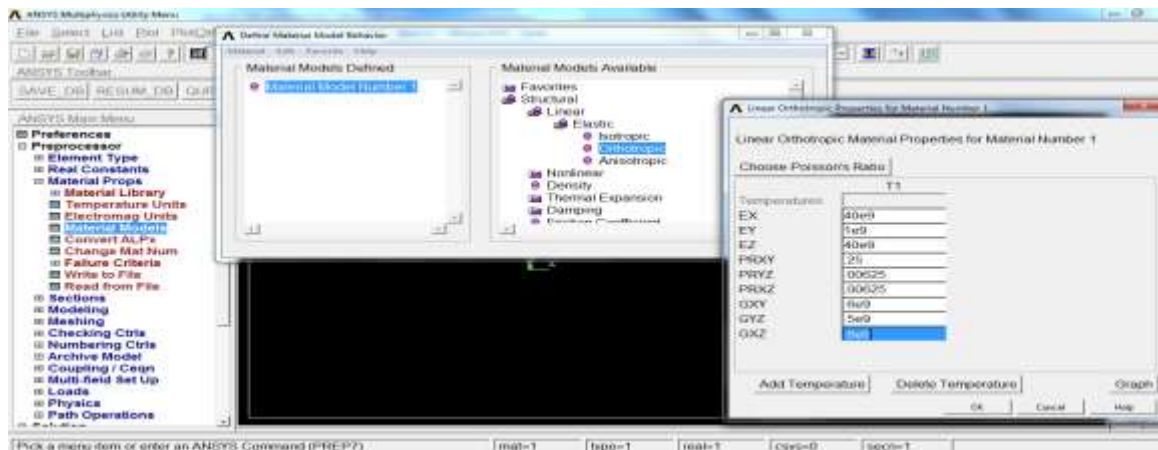
1. Preferences → Structural → Ok



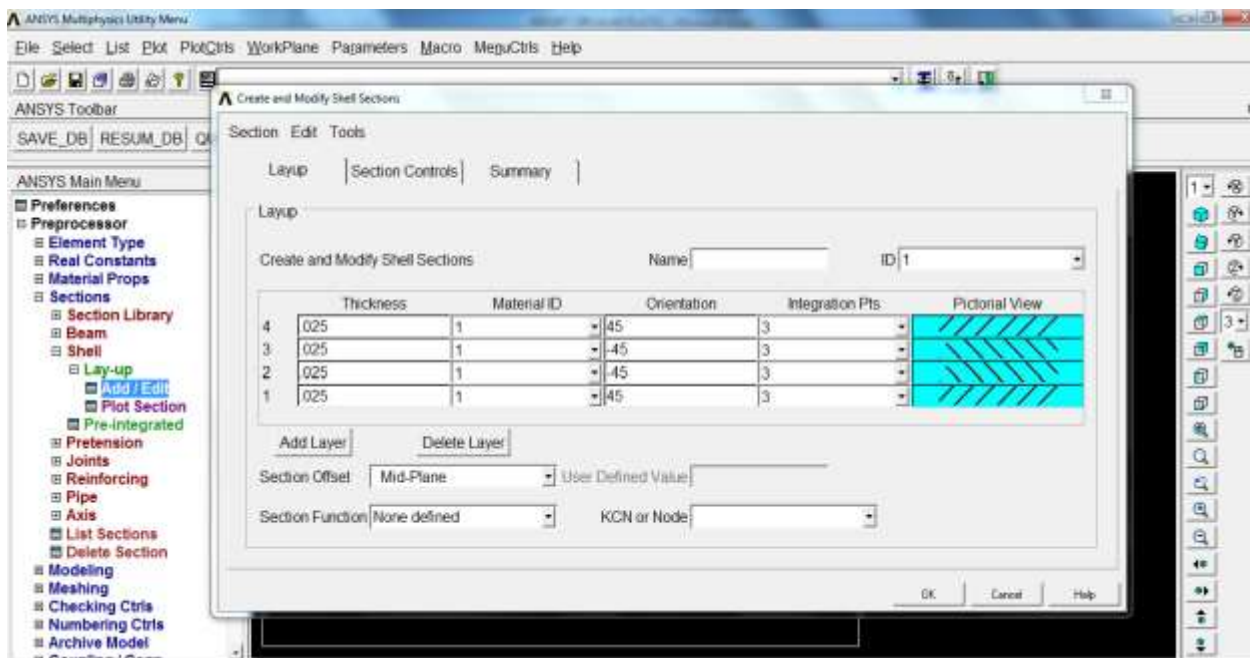
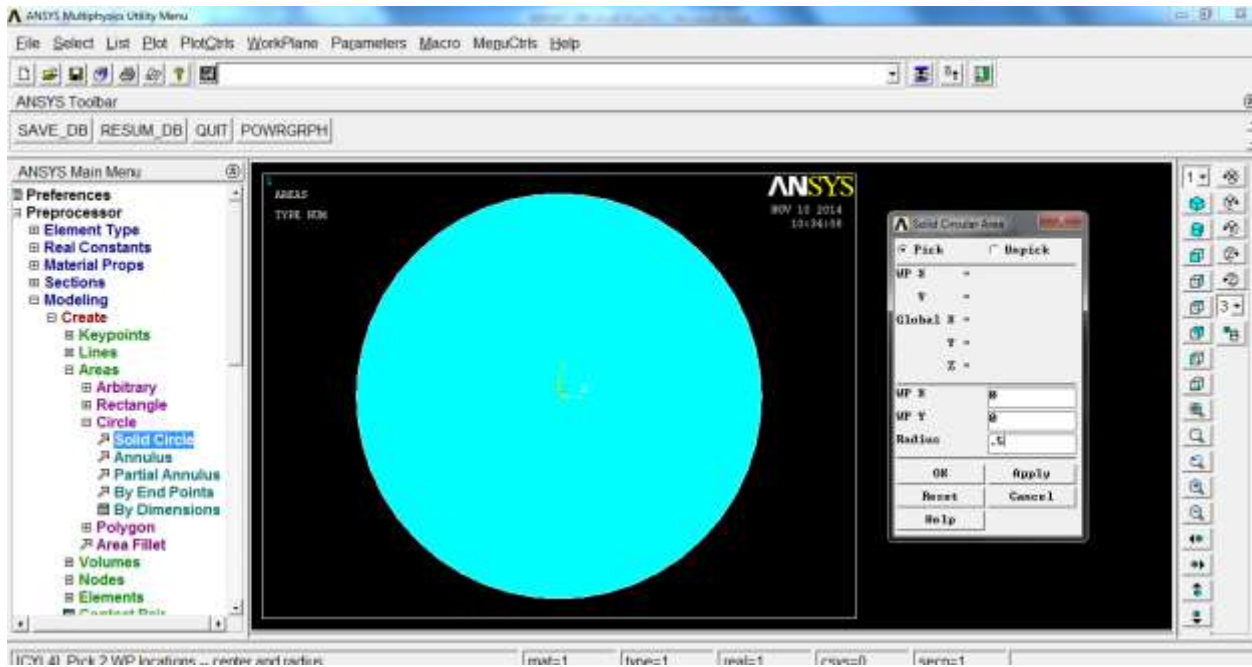
2. Pre-processor → Element type → Add → Structural Mass → Shell → 8node 281 → Ok



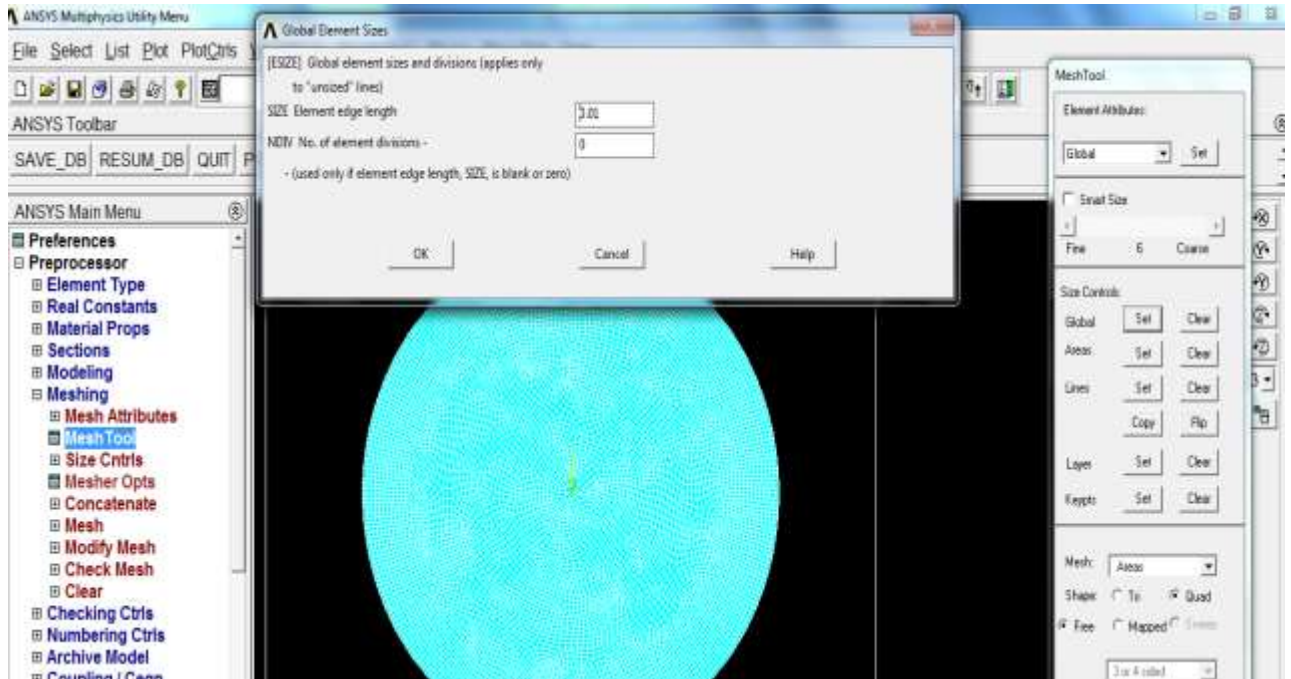
3. Pre-processor → Material Properties → Material Models → Structural → linear → Elastic → Orthotropic → Density → Enter the density of composite laminate → Ok



4. Preprocessor → Modelling → Create → Areas → solid circle → Sections → Shell → Lay-up → Add/Edit → Ok

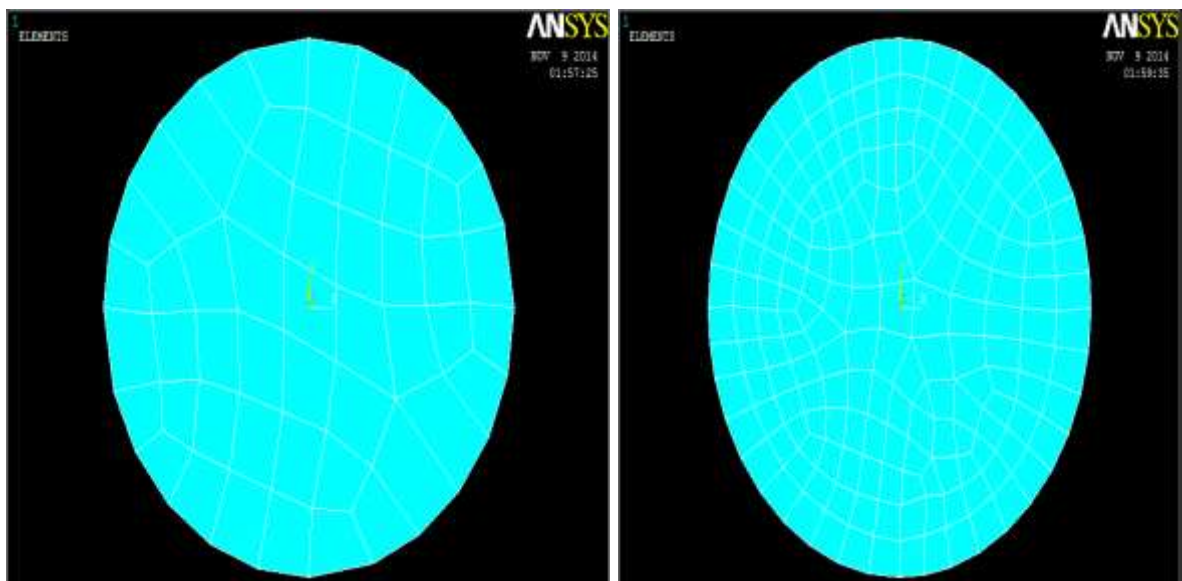


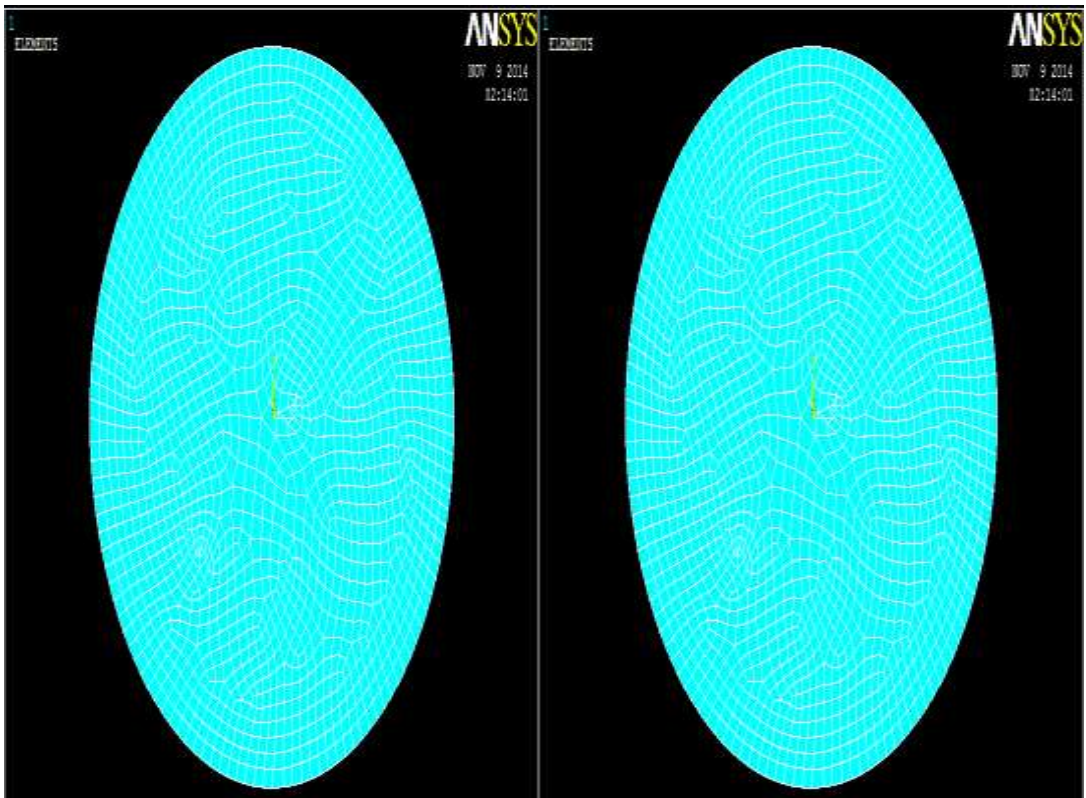
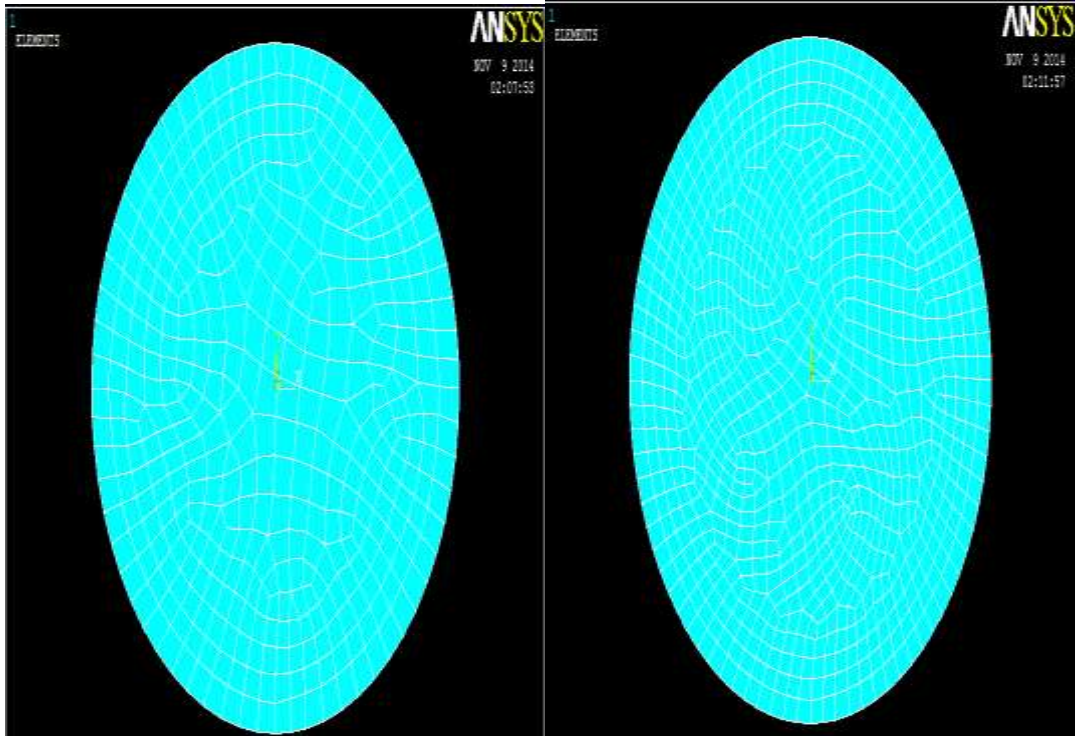
5. Preprocessor → Meshing → Mesh Tool → Size Controls → Enter length of element → Ok



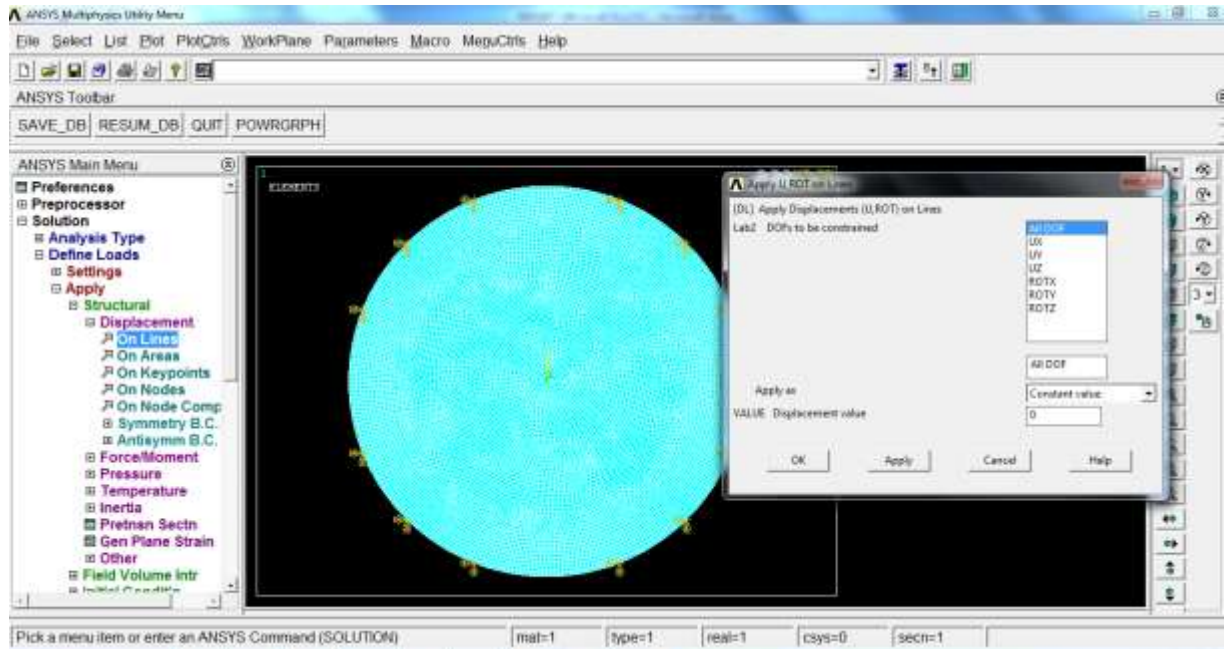
MESHING USING SMART SIZE

MESH 6 → MESH 5 → MESH 4 → MESH 3 → MESH 2 → MESH 1

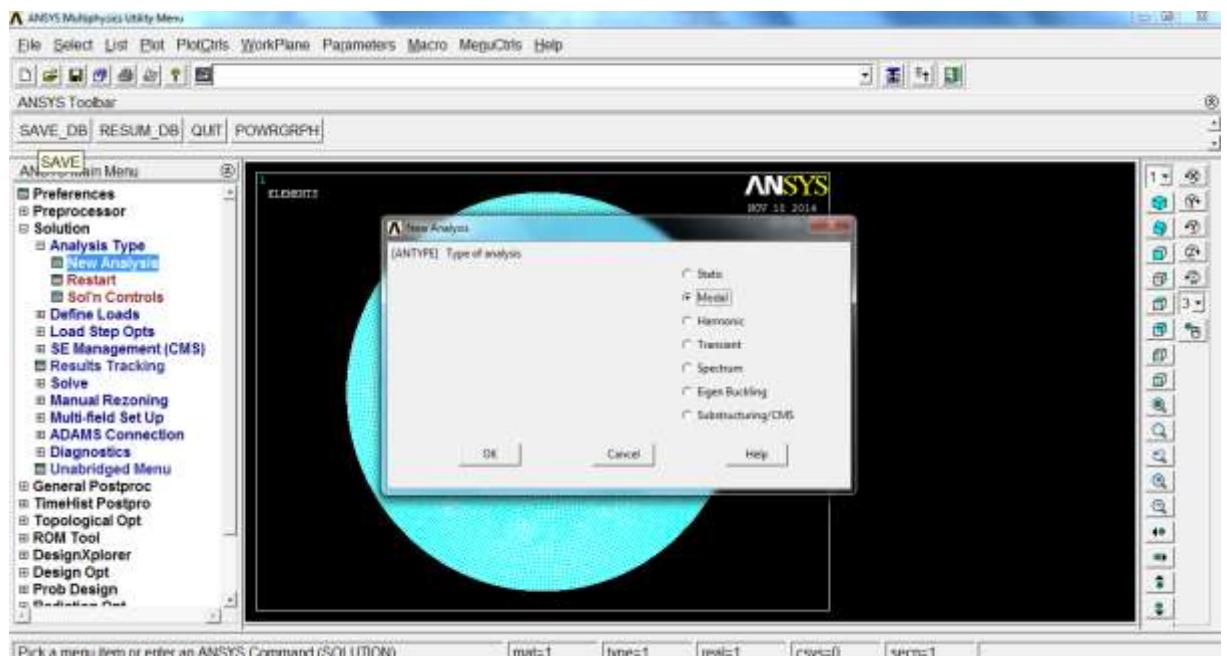




6. Preprocessor → Loads → Define Loads → Apply → Structural → Displacement



7. Preprocessor → Loads → Analysis Type → New Analysis → Type of analysis – Modal → Ok → Analysis Options → Mode extraction method – Block Lanczos → No. of modes to extract - 8 → Enter the start and end frequency → Ok

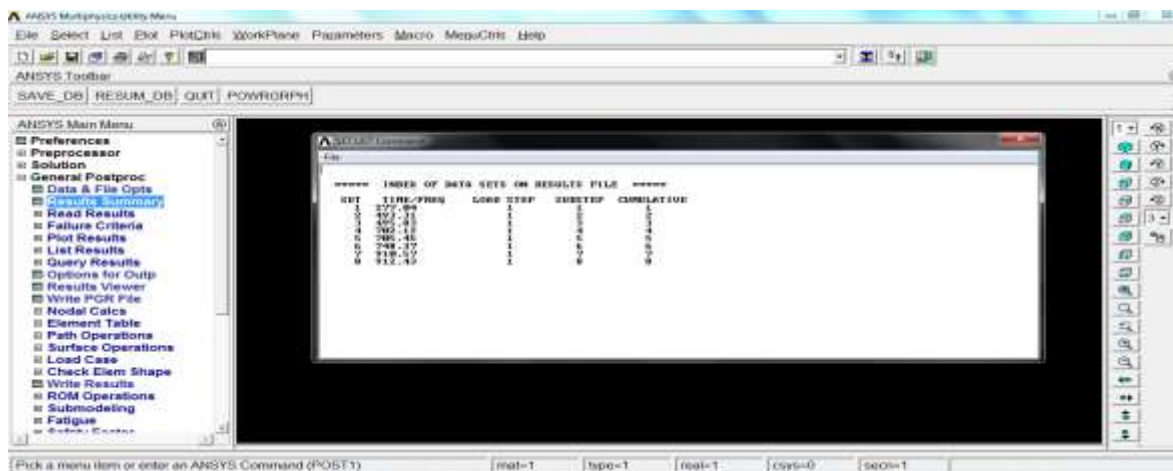




8. Solution → Solve → Current LS



9. For viewing the results, follow the step given below: General Post processing → Results Summary



Chapter 5

RESULTS AND DISCUSSIONS

5.1 INTRODUCTION

A laminated composite circular plate with a central hole (annular plate) is studied for its dynamic characteristics. First the validity of the ANSYS formulation is checked for its accurateness for a solid plate by comparing the results obtained with that presented by Cong et al [6]. Then the dynamic characteristics for an annular laminated composite plate are studied.

5.2 VALIDATION WITH PREVIOUS RESULTS

A plate with the following parameters as proposed by Cong et al. [6] was considered for the analysis. The non-dimensional natural frequency is expressed as,

$$w' = \frac{wb^2}{h} \sqrt{\frac{\rho}{E_2}}$$

BOUNDARY CONDITIONS:

Clamped Edge

No displacements or rotations

PARAMETERS INVOLVED:

Density, $\rho = 1580\text{kg/m}^3$

Diameter, $b = 1 \text{ OR } 2 \text{ m}$

Thickness, $h \text{ or } t = 0.1b$

The Young's moduli, $E_1 = 40 \text{ GPa}$ and $E_2 = 1 \text{ GPa}$

Shear moduli, $G_{23} = 0.5 \text{ GPa}$, $G_{13} = 0.6 \text{ GPa}$, and $G_{12} = 0.5 \text{ GPa}$

Poisson's ratios, $\mu_{12}=\mu_{23}= 0.25$, $\mu_{13}= .25*(E_2/E_1) = 0.00625$

TABLE 5.1: Comparison of natural frequencies for clamped four-ply circular laminated plates

Diameter = 1m, lamination scheme $[45^\circ/-45^\circ/-45^\circ/45^\circ]$, $(w' = \frac{wb^2}{h} \sqrt{\frac{\rho}{E_2}})$, $h/b=0.1$, $E_1/E_2=40$

Modes	1	2	3	4	5	6	7	8
Present study	22.74	38.60	39.08	54.79	55.77	59.18	71.67	71.77
Ngo-cong et al.[6]	24.737	39.112	43.638	57.190	57.254	65.693	74.254	75.149

TABLE 5.2: Comparison of natural frequencies for clamped four-ply circular laminated plates

Diameter = 1m, lamination scheme $[0^\circ/-0^\circ/-0^\circ/0^\circ]$, $(w' = \frac{wb^2}{h} \sqrt{\frac{\rho}{E_2}})$, $h/b=0.1$, $E_1/E_2=40$

Modes	1	2	3	4	5	6	7	8
Present study	25.68	38.72	40.2	50.34	51.80	60.70	62.48	63.42
Ngo-cong et al.[6]	22.198	29.658	40.987	42.764	50.582	55.075	61.568	65.849

TABLE 5.3: Comparison of natural frequencies for clamped four-ply circular laminated plates

Diameter = 2m, lamination scheme $[45^\circ/-45^\circ/-45^\circ/45^\circ]$, $(w' = \frac{wb^2}{h} \sqrt{\frac{\rho}{E_2}})$, $h/b=0.1$, $E_1/E_2=40$

Modes	1	2	3	4	5	6	7	8
Present study	22.41	38.47	38.10	55.26	56.06	59.86	71.91	72.46
Ngo-cong et al.[6]	24.737	39.112	43.638	57.190	57.254	65.693	74.254	75.149

TABLE 5.4: Comparison of natural frequencies for clamped four-ply circular laminated plates

Diameter = 2m, lamination scheme $[0^\circ/-0^\circ/-0^\circ/0^\circ]$, $(w' = \frac{wb^2}{h} \sqrt{\frac{\rho}{E_2}})$, $h/b=0.1$, $E_1/E_2=40$

Modes	1	2	3	4	5	6	7	8
Present study	25.78	38.66	39.96	51.03	51.88	59.52	63.66	64.02
Ngo-cong et al. [6]	22.198	29.658	40.987	42.764	50.582	55.075	61.568	65.849

It is seen from Tables 5.1 to 5.4 that the results of the present formulation are quite similar to that of Cong et al [6]. Some variations may be observed due to the assumption of the density. We can observe that as the diameter is increased from 1m to 2 m, the frequency increases for both the orientation of the fibres.

In the next study, the natural frequency of a solid orthotropic plate is analyzed by varying the thickness to radius ratio from 0.001 to 0.2.

TABLE 5.5: Comparison of natural frequencies for clamped four-ply circular laminated plates with varying thickness:

$$\text{Diameter} = 1\text{m}, [45^\circ/-45^\circ/-45^\circ/45^\circ], (w' = \frac{wb^2}{h} \sqrt{\frac{\rho}{E_2}}), E_1/E_2=40$$

t/b ratio	Modes	1	2	3	4	5	6	7	8
0.001	Present study	43.9	86.95	89.78	137.46	146.1	153.35	202.87	207.14
	Ngo-cong et al.[6]	46.435	70.615	110.019	115.873	143.115	163.166	184.000	218.970
0.050	Present study	31.63	57.48	58.63	85.162	87.97	92.00	116.35	116.98
	Ngo-cong et al.[6]	35.506	55.366	70.743	84.208	89.932	112.326	117.096	117.614
0.1	Present study	22.74	38.60	39.08	54.79	55.77	59.18	71.67	71.77
	Ngo-cong et al. [6]	24.737	39.112	43.638	57.190	57.254	65.693	74.254	75.149
0.15	Present study	17.36	28.53	28.75	754.1	39.7	43.26	50.82	50.9
	Ngo-cong et al. [6]	18.580	29.222	31.328	41.286	41.855	46.084	53.062	53.211
0.2	Present study	13.9	22.24	22.58	30.88	31.14	33.72	39.17	39.25
	Ngo-cong et al.[6]	14.754	23.093	24.359	32.151	32.686	35.454	41.023	41.079

With an increase in the t/b ratio, natural frequency decreases significantly. The results are also quite comparable. In Table 5.6, the results are presented for another lamination scheme.

TABLE 5.6: Comparison of natural frequencies in Hz for clamped four-ply circular laminated plates with varying thickness

$$\text{Diameter} = 1\text{m}, [0^\circ/-0^\circ/-0^\circ/0^\circ], (\mathbf{w}' = \frac{wb^2}{h} \sqrt{\frac{\rho}{E_2}}), E_1/E_2=40$$

t/b ratio	Modes	1	2	3	4	5	6	7	8
0.001	Present study	48.41	96.67	105.83	139.16	150.21	183.07	188.28	195.4
	Ngo-cong et al.[6]	45.245	56.510	72.646	93.440	118.647	119.399	134.972	146.804
0.050	Present study	32.73	57.76	62.03	77.84	82.28	97.91	98.59	101.56
	Ngo-cong et al. [6]	33.401	42.190	55.649	70.957	73.602	80.909	94.742	95.602
0.1	Present study	25.68	38.72	40.2	50.34	51.80	60.70	62.48	63.42
	Ngo-cong et al. [6]	22.198	29.658	40.987	42.764	50.582	55.075	61.568	65.849
0.15	Present study	18.68	28.42	29.15	36.83	37.7	43.89	44.64	46.08
	Ngo-cong et al. [6]	16.424	23.040	30.528	32.359	37.045	43.081	45.728	45.907
0.2	Present study	14.93	22.7	22.82	29.16	29.71	33.55	34.388	36.67
	Ngo-cong et al. [6]	13.111	18.923	23.801	26.641	29.339	35.1..09	35.246	36.465

As can be observed, the results match quite well. With the increase in the thickness to radius ratio, the natural frequency decreases significantly for various modes.

Next, an isotropic annular plate free on outside and clamped on inside was considered and the frequency was calculated and the results compared with that of Leissa [11]. The non-dimensional frequency is as given below,

$$\lambda^2 = \omega \alpha^2 \sqrt{\frac{\rho}{D}}$$

BOUNDARY CONDITIONS:

Annular Plates Free on Outside and Clamped on Inside

PARAMETERS INVOLVED:

Material= steel

Mass density per unit area of the plate, $\rho = 7850 \text{ kg/m}^3/\text{area}$

Outer radius, $a = 0.5 \text{ m}$

Inner radius, $b = 0.1 \text{ to } 0.8 \text{ of } a$

Thickness, $h = 0.1 \text{ to } 0.8 \text{ of } a$

Young's modulus, $E = 210 \text{ GPA}$

Poisson's ratio, $\mu = .33$

Where D is the flexural rigidity defined by $D = \frac{Eh^3}{12(1-\nu^2)}$

TABLE 5.7: Comparison of natural frequencies for annular isotropic plate with free-clamped condition:

$$\text{Diameter}=1\text{m}, \lambda^2 = \omega\alpha^2 \sqrt{\frac{\rho}{D}} \text{ \& } h = 0.05\text{m}$$

b/a ratio	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
Present study	1.6	1.91	2.24	2.64	3.16	3.94	5.19	7.51
Leissa[11]	2.04	2.25	2.52	2.53	3.35	4.044	5.08	6.96

As b/a ratio increases, frequency of the plate increases. This happens due to the increase in radial stiffness. It can be observed that the results are quite agreeable. The discrepancies may be due to the fact that the thickness was not defined and hence was assumed in the present formulation.

TABLE 5.8: Comparison of natural frequencies in Hz for annular isotropic plate with varying element length of mesh

$$\text{Diameter}=1\text{m}, [45^\circ/-45^\circ/-45^\circ/45^\circ], b/a=0.4, t/a=0.2$$

Modes	1	2	3	4	5	6	7	8
Element length								
.2	282.49	500.77	506.85	721.88	729.19	784.06	936.19	942.17
.15	281.9	502.18	514.2	710.09	729.72	773.31	919.35	937.2
.1	287.61	494.91	499.89	702.55	712.84	759.83	912	921.61
.05	282.16	491.82	498.14	711.02	713.98	762.63	915.56	923.13
.01	277.14	493.31	495.83	702.12	705.45	748.37	910.57	912.43

From Table 5.8, it is seen that there is a large variation in results as the mesh element length is increased; hence this method of meshing is not adopted. Automatic meshing was done in ANSYS using smart meshing tool.

5.3 RESULTS AND DISCUSSIONS

5.3.1 LAMINATED ORTHOTROPIC CIRCULAR PLATE

Natural frequencies for annular four-ply circular laminated plates with clamped free conditions are analyzed using ANSYS software.

TABLE 5.9: Natural frequencies in Hz for an annular laminated composite plate with varying b/a ratio

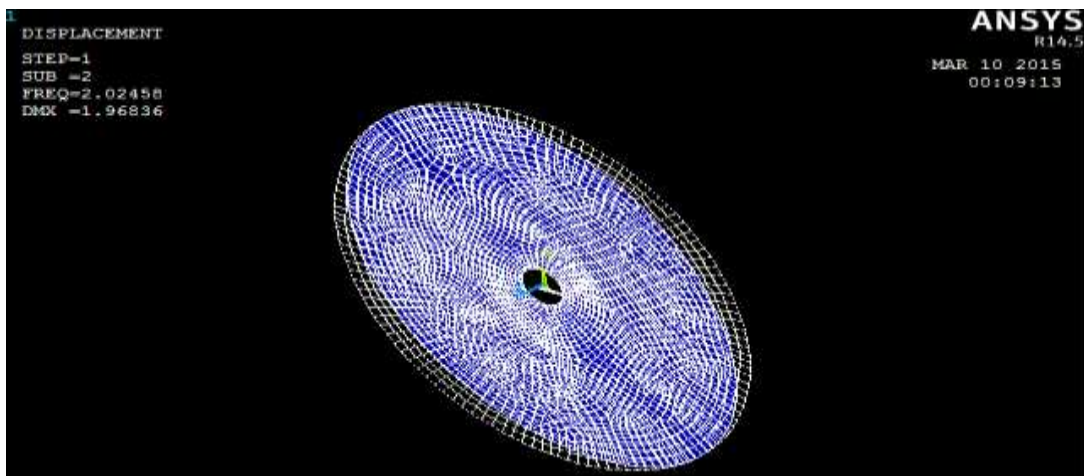
Diameter =1m, $[45^\circ/-45^\circ/-45^\circ/45^\circ]$, $\nu = 0.25$, $E_1/E_2=40$, $t/a=0.2$

Modes	1	2	3	4	5	6	7	8
b/a ratio								
0.1	133.35	148.88	164.28	177.5	260.2	296.58	435.33	492.5
0.2	190.74	192.87	206.4	279.9	296.1	337.6	485.84	495.75
0.3	242.53	244.58	255.08	303.26	315.88	488.22	499.82	501.41
0.4	314.9	319.9	322.6	365.17	382.9	518.12	524.17	700.78
0.5	408.9	411.1	419.48	446.57	468.74	556.87	591.9	740.24
0.6	542.7	557.02	557.6	604.2	610.5	697.54	699.33	817.7
0.7	770.6	780.98	786.9	825.34	825.8	894.15	896.33	989.13
0.8	1210.4	1221.3	1224.1	1254	1256	1306.4	1308.7	1377.6

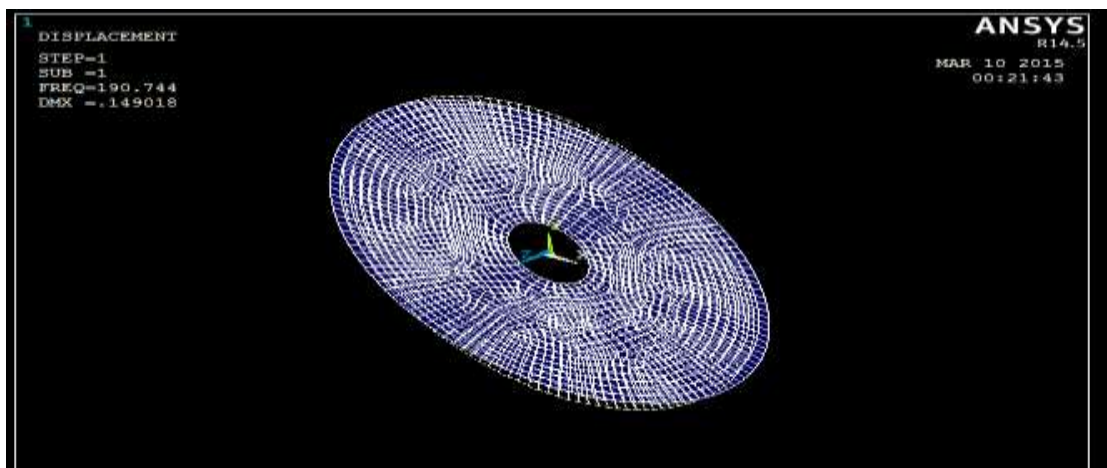
Keeping the outer radius of a circular plate constant if we increase the inner radius, the length of the composite circular plate along the radii decreases and there is an increase in the plate's stiffness. Hence the frequency of the system increases [2].

FIG 5.1 DEFORMED SHAPES FOR VARYING b/a RATIOS (0.1-0.8)

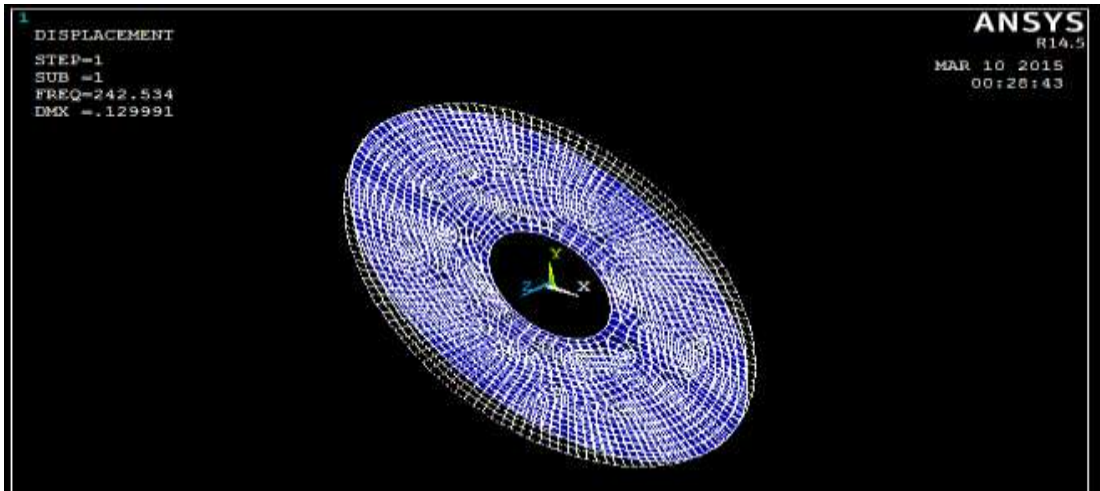
b/a=0.1



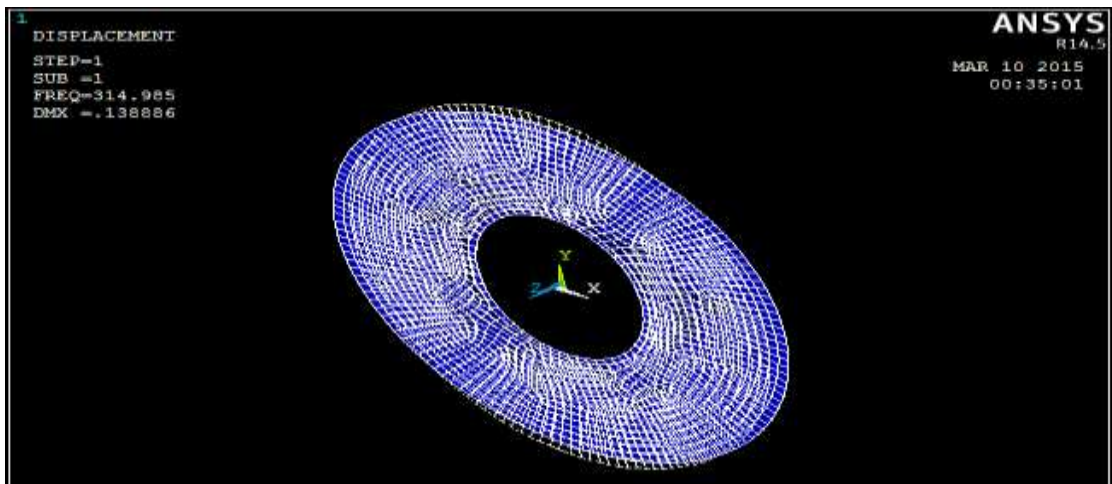
b/a=0.2



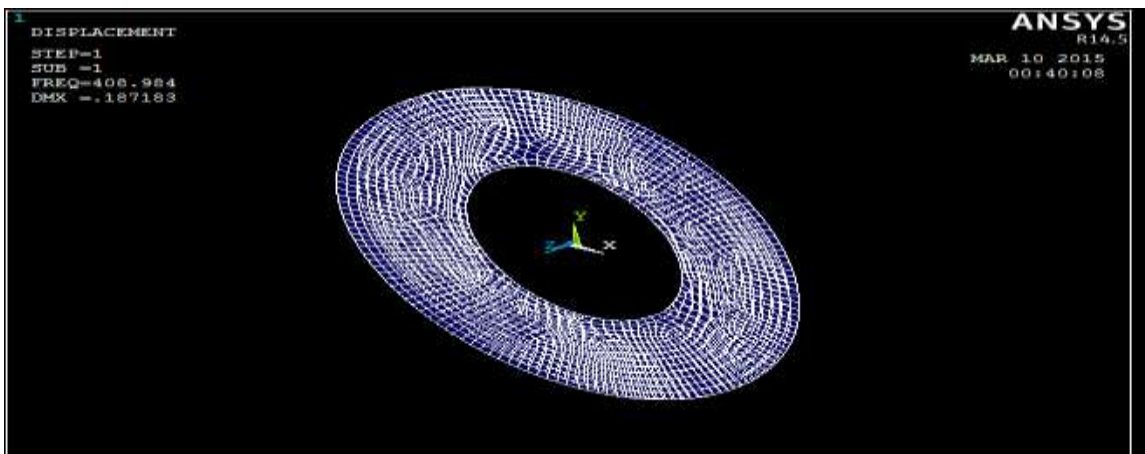
b/a=0.3



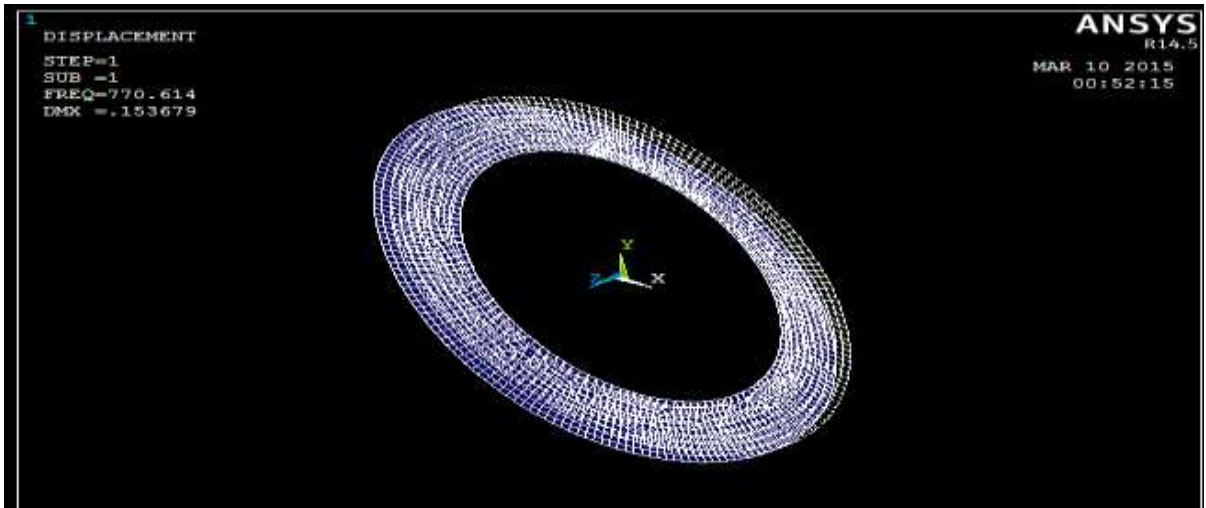
$b/a=0.4$



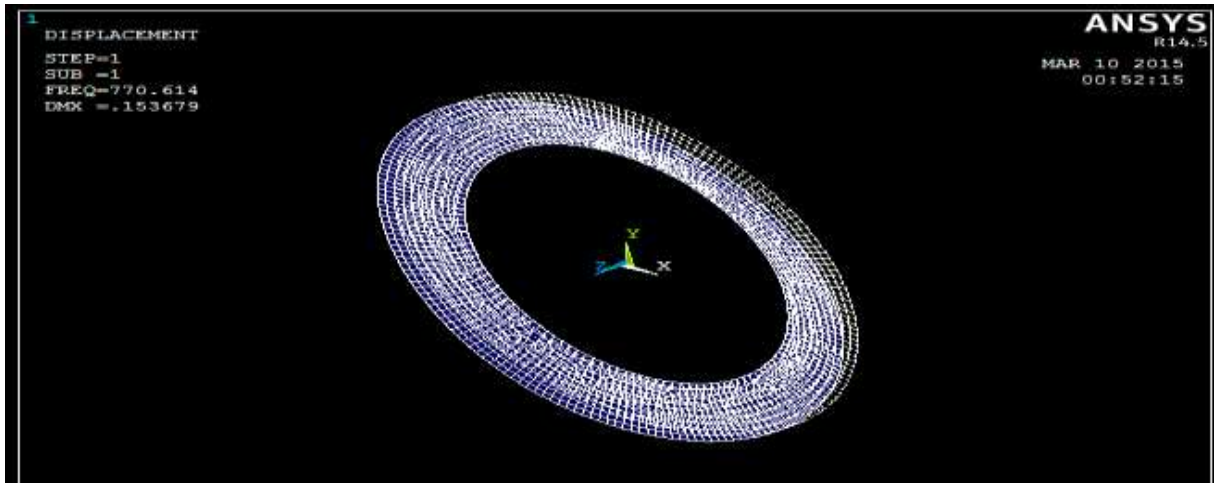
$b/a=0.5$



$b/a=0.6$



$b/a=0.7$



$b/a=0.8$

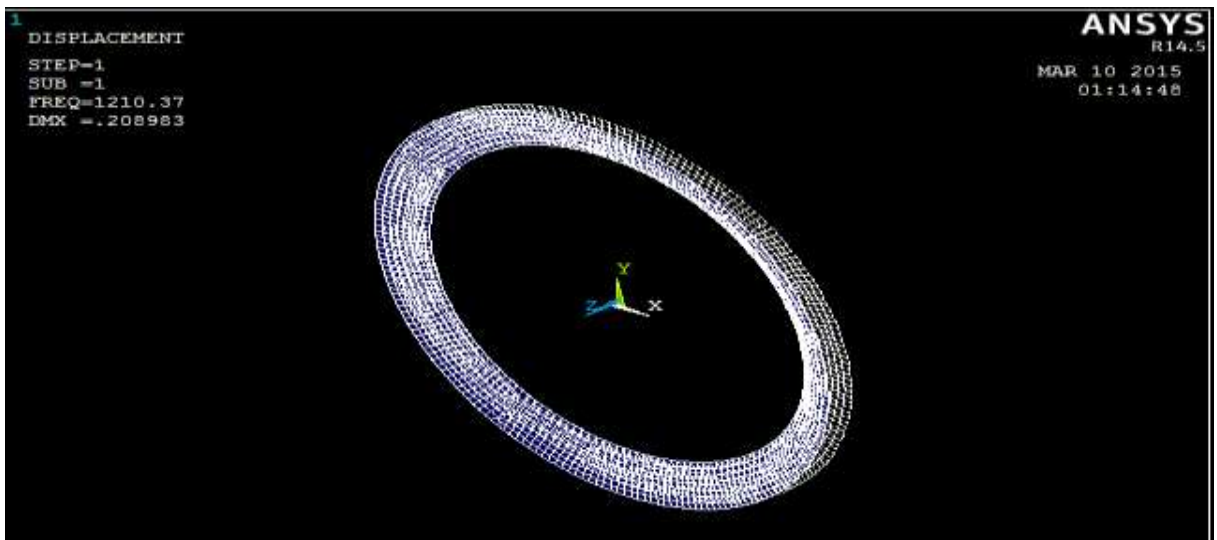


TABLE 5.10: Natural frequencies in Hz for annular laminated composite plate with varying layer orientation

Diameter (a) =1m, t/a =0.2 $E_1/E_2=40$, b/a=0.4

Modes	1	2	3	4	5	6	7	8
Layers								
[0°/-0°/-0°/0°]	64.74	141.4	158.7	167.9	223	240	334.84	360.4
[0°/90°/0°]	74.73	139.6	159.3	165.72	235.1	258.4	409.66	422.9
[0°/90°/90°/0°]	76.6	143.23	165.61	169.8	256.8	286.3	448.92	466.68
[45°/-45°/-45°/45°]	133.35	148.88	164.28	177.5	260.2	296.58	435.33	492.5

The natural frequencies of the annular plate was studied for varying layer orientations. This is shown in Table 5.10. The results for [0°/-0°/-0°/0°] show the lowest frequencies and for [45°/-45°/-45°/45°] show the highest frequencies.

5.3.2 LAMINATED COMPOSITE ANNULAR PLATES WITH HOLES

The natural frequency of an orthotropic laminated circular plate was analysed with inner edge of central hole clamped and various symmetric holes.

TABLE 5.11: Natural frequencies in Hz for annular laminated composite plate with different hole locations

Diameter (a) =1m, $\nu = 0.25$, $E1/E2=40$, $[45^\circ/-45^\circ/-45^\circ/45^\circ]$, $t/a=0.4$

MODES	1	2	3	4	5	6	7	8
TYPES OF HOLES								
ONE CENTRAL HOLE	314.9	319.9	322.6	365.17	382.9	518.12	524.17	700.78
ONE CENTRAL HOLE+ TWO SYMMETRIC HOLES	237.67	242.86	248.93	251.06	266.77	346.19	367.97	503.00
ONE CENTRAL HOLE +FOUR SYMMETRIC HOLES	229.32	235.96	240.70	244.73	272.94	334.25	369.44	495.13

With increase in number of symmetric holes the natural frequency decreases.

FIG 5.2 MODELLING (left) AND DEFORMED SHAPE (right) OF CIRCULAR PLATE WITH ONE CENTRAL HOLE AND TWO ECCENTRIC SYMMETRIC HOLES

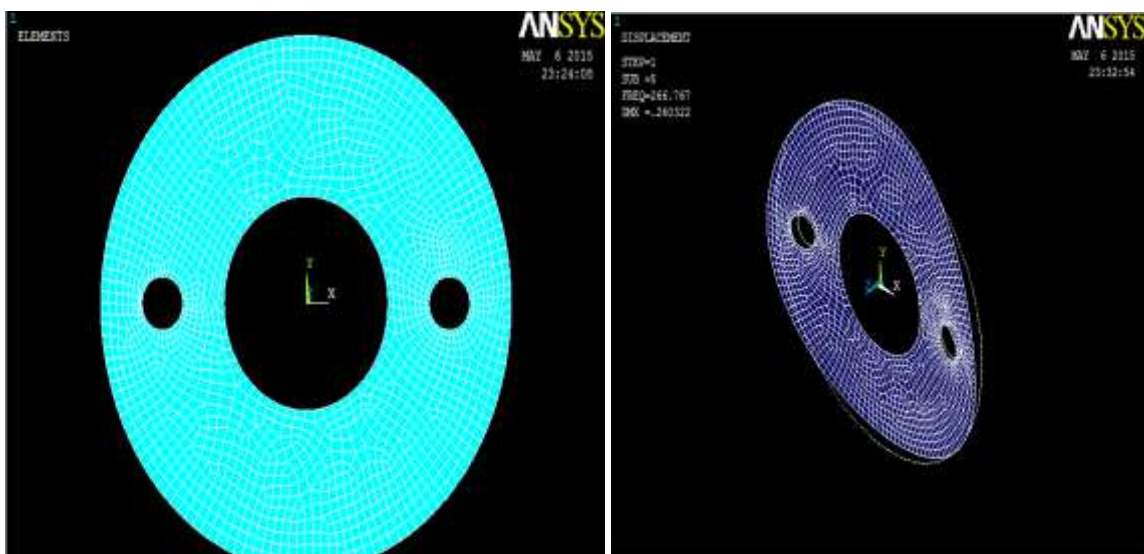
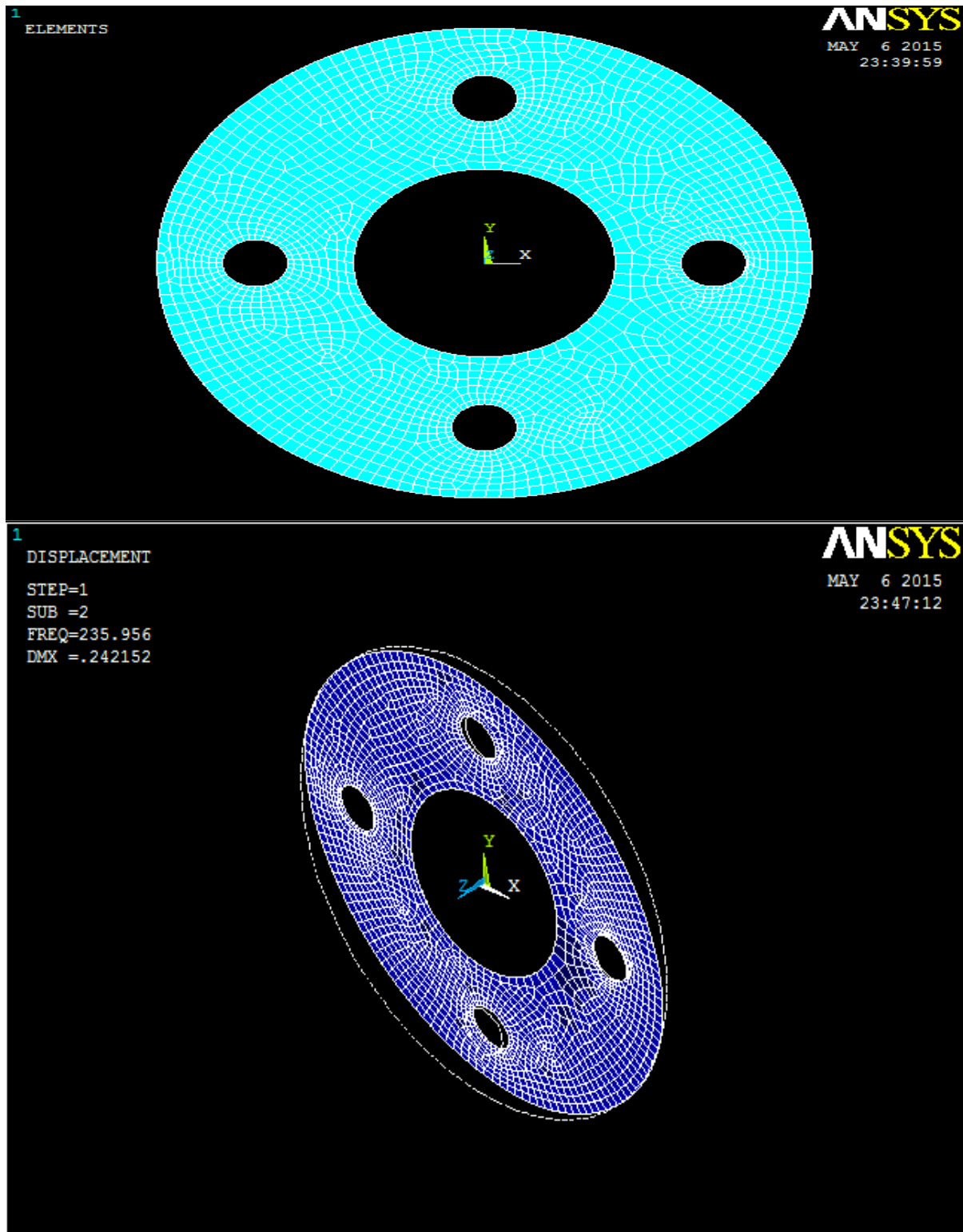
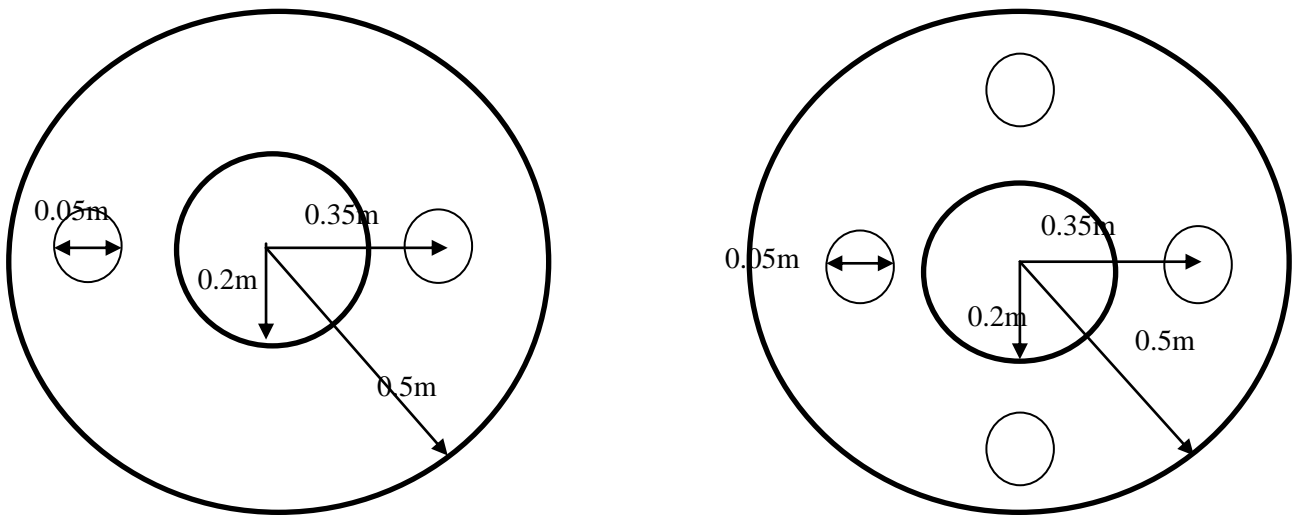


FIG 5.3 MODELLING AND DEFORMED SHAPE OF CIRCULAR PLATE WITH ONE CENTRAL HOLE AND FOUR ECCENTRIC SYMMETRIC HOLES





**FIG 5.4 DIMENSIONS OF CIRCULAR
PLATE WITH 2 AND 4 SYMMETRIC
HOLES AS TAKEN IN TABLE 5.11**

CHAPTER 6

CONCLUSIONS

6.1 CONCLUSION

1. In case of solid clamped four-ply circular laminated plate, the results are quite similar to the research paper [6]. Some variations may be observed due to the assumption of the density. As the diameter of hole increases, the natural frequency increases.
2. From the results we conclude that for a solid plate as radius increases, mass and stiffness increases and hence the natural frequency of the system being directly proportional to the stiffness increases.
3. The natural frequency decreases significantly with increase in the thickness to radius ratio.
4. There is an increase in the natural frequency of the plate as the inner radius to outer radius ratio increases. This happens because of the increase in radial stiffness of the system.
5. The discrepancies in case of an annular isotropic plate with free-clamped condition may be due to the fact that the thickness was not defined and hence was assumed in the present formulation.
6. There is a large variation in results when the mesh element length method is considered for meshing; hence this method of meshing is not suitable to obtain accurate results.
7. Keeping the outer radius of a circular plate constant if we increase the inner radius, the length of the composite circular plate along the radius decreases and there is an increase in the plate's radial stiffness. Hence the frequency of the system increases [2].
8. It is seen that as thickness increases, the natural frequency of the plate increases.
9. With increase in number of symmetric holes the natural frequency decreases. The mass of the system decreases, stiffness decreases and hence natural frequency decreases.

CHAPTER 7

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7.1 REFERENCES

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