Performance Improvement in Multi-user MIMO Networks via Interference Alignment

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Performance Improvement in Multi-user MIMO Networks via Interference Alignment

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Dr. Siddharth Deshmukh

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May 26, 2016

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Supervisor's Certificate

This is to certify that the work presented in the dissertation entitled *Performance Improvement in Multi-user MIMO Networks via Interference Alignment* submitted by *Deexith Rathna*, Roll Number 711EC4108, is a record of original research carried out by him under my supervision and guidance in partial fulfillment of the requirements of the degree of *Masters of Technology* in *Electronics and Communication Engineering.* Neither this thesis nor any part of it has been submitted earlier for any degree or diploma to any institute or university in India or abroad.

Siddharth Deshmukh

Dedication

అమ్మకీ నాన్నకీ ...
గురువుకీ దైవానికీ ...
పుడమికీ పుస్తకానికీ ...
నమస్కరిస్తూ ...

" To Mom and Dad, Teacher and God, Earth and the Book"

Signature

Declaration of Originality

I, *Deexith Rathna*, Roll Number *711EC4108* hereby declare that this dissertation entitled *Performance Improvement in Multi-user MIMO Networks via Interference Alignment* presents my original work carried out as a postgraduate student of NIT Rourkela and, to the best of my knowledge, contains no material previously published or written by another person, nor any material presented by me for the award of any degree or diploma of NIT Rourkela or any other institution. Any contribution made to this research by others, with whom I have worked at NIT Rourkela or elsewhere, is explicitly acknowledged in the dissertation. Works of other authors cited in this dissertation have been duly acknowledged under the sections "Reference" or "Bibliography". I have also submitted my original research records to the scrutiny committee for evaluation of my dissertation.

I am fully aware that in case of any non-compliance detected in future, the Senate of NIT Rourkela may withdraw the degree awarded to me on the basis of the present dissertation.

May 26, 2016
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Abstract

Almost all wireless networks are interference limited. Interference management has been always a primary concern for large section of current wireless networks with exponentially growing devices, lack of centralized medium access, power management. Because of broadcast nature of the wireless channel, all signals from simultaneous transmissions from devices apart in the same space, are added to the desired signal at the receiver end. Therefore optimal spectrum efficiency in such systems mandates distributed, low complexity interference management strategies with very less overhead which should be far more superior than existing successive interference cancellation, highly complex multiuser detection techniques. In this thesis, a novel interference management scheme- "Interference alignment" scheme for multi user scenario is investigated and analysed supporting the arguments with numerical results for most scenarios.

Firstly, the concept of interference channel, Degrees of Freedom were well established which are prerequisite in understanding the predicament of multi user wireless channels. Later on, interference alignment concept has been put forward stating its origin back from linear algebra. IA for K-user MIMO is studied. In a fully connected K-user network with perfect channel state information, IA minimizes the interference space dimension at intended receivers thus maximizing the achievable capacity of the entire channel and increasing the Multiplexing gain.

Later on the idea of IA is extended to multi-hop networks. A practical cellular multi-hop wireless network is considered and distributed interference alignment technique is implemented which shows superior performance even in high interference case. All IA schemes assume that the channels are full rank richly scattered environments which in practise is not always possible. The idea of using relays to act as external scatters which increase the rank of effective channel observed is considered. So two novel distributed relaying schemes have been proposed modifying the existing IA scheme to fit the case for rank deficient channels and still achieve multiplexing gain on par with full rank channels. The proposed algorithms doesn't require global channel state information at all nodes except at relay nodes, doesn't need large symbol extensions, and still are able to enhance the sum capacity of the network.

Keywords: **Interference alignment; Degrees of Freedom; Multi user MIMO networks. Cooperative relaying; Rank deficient channels;**

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Chapter 1

Introduction

In the last decade, phenomenal advancement in Electronics industry made the devices economical and available to very large section of world. State of the art innovation and very less production time in the world of technology enthusiasts led to exponential increase of the high-technology devices such as smart phones, tablets, laptops and Internet of Things (IoT) most of which are portable and rely on Wireless communication. These smart devices provided many different data application services like high definition video services, interactive gaming services, and high quality voice and multimedia services. The requirements for Wireless communication application has been experiencing a tremendous growth and the demand is expected to increase even further is predicted. Huge traffic demands are to be satisfied with the ever increasing density of user. Therefore, service providers have to optimally utilize the limited available spectrum and ambitious to serve as many users as they can with a sufficiently high data rate.

1.1 Motivation

Meeting the expectations in reality is very challenging task due to limitation of available wireless resources such as spectrum allocation, transmit power per user and the characteristics of wireless communication channel. Among these limitations, the most challenging phenomenon which deteriorate the achievable data rates are channel fading characteristics and interference of wireless networks. Extensive research has been done addressing negative impacts of Channel fading and many efficient and excellent techniques have been developed which are currently used in telecommunication industry. For example, with the help of multiple antennas to realize spatial diversity using transmit diversity techniques, receiver diversity techniques, Error Control coding techniques, Frequency-Time interleaving diversity etc., have considerably improved performance over the traditional single antenna techniques. However, the research of dealing with users' interference in wireless environment is not extensive owing to the increase of wireless networks occurred years later the commencement of exploiting radio medium. In wireless networks, inter-user interference is in general far more challenging to tackle compared to fading.

Therefore, recent trend of research shifted in addressing and devising the efficient strategies for interference mitigation problems. In this thesis work, we are primarily focusing on the methods that are used for coping with the second drawback of wireless medium i.e. interference. To be more specific the entire research carried out is focused on exploring Interference Alignment which was very recent novel scheme and has been proved as an excellent technique to achieve the Capacity bounds of various network schemes which remained unsolved till then.

1.2 Objective

The main objective of the research is to increase the performance of multi-user, multiple access MIMO networks in terms of the total Capacity offered by the network and in specific use Interference alignment for such improvement.

- Study Interference Alignment for simple networks, K – User full connected MIMO and Validate the benefits of IA with numerical results.
- Extend the scheme for Multi-hop networks , Cooperative Relay networks
- Extending Interference alignment scheme for rank deficient channels with least possible assumptions and achieve capacity on par with full rank channels.

1.3 Thesis Outline

This section outlines the organisation of remaining chapters of the thesis and summarizes the main contributions.

i. **Chapter 2**
This chapter is a review of results, concepts and definitions which are fundamental for the presentation of the materials in the further chapters. We start our presentation from defining Interference Channel, Capacity and Degrees of Freedom. Next, we review the main topic of discussion, Interference Alignment presenting the various types of it, practical constraints in implementing it, diligent Literature survey dealing with IA and results of Single hop Interference Alignment for Bit Error Rate performance (BER), Sum-capacity of the network for different scenarios. We analyse its performance comparing with conventional orthogonal schemes like Time division multiple access scheme and validate its performance.

ii. **Chapter 3**
This chapter covers extending the IA scheme for multi-hop multi user MIMO networks and provides the sufficient previous research work underwent. The chapter focuses on proving the power efficiency comparison of multi-hop IA schemes over direct link IA for different scenarios considered.

iii. **Chapter 4**

In this chapter, the rank deficient nature of the channel is considered and stating all the possible situations leading to rank deficiency. It also presents Literature survey on the poorly scattered nature of wireless environments and the available relaying schemes to mitigate this effect. This chapter is further divided into two sub section, each section has a unique Cooperative relaying scheme to mitigate the rank deficiency and inter user interference simultaneously.

iv. **Chapter 5**

The last chapter, Conclusion of the thesis presents a brief summary of all the research work done highlighting the major contributions of the work. This chapter further also provides the challenging areas in this topic where further research is required.

Chapter 2

Interference Alignment

2.1 Introduction

Interference alignment is a revolutionary idea that has recently came forth out of the capacity analysis of interference networks. In a very short span, this concept has questioned much of the conventional study about the throughput limits of wired and wireless networks. For example in the wireless interference channel with K transmitter receiver pairs, through IA technique each user can communicate with its intended receiver simultaneously at a data rate equal to half of the Channel capacity when the channel is free from interference irrespective of the number of users involved in the network. However all the path breaking benefits of interference alignment concept are shown under idealised conditions such as global channel state information availability at all nodes, high signal powers and large delays. This radical idea has attracted increasing interest in communication and signal processing research fraternity and produced surprising insights of DoF, the number of available signalling dimensions in wired and wireless media.

Extensive tools from Linear algebra, Diophantine approximation theory, Information theory led to large variety of interference alignment schemes such as spatial alignment scheme, Opportunistic Interference Alignment, Ergodic Alignment scheme, Asymptotic Interference Alignment, Asymmetric complex signal alignment, lattice Interference alignment, Blind Interference alignment and Retrospective Interference Alignment. Owing to advantage of increased throughput of the network brought by IA, it is applicable to diversified network scenarios like wireless interference networks, X-networks, Multi cast, Broadcast networks, Multi-hop networks, Cooperative communication networks, cognitive radio networks etc.

2.2 Literature Review

The idea of interference alignment can be found in literature back in 1998 in paper of Birk and Kol [1] where they addressed indexed coding problem. Later in 2006, the idea was familiarised by Maddah-Ali [2] for specific scenario of MIMO X channel and inherently used for multiple input single output broadcast channel by Weingarten et al. [3]. The idea was further backed by Jafar and Shamai in [4] and its advantages and general method of implementation by Cadambe and Jafar [5] in for arbitrarily large number of users, whose results concluded that interference channel are not limited by interference. From then, it was applied to many network scenarios [6],[7],[8].

Types of Interference alignment schemes

The next section is brief study of the various interference alignment schemes and the insights of previous research work done in that particular schemes.

Interference Alignment over Time/Frequency

When transmitter and receiver antennas do not have multiple antennas, MIMO interference alignment is not possible, therefore to apply interference alignment we have to increase dimensionality at each node by symbol extensions over frequency or time. In simple words, symbol extensions over time or frequency can be interpreted as pre-coding over block diagonal channel realisations by sending symbols over multiple time slots or multiple frequency slots. As capacity analysis of single antenna networks are relatively easy, the earlier research on interference alignment is focused on this kind of networks [9], [10]. In fast fading channel, to better understand the capacity bounds of MIMO networks, pre-coding over multiple instances of time or frequency is done along with pre-coding for multiple antennas. The main setback of this approach is it offers optimal DoF only at asymptotic number of channel usage as number of users in network increases and thus offering very high delays in data transmission far from practical realizations [4] [11]

Ergodic Interference Alignment

Instead of imposing a structure to the interference through using pre-coding filters, one could exploit the fact that in any time-varying channel (with some assumptions on the distribution of the channel values), over a long enough period, one will ultimately encounter a set of desired channel values that produce such a structure. This is the basis for what is called ergodic interference alignment [12] [13] [14]. In contrary to MIMO or time/frequency interference alignment, ergodic interference alignment obtains its DoF at any SNR value but the main drawback is the practically unacceptable delays even when some rate is sacrificed to obtain the desired channel values faster [15] [16]

Blind Interference Alignment

In practice, each user can experience a different physical channel autocorrelation (different coherence time and coherence bandwidth). It is shown in [17] that even without full CSIT, if the transmitters only know these autocorrelation functions as experienced by each user, it is possible to align the interference over multiple time and frequency channel values using multiple transmit antennas. In general, blind interference alignment approaches work on the basis that if a receiver is given multiple linear combinations of its desired signal and interference, by carefully planning the transmissions over multiple time/frequency/spatial dimensions, each receiver can simply subtract the aggregate interference and then resolve the multiple desired messages interference free. Instead of relying on physical autocorrelation values, it is shown in [18] that the necessary channel patterns can be created using staggered antenna switching or reconfigurable antennas. Blind interference alignment techniques depend heavily on predictable channel patterns and the obtained benefits are lost if channel values are assumed to be i.i.d. over all the time/frequency/spatial dimensions . Therefore, when the coherence time and bandwidth of the channels decrease, so does the gains of blind interference alignment. In retrospective interference alignment [19], delayed CSI is used at the transmitters to send a combination of the received desired signals and interferences in previous time slot. This approach is very similar to the blind interference alignment techniques and enables the receivers to eliminate the interference and obtain interference-free linear combinations of potentially multiple desired messages.

Lattice Interference alignment

Lattice alignment harvests the special property of lattice codes that sum of any two lattice points is also a lattice point. Based on this property, lattice alignment was first proposed in [20] In lattice alignment, as the aggregate interference is a valid codeword itself, it can be decoded and subtracted from the received signal to obtain an interference-free desired signal. It is interpretive to note that unlike previously mentioned alignment techniques that relied on the received signal vector properties, in lattice alignment the properties of signal levels in structured codes is exploited. Although signal-level space alignment has been proven to be a very powerful theoretical tool in establishing the DoF characteristics of many networks [11], the extreme sensitivity of lattice codes to channel uncertainties and a different signalling technique than what is widely used in industry restricts its practicality, at least in the near future

Considering the potential of interference alignment and its numerous favours each suited for a different system scenario, recent work has further explored its limitations and applications. As a powerful theoretical tool, interference alignment has been successfully used to characterize the DoF of the MIMO interference channel with time varying/frequency selective channels [5] or constant channel gains , the X channel [8], the MIMO X channel [9], as well as X and MIMO X networks . The DoF of MISO and MIMO broadcast channels with delayed CSIT are considered in [21]. In addition, employing interference alignment in real-world interference networks such as cellular, relay-aided, and ad hoc networks has been an active area of research. Interference alignment for cellular networks is considered in [22]. Transmission techniques for cognitive radios inspired by interference alignment have been proposed in [23]. Finally, extending interference alignment to relay-aided networks is considered in [24].

2.3 Preliminaries

Before discussion on Interference Alignment, some fundamental definitions of performance metrics of channel are stated which are highly essential in establishing the advantages of IA.

2.3.1 Interference Channel

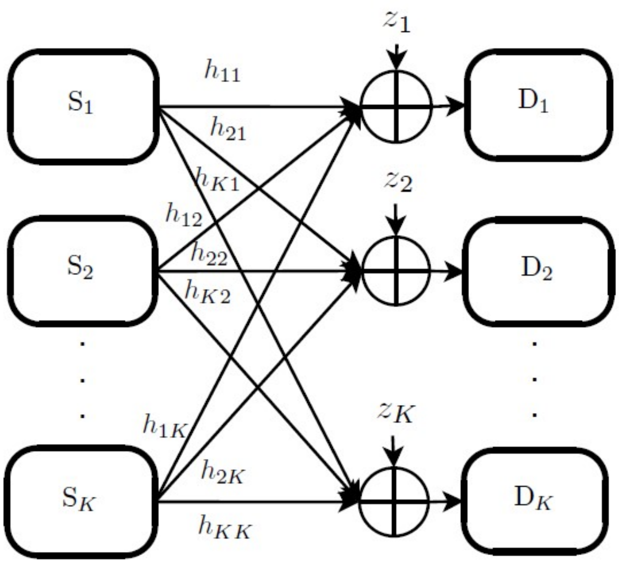

Figure 2.1: K User Interference Channel

Wireless Interference Channel is a network topology to model communication scenario with multiple source and destination nodes. All the nodes share the same transmission medium and each source node expects to communicate with its desired receiver simultaneously. Because of the broadcast character of wireless channel, each destination node also overhears the signals from undesired source nodes. Therefore each destination node receives a noisy combination of the signals from intended and unintended source nodes, weighted by the corresponding channel gains. 2.3 is shows Interference Channel for general K-user scenario where S_k and $D_k(\forall k \in 1, 2, 3, ..., K)$ represents source and destination nodes and $h_{ij}(\forall i, j \in 1, 2, 3, ..., K)$represents the channel gain . This model can be used to study many practical scenarios such as cellular networks, cognitive radio networks etc.

2.3.2 Channel Capacity

Channel Capacity is defined as the maximum rate at which information can be transmitted across a noisy channel with arbitrary small bit error probability.It is the basic quantitative measure of performance of the channel. The notion of Capacity is introduced through a simple example of Additive white Gaussian noise (AWGN) channel through heuristic argument. The AWGN channel model acts as building block in defining and analysing capacity for other wireless scenarios. In a Gaussian channel shown in Fig 2.2, the capacity of the channel depends

Figure 2.2: Gaussian point to point channel

on the additive white Gaussian noise at each receiver node, the signal power available at each transmitter node. The capacity of AWGN channel is given by Shannon's formula.

$$C = \log(1 + SNR) \text{ bits/s/Hz} \tag{2.1}$$

where $SNR = P/\sigma^2$, P is the power constraint at Transmitter node and σ^2 is the noise variance of AWGN channel. There is no single definition of Capacity for fading channel unlike AWGN channels which is applicable to all scenarios. In fast fading environments for a point to point wireless channel , the Channel capacity is given by

$$C = \mathbb{E}[log(1 + |h|^2 SNR)] \text{ bits/s/Hz} \tag{2.2}$$

where h is the channel gain of receiver to transmitter link. There are many other wireless scenarios such as Broadcast(BC) channel, Multiple access channel (MAC), Interference Channel(IC) for which definition of Capacity region is not known in general. The sum capacity of IC is known for specific conditions such as by treating the received interference as noise, for weak interference case. Therefore in recent years of research, there was a strong need of performance metric of channel which exclusively depends on channel topology unlike capacity which depends on many parameters like strength of Interference, noise variance of the channel, Transmitter power.

2.3.3 Degrees of Freedom (DoF)

The Degrees of Freedom in communication networks also known as Pre-log factor or Multiplexing gain is defined as

$$DoF = \lim_{P \to \infty} \frac{C_{\sum}(P)}{\log(P)} \tag{2.3}$$

where $C(P)$ is the sum capacity of the channel with total transmit power P. The DoF metric is mainly concerned with the limit of the channel Capacity as the total transmit power tends to infinity and the noise power remains unchanged. The equation 2.3 can be re written as which is evident why DoF is called pre-log factor

$$C(P) = DoF * \log(P) + o(\log(P)) \tag{2.4}$$

where $o(\log(P))$ is a function in P such that

$$\lim_{P\to\infty} \frac{o(\log(P))}{\log(P)} = 0$$

Next we focus further on the interpretation of DoF for few simple networks.
Consider a Gaussian point to point channel with fading,

$$y = hx + z,$$

where for a channel usage, y is the output symbol,h is the channel gain, x is the input symbol and z is AWGN term. The input is subjected to power constraint $\mathbb{E}[|x|^2] \leq P$ and z is independent and identically distributed (i.i.d) circularly symmetric Gaussian noise with $\mathcal{N}(0, \sigma^2)$. The capacity of this channel as given by Shannon is

$$C = \log(1 + P\frac{|h|^2}{\sigma^2})$$

which induces

$$C = \log(P) + o(\log(P))$$

Therefore this channel has DoF = 1. It is important to notice that channel gain h and noise power σ^2 is irrelevant to DoF metric as they do not scale with P. If we have M parallel AWGN P2P channels, with each channel expressed as

$$y_m = h_m x_m + z_m \; \forall m \in \{1, 2...M\}$$

The power constraint is $\mathbb{E}[|x|^2] \leq P$, it is obvious for the sum capacity of the channel to be stated as

$$C_{\sum} = \sum_{m=1}^{M} \log(1 + P\frac{|h|^2}{\sigma^2})$$

which can be written as

$$C = M\log(P) + o(\log(P))$$

Therefore we have M DoF. Once again the DoF is quantitative measure of number of channels and not the channel strength or the noise power.It is easy to interpret DoF as the number of signalling dimensions per channel usage, where 1 signal dimension corresponds to one interference free Gaussian channel. DoF also known as multiplexing gain as it quantifies the number of signals multiplexed over channel. Thus, DoF can be effectively stated as the multiplexing gain, number of signalling dimensions, or the capacity pre-log factor. From the discussion, it is evident how much DoF metric is fundamental and significant.

2.4 Interference Alignment - Origins from Linear Algebra

Although many of the Interference alignment schemes are presented in complicated manner, but the basic idea and origin of IA lie in elementary linear algebra. The present subsection focuses on explaining the IA scheme using very primal language and advantages of it in a simplified manner.
Consider the following system of linear equations.

$$\begin{aligned} y_1 &= h_{1,1}x_1 + h_{1,2}x_2 + ... + h_{1,K}x_K \\ y_2 &= h_{2,1}x_1 + h_{2,2}x_2 + ... + h_{2,K}x_K \\ &. \\ &. \\ &. \\ y_B &= h_{B,1}x_1 + h_{B,2}x_2 + ... + h_{B,K}x_K \end{aligned} \tag{2.5}$$

Where we have B number of equations, $y_1, y_2, ..., y_B$each is in the form of linear equation with K unknowns $x_1, x_2, ..., x_K$ with $h_{i,j}$are the coefficients. As we are interested in Interference channel, the term can be interpreted in wireless environment as following. The B number of equations , $y_1, y_2, ..., y_B$are observations at receiver with B antennas or B signalling dimensions accessible at receiver, $x_1, x_2, ..., x_K$ are K information symbols from K different users and $h_{i,j}$ is the channel gain from receiver i to transmitter j.

If the equations are unique i.e. if the channel gains are independent and drawn from continuous distribution, all transmitted symbols can be extracted, provided that there are at least as many equations as the unknowns. But in interference networks, each receiver in interested in only subset of entire symbol set and rest of them are unintended i.e. they cause interference. As a specific example, assume the receiver is interested only in symbol x_1 and rest of the symbols are undesired. The fundamental question is how many signalling dimensions are required to extract the symbol x_1 at receiver. Generally, K signalling dimensions are required to resolve the 1 symbol desired by this receiver. Since there are presumably K such receivers, each interested in a different desired symbol, and each have access to a different set of K linear equations dictated by its channel gains from the respective transmitters, each receiver will be able to solve the system of equations and extract its desired symbol. Thus, a total of K signalling dimensions (or Bandwidth of K) are used so that each receiver is able to resolve its desired one dimensional signal. In the analogy of wireless interference networks, this strategy corresponds to the cake cutting interpretation of spectrum allocation — the total number of signalling dimensions, i.e., the total bandwidth, is divided among the K users so that each user is able to use $\frac{1}{K}$ fraction of the channel resource. However, cake cutting spectrum division is not optimal and it is possible to recover the desired symbol even when the no of signalling dimensions are much less than the number of unknown symbols.To understand this, the equations 2.5 observed by receiver can be re written as

$$\boldsymbol{Y} = \boldsymbol{H_{*1}}x_1 + \boldsymbol{H_{*2}}x_2 + ... + \boldsymbol{H_{*K}}x_K \tag{2.6}$$

where

$$\boldsymbol{Y} = \begin{bmatrix} y_1 \\ y_2 \\ . \\ . \\ . \\ y_B \end{bmatrix} \boldsymbol{H_{*k}} = \begin{bmatrix} h_{1,k} \\ h_{2,k} \\ . \\ . \\ . \\ h_{B,k} \end{bmatrix} \tag{2.7}$$

are the received signal vector and the signalling dimension along which the symbol x_k is received respectively. In parallel with Multiple Input Multiple Output (MIMO) literature, the $\boldsymbol{H_{*k}}$ is the received beam direction for the symbol x_k. In order to recover the desired symbol x_1 at receiver from the received signal vector $\boldsymbol{Y}$ is simply that received beam direction of x_1, i.e., $\boldsymbol{H_{*1}}$ should not be inside the space spanned by the interfering beams $\boldsymbol{H_{*2}}$, $\boldsymbol{H_{*3}}$, ...,$\boldsymbol{H_{*K}}$. The receiver can recover the desired symbol x_1 if and only if

$$\boldsymbol{H_{*1}} \notin span(\boldsymbol{H_{*2}}, \boldsymbol{H_{*3}}, ..., \boldsymbol{H_{*K}}.) \tag{2.8}$$

The K $-$ 1 interference beams$\boldsymbol{H_{*2}}, \boldsymbol{H_{*3}}, ..., \boldsymbol{H_{*K}}$ will typically span a vector space of $min(B, K{-}1)$ dimensions. Thus, if B <K, i.e., the number of observation at receiver is smaller than the number of users, the interference will span $min(B, K{-}1) = B$ dimensions. As all the available dimensions B are spanned by undesired signal beams, the desired signal beam will contain inside the interference space as well and therefore cannot be recovered. However, if the interference beams can be merged into a smaller subspace so that they do not cover the entire available signal dimensions and the desired signal beam could avoid to fall in interference space and thus receiver could resolve the desired symbol. This is exactly interference alignment for K-user interference channel.

For sake of understanding, consider following system of equations

$$\begin{aligned} y_1 &= 3x_1 + 2x_2 + 3x_3 + x_4 + 5x_5 \\ y_2 &= 2x_1 + 4x_2 + x_3 - 3x_4 + 5x_5 \\ y_3 &= 4x_1 + 3x_2 + 5x_3 + 2x_4 + 8x_5 \end{aligned}$$

From observation, we can say that $\boldsymbol{H_{*4}} = \boldsymbol{H_{*3}} - \boldsymbol{H_{*2}}, \boldsymbol{H_{*5}} = \boldsymbol{H_{*3}} + \boldsymbol{H_{*2}}$. The received signal vector can be projected along with the interference free dimension to decode the desired symbol x_1. To be clear $\boldsymbol{U} = [17 - 1 - 10]^T$ is orthogonal to both the interference beams, and therefore the projection of Y along U eliminates the interference. $\boldsymbol{U}^T\boldsymbol{Y} \implies 17y_1 - y_2 - 10y_3 = 9x_1$, therefore x_1 is decoded at receiver from the observed equations y_1, y_2, y_3 in the three dimensional signal space In spite of the number of unknowns are 5 > 3.Summing up, interference alignment can be used in a scenario where many interfering users could communicate simultaneously over limited number of signalling dimensions by minimizing the space spanned by the interference to small number of dimensions, while making the desired signal space distinct from interference space so that they can be projected to orthogonal space of interference and thereby extract the desired symbols which are free from interference.

The intended outcome expected from interference alignment is demonstrated using the above example. However to accomplish this objective within constraints mandated by network architecture, the main problem lies in designing linear alignment schemes. The present example is focused on just one receiver that intends to resolve symbol $x1$. Presumably there are other such receivers which simultaneously receive the outputs of their respective channel with in the same spectrum B. i.e., each of the receiver have B linear equations but each interested in different symbol. For example Receiver 4 may intend to extract only symbol $x4$ and the symbols $x1, x2, x3, x5$ cause interference at that receiver. In order for receiver 4 to extract symbol $x4$, the interference beams of $x1, x2, x3$ and $x5$ should be consolidated to two dimensional space and remaining dimension for desired symbol $x4$. Interests of receiver 1 do not conflict with the interests of receiver 4 because the channels observed by the receiver are different i.e., the equation at each receiver are different. This is called relativity of alignment. That is alignment of signal space and interference space are different at each receiver and by careful design of the transmitted symbols, it is possible to satisfy the alignment constraints at all the receivers in the network. Relativity of alignment is the promising phenomenon of wireless networks which enables us to design proper interference alignment schemes.

The main advantage offered by Interference alignment is Sum of Degrees of Freedom of the network per channel usage. If there are K users SISO network, the degrees of freedom offered by IA is $\frac{K}{2}$. In K- user MIMO networks with M- antennas at each user, the SDoF is given by $\frac{KM}{2}$ where the traditional TDMA, FDMA and other multi user schemes could offer us only 1 DoF making the IA scheme by far the most effective scheme for multi user case offering the highest Sum capacity.

Therefore the most critical challenge in this scheme is the design of precoding signal vectors to satisfy the intended alignment constraints. In single hop networks, Nature mandates the behaviour of the channel and interference alignment vectors are to be designed inspite of there is no control over the channel coefficients.

2.5 Interference Alignment for 3 user MIMO network

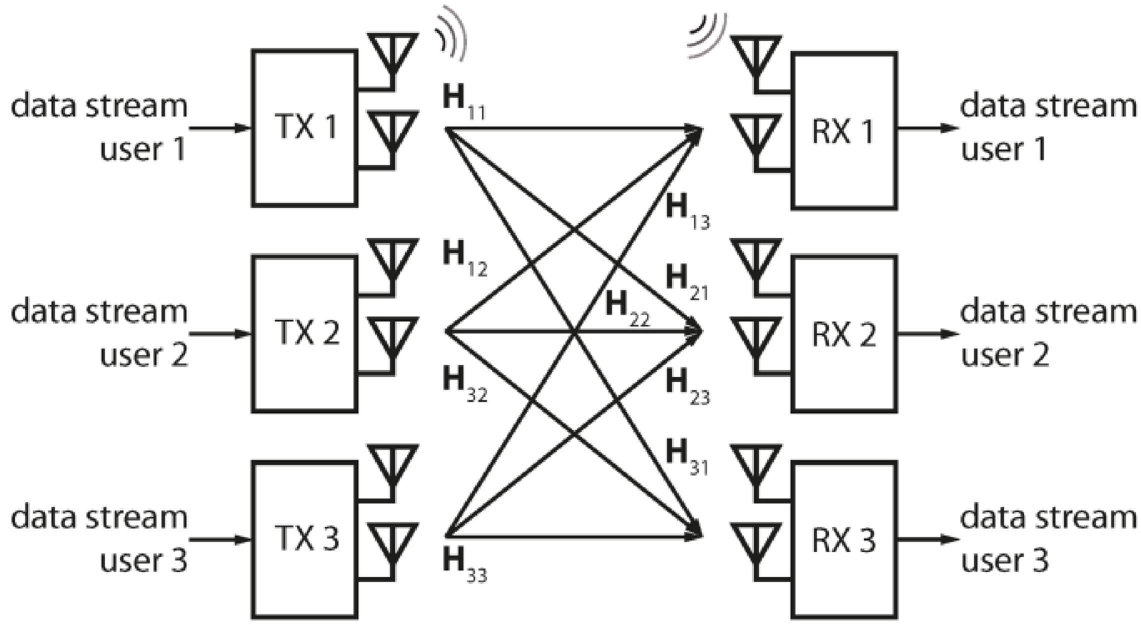

Figure 2.3: 3 User Interference Channel

2.5.1 System Model

Consider a 3 User MIMO network with 2 antennas at each user. User i i.e., Trasmitter TX_i sends message $\boldsymbol{X}^{[i]}[t]$ to Receiver RX_i through $M/2$ independently encoded vector streams.

$$\boldsymbol{X}^{[i]}(t) = \sum_{m=1}^{M/2} s^{[i]}{}_m(t) v^{[i]}{}_m(t) = \boldsymbol{V}^{[i]} \boldsymbol{X}^i(t), i = 1, 2, 3. \tag{2.9}$$

The signal received at receiver RX_i can be written as

$$\boldsymbol{Y}^{[i]}(t) = \boldsymbol{H}^{[i1]}\,\boldsymbol{V}^{[1]} X^1(t) + \boldsymbol{H}^{[i2]}\,\boldsymbol{V}^{[2]} X^2(t) + \boldsymbol{H}^{[i3]}\,\boldsymbol{V}^{[3]} X^3(t) + \boldsymbol{Z}^{[i]}(t), i = 1, 2, 3. \tag{2.10}$$

where $\boldsymbol{H}^{[i1]}$ is the channel observed by RX_i from TX_1 , $V^{[i]}$ is the pre-coding vector at TX_i, $Z^{[i]}(t)$ is additive white Gaussian noise at RX_i with 0 mean and unit variance. The channel fading coefficients are assumed to be i.i.d. and follow Rayleigh distribution. At RX_i, all the symbols from $TX_j j \neq i$ cause interference. The pre-coding ensures us the Interference is spanned into smaller space leaving half of the signalling dimensions free from interference. To resolve the M/2 streams of desired symbols along the pre-coding vector $\boldsymbol{V}^{[i]}$ from the M dimensions of the received vector, the interference space has to be within a space of dimension $M/2$. The following are the alignment conditions for the interference to be consolidated to

$M/2$ dimensions at all receivers and closed form solution for designing pre-coding vectors.

$$\begin{aligned} span(\boldsymbol{H}^{[12]}\boldsymbol{V}^{[2]}) &= span(\boldsymbol{H}^{[13]}\boldsymbol{V}^{[3]}) && (2.11)\\ \boldsymbol{H}^{[21]}\boldsymbol{V}^{[1]} &= \boldsymbol{H}^{[23]}\boldsymbol{V}^{[3]} && (2.12)\\ \boldsymbol{H}^{[31]}\boldsymbol{V}^{[1]} &= \boldsymbol{H}^{[32]}\boldsymbol{V}^{[2]} && (2.13) \end{aligned}$$

where $span(A)$ indicates the vector space formed by column vectors of matrix A. We now have to design $\boldsymbol{V}^{[i]}, i = 1, 2, 3.$ so that the above alignment constraints are satisfied. Presumably $\boldsymbol{H}^{[ij]}, i, j \in \{1, 2, 3\}$ are full rank channels of rank M, the above equations can be re written as

$$\begin{aligned} \boldsymbol{V}^{[1]} &= \boldsymbol{E}\boldsymbol{V}^{[1]} && (2.14)\\ \boldsymbol{V}^{[2]} &= \boldsymbol{F}\boldsymbol{V}^{[1]} && (2.15)\\ \boldsymbol{V}^{[3]} &= \boldsymbol{G}\boldsymbol{V}^{[1]} && (2.16) \end{aligned}$$

where

$$\begin{aligned} \boldsymbol{E} &= (\boldsymbol{H}^{[31]})^{-1}(\boldsymbol{H}^{[32]})(\boldsymbol{H}^{[12]})^{-1}(\boldsymbol{H}^{[13]})(\boldsymbol{H}^{[23]})^{-1}(\boldsymbol{H}^{[21]})\\ \boldsymbol{F} &= (\boldsymbol{H}^{[32]})^{-1}(\boldsymbol{H}^{[31]})\\ \boldsymbol{G} &= (\boldsymbol{H}^{[23]})^{-1}(\boldsymbol{H}^{[21]}) \end{aligned}$$

For example, at RX_1 the columns of $\boldsymbol{H}^{[11]}\boldsymbol{V}^{[1]}$ to be linearly independent of interference $\boldsymbol{H}^{[12]}\boldsymbol{V}^{[2]}$, the rank of the below matrix needs to be a full rank matrix

$$[\boldsymbol{H}^{[11]}\boldsymbol{V}^{[1]}, \boldsymbol{H}^{[12]}\boldsymbol{V}^{[2]}]$$

similarly at other receivers RX_2, RX_3 the following matrices should be of full rank.

$$[\boldsymbol{H}^{[22]}\boldsymbol{V}^{[2]}, \boldsymbol{H}^{[21]}\boldsymbol{V}^{[1]}]$$

$$[\boldsymbol{H}^{[33]}\boldsymbol{V}^{[3]}, \boldsymbol{H}^{[31]}\boldsymbol{V}^{[1]}]$$

The alignment schemes makes sure the above matrices are full rank so that all the receivers RX_1, RX_2, RX_3 can extract the symbols $M/2$ streams of V^1, V^2, V^3 using zero forcing technique. Thus total $\frac{3M}{2}$ interference free transmissions can be done per channel usage and for number of antennas at user M = 2, the Sum DoF by interference alignment scheme is $3{*}2/2 = 3$ i.e., each user is able to transmit one symbol simultaneously.

2.5.2 Simulation Environment

The Montecarlo simulations for the above scenario has been performed in **MATLAB™**till the number of error bits received are 1000 by each user and the transmitted power is varied from 0 dB to 30 dB. The results of IA scheme for 3 User MIMO network with 2 M = 2 has been compared with conventional multi user scheme (Time division multiple access.

2.6 Results and Discussion

For 3 User MIMO with 2 antennas at each node

From the results it is very much evident from Fig. {2.6} that at high SNR, Interference alignment technique gives 3 Dof per channel usage where as conventional scheme gives us only 2 DoF. The advantage of using Interference alignment can also be clearly seen from Sum Capacity curve Fig {2.5} where IA Capacity overthrows the Capacity offered by tdma scheme.

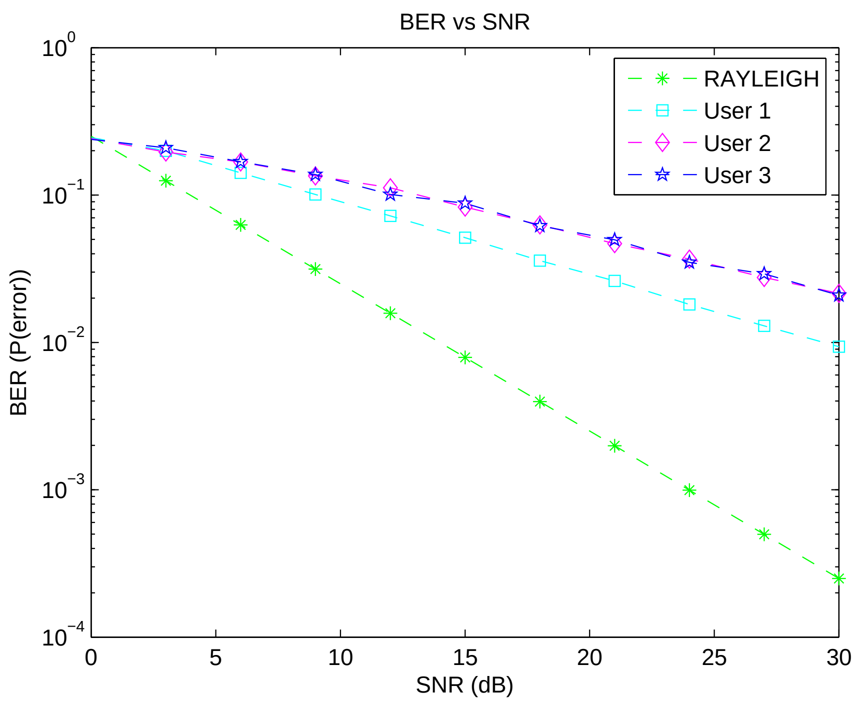

Figure 2.4: Bit error rate curve vs SNR for $(2 \times 2, 1)^3$ system

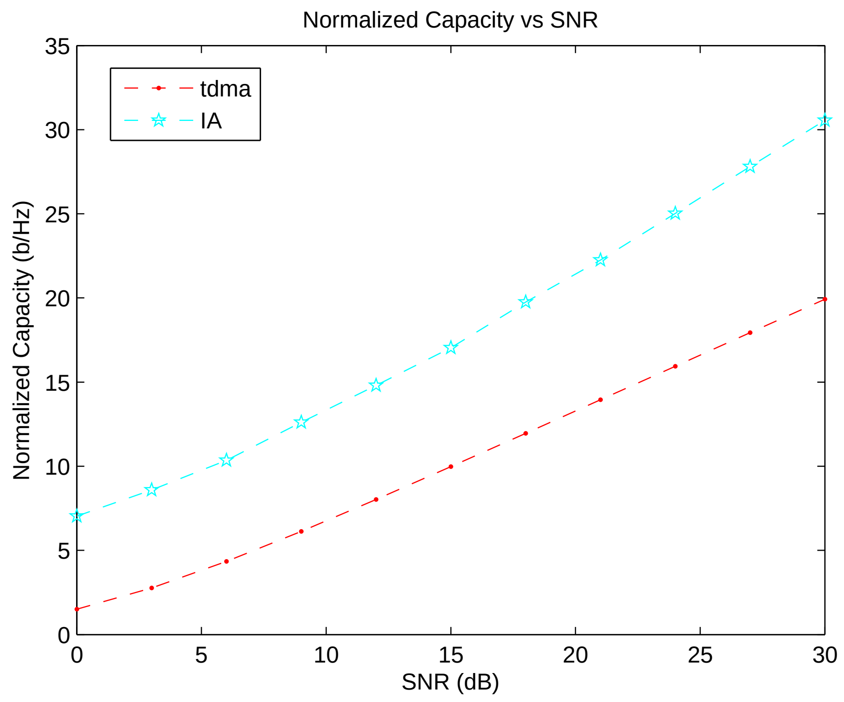

Figure 2.5: Sum Capcity of the network vs SNR for $(2 \times 2, 1)^3$ system

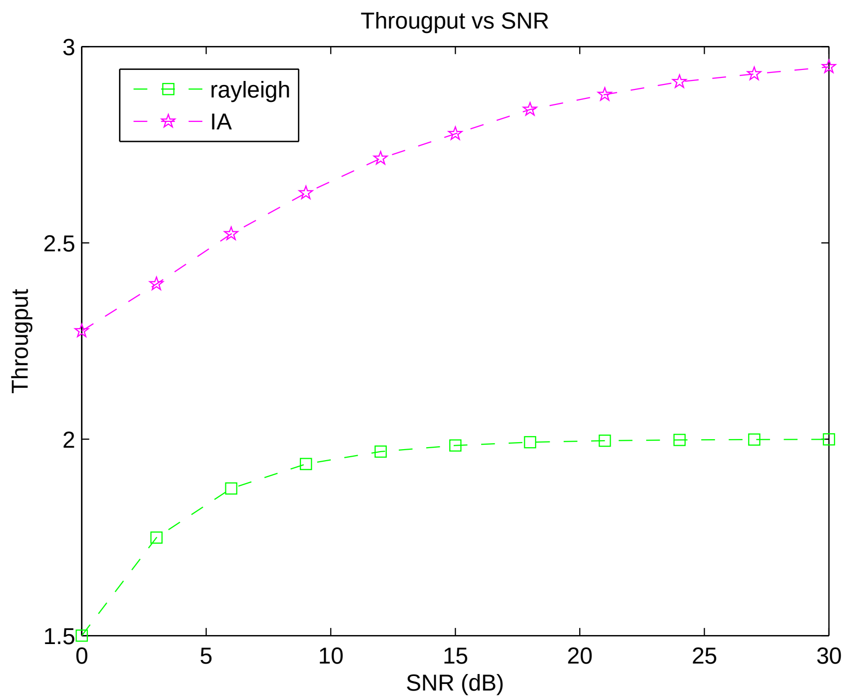

Figure 2.6: Throughput vs SNR for $(2 \times 2, 1)^3$ system

For 3 User SISO network

Given below is 3 User Single Input and Single output network for SDoF given by IA scheme is 4/3. The results have been verified for the same simulation environment as stated above. From

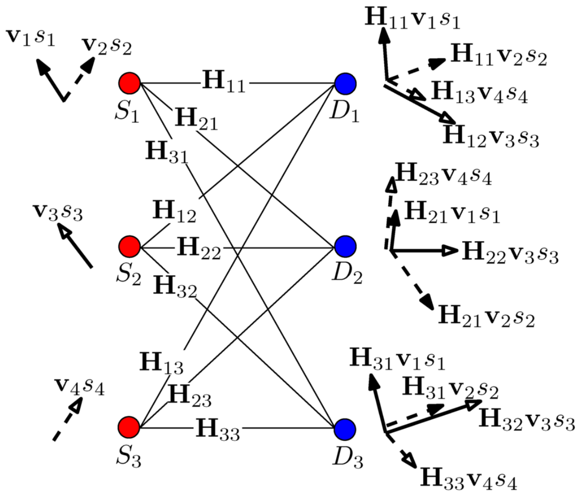

Figure 2.7: 3 User SISO Network and the manner in which desired and interference beams are aligned

the results we can observe from Fig. {2.6} that at high SNR, Interference alignment technique tends to 1.333 $\Longrightarrow$ 4/3 Dof per channel usage where as conventional scheme gives us only 1 DoF. The advantage of using Interference alignment can also be clearly seen from Sum Capacity curve Fig {} where IA Capacity overthrows the Capacity offered by tdma scheme.

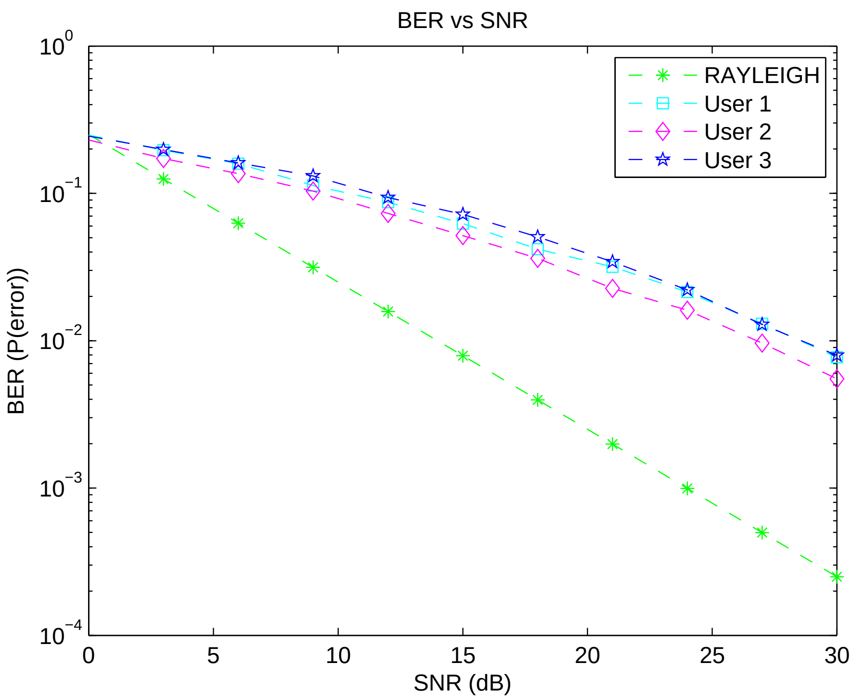

Figure 2.8: Bit error rate vs SNR for 3 user SISO network

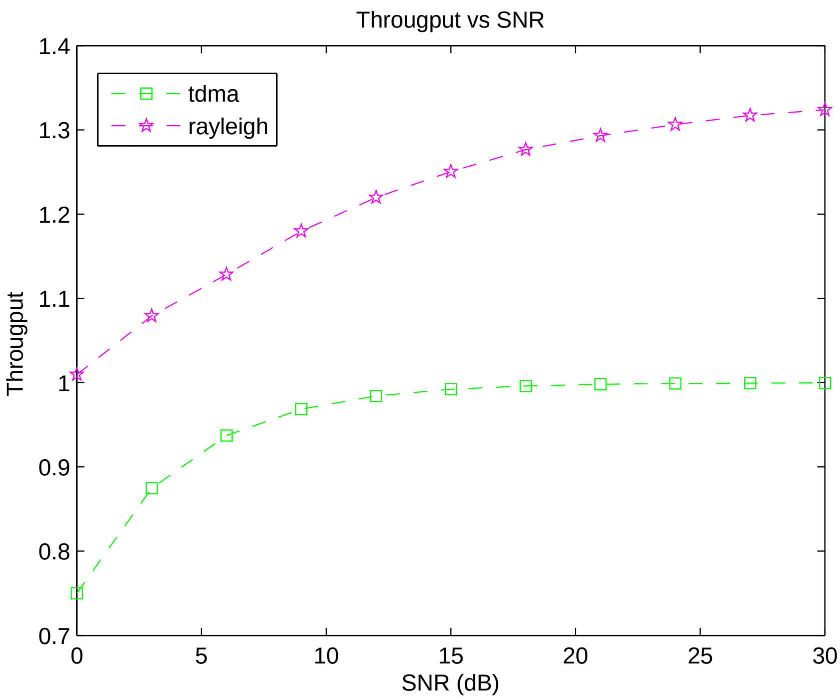

Figure 2.9: Througput vs SNR for 3 user SISO network

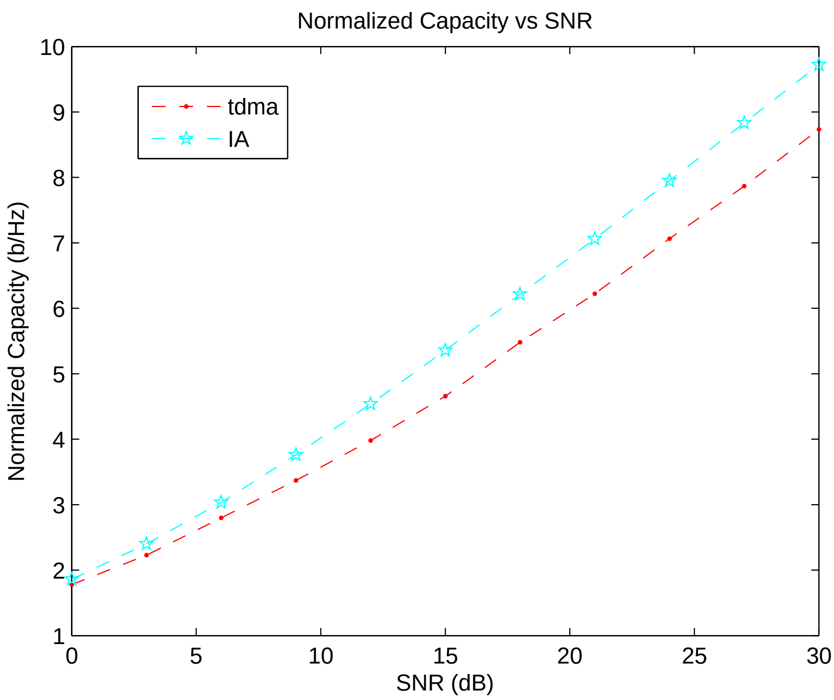

Figure 2.10: Capacity vs SNR for 3 user SISO network

2.7 Conclusion

From the numerical results, we can clearly infer that Interference Alignment is far more superior than the conventional multiple access schemes in terms of DoF and offers $KM/2$ DoF in K-user MIMO case. Owing to its computational complexity as increase in number of users in the network, there is strong need to extend or find an alternate way in developing a distributed alignment scheme rather than centralized manner. Also, the scope to extend Interference alignment scheme for multi-hop networks. The next chapter deals in analysing the Distributed multi-hop alignment schemes and validate the numerical results with simulations.

Chapter 3
Multihop Interference Alignment

3.1 Introduction

Interference alignment is considered as the most path breaking advancement in Information theory and Coding in the recent years of research [5]. IA in short is design of signals at source nodes so that they overlap at undesired destinations while they remain resolvable at desired destination node. From the previous chapter we have seen each user in K-user interference channel using IA can be able achieve $\frac{1}{2}$ of the interference capacity irrespective of the number of users in the network. That is it could scale the sum capacity of the network with the factor $\frac{K}{2}$ and in MIMO IC where all nodes are equipped with M antennas, $\frac{KM}{2}$ DoF could be achieved by IA with the help of symbol extensions. But the use of symbol extensions is not always feasible in practise owing to large delays in the network and the computational complexity involved. One possible way to address this drawback is to use relay which opens the doors for multi-hop Interference alignment.

Although there is much advance in information theory, major portion of the results on IA are concentrated only on point to point, single hop MIMO networks and there is clear lack in the progress with respect to Multi-hop networks. The main hindrance is due to complicated interference space inherently involved in multi-hop environments.

3.2 Literature Review

In spite of showing phenomenal enhancement in Sum capacity of the network, All the IA optimal techniques proposed are entangled with huge computational complexity and large delays owing to multiple orthogonal time slots involved. Recently, relays have been employed to address this problem [25]. Not only relays help the direct link communication assisting transmitter-receiver nodes via relaying, they have also been proved to mitigate interference at receiver by innovative approach called interference forwarding. However, even this technique require successive interference cancellation (SIC) which is again brings considerable complexity to the system as a whole.

This problem has been investigated considering with and without the direct link communication. In [26], it was showed that with the direct links present, using amplify forward relaying scheme can significantly reduce the number of symbol extensions, but does not contribute to any increase in DoF. In the absence of direct links, a network 2 x 2 x 2 with 2 sources, 2 relays and 2 destinations with single antennas at all nodes is analysed for theoretic bounds of SDoF in [27], results were extended for multi antenna scenario in [28]. In, study on DoF region for more than 2 user pairs in the scenario of multi hop interference channel. In [29], a K x K x K full connected network was considered and with the help of real interference alignment, proved theoretically that K DoF could be achievable. However, the tolerance of precision needed for channel coefficients in real interference alignment scheme is not practically feasible. Therefore, K DoF is not achievable in reality. There was also parallel branch of study investigating the possibility of optimal design of beam-forming vectors at source and relay which could enhance the sum rate capacity of the network and at the same time minimize the total transmit power of the network per channel usage.

3.3 Dual-hop Interference alignment in Multi node Network

A Distributed Interference alignment scheme is investigated to mitigate the effect of interference in multi node source – destination network. All the destination nodes (Di) do not use SIC and all base stations BS do not share information and hence cannot perform interference alignment in the direct link communication. Therefore relay nodes are used to implement such alignment at destination nodes in a distributed way. Half duplex decode and forward MIMO relaying scheme is considered and it is been shown that amalgamation of desired and interfering signals collected over two slots can be aligned such a way that on zero forcing the composite signal, interference component is nullified completely. Moreover, the pre-coding matrix at relay to perform this operation can be obtained in a closed form. For comparison purpose, the investigated scheme is compared with a benchmark scheme in which the relay doesn't have channel state information (CSI) and hence forward the message with a predetermined matrix at relay. Numerical results supporting the investigated method for outperforming the benchmark scheme have been presented for different relay power.

3.3.1 System Model

A two cell downlink framework is considered where the Base station BS_i for $i \in \{1, 2\}$ in each cell is to transmit the symbols with their assigned user equipment UE_i for $i \in \{1, 2\}$. We consider the scenario that the adjacent cells are operating the entire communication process within the same resources viz. time and frequency, the BS and UE s are equipped with one antenna each. . BS_i wishes to transmit a message M_i drawn uniformly from the set $[1; 2^{nR_i}]$ to its desired receiver UE_i, where n represents the number of usages of channel and R_i denotes the achievable rate. The relay node used to assisting both the UE_i and BS_i s is an in-band half duplex relay with two antennas operating in the same spectrum allocation as of BS_i and UE_i. In the times slot 1 as $[0 - T_0]$, the channel is assumed to be Rayleigh fading channel with

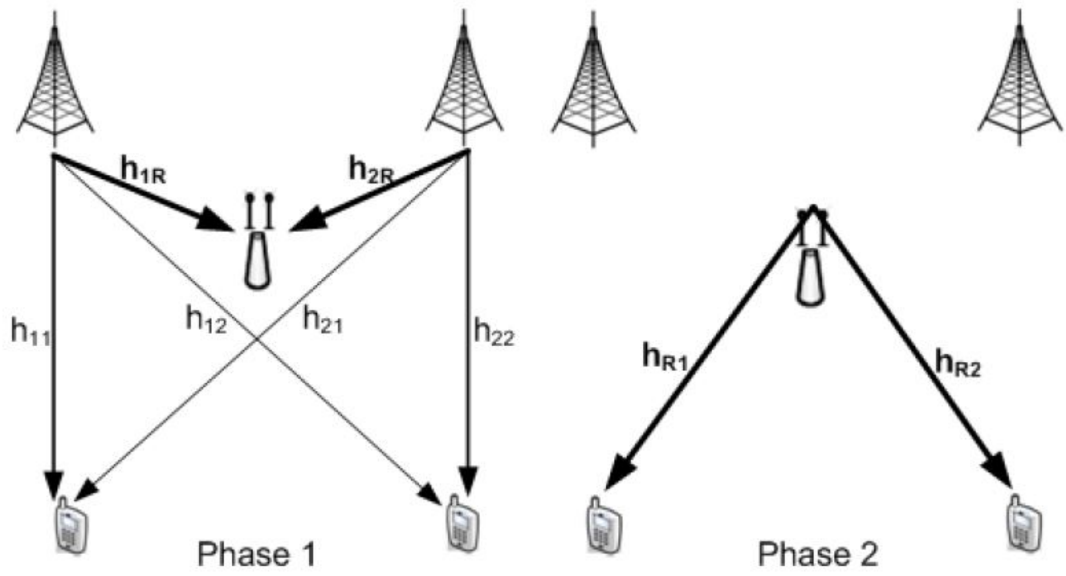

Figure 3.1: System Model - Dual-hop Distributed interference alignment

AWGN noise, and the received signals at relay and user equipment are as follows

$$\boldsymbol{Y_R} = \boldsymbol{h_{1R}X_1} + \boldsymbol{h_{2R}X_2} + \boldsymbol{Z_R} \tag{3.1}$$

$$Y_1 = h_{11}X_1 + h_{12}X_2 + Z_1 \tag{3.2}$$

$$Y_2 = h_{21}X_1 + h_{22}X_2 + Z_2 \tag{3.3}$$

Where X_i for $i \in 1, 2$ is the symbol to be transmitted from base station BS_i to UE_i within the power constraints $\mathbb{E}[|X_i|^2] <= P_i$ and $Z^{[i]}$ is additive white Gaussian noise at UE_i with 0 mean and unit variance. The channel fading coefficients are assumed to be i.i.d. and follow Rayleigh distribution. Here $\boldsymbol{h_{1R}} = [h_{1R,1}, h_{1R,2}]$ and $\boldsymbol{h_{2R}} = [h_{2R,1}, h_{2R,2}]$ are the channel matrices from BS_1, BS_2 to relay node and h_{ij} for $i, j \in \{1, 2\}$ is the channel gain from UE_i to BS_j.

In the time slot 2 i.e., $[T_0 - T]$, BS s are inactive and only the relay transmits the message $\boldsymbol{X}_R$ to both the UEs and the signals received by UEs are as follows

$$Y_1^{'} = \boldsymbol{h_{R1}X_R} + Z_1^{'} \tag{3.4}$$

$$Y_2^{'} = \boldsymbol{h_{R2}X_R} + Z_2^{'} \tag{3.5}$$

Where $Z^{[i]}$ is additive white Gaussian noise at UE_i with 0 mean and unit variance in the second time slot. The channel fading coefficients are assumed to be i.i.d. and follow Rayleigh distribution. Here $\boldsymbol{h_{R1}} = [h_{R1,1}, h_{R1,2}]$ and $\boldsymbol{h_{R2}} = [h_{R2,1}, h_{R2,2}]$ are the channel matrices from relay node to UE_1, UE_2.

The relay sends message $\boldsymbol{X}_R$ to UE_i within the power constraints $tr\{\mathbb{E}[\boldsymbol{X_R X_R^*}]\} <= P_R$.For the entire discussion, $T_0 = T/2$ is assumed.Unlike other IA schemes, this scheme requires only global channel state information at relay node.

3.3.2 Distributed Interference Alignment scheme

In the investigated scheme, the BS completes the transmission independently without any coordination in the first time slot and therefore transmitted signals interfere with each other at all nodes in the network. Relay also receive a copy of the signal from both the base stations due to the broadcast nature of the transmission channel. The first phase of the communication between Base station and relays can be interpreted as multiple access channel where relay needs to decode the message. Relay has to perform some mathematical transformation over the received signal in time slot 1 such that desired and undesired signals can be separated at respective receivers at the end of second phase of communication initiated by relay to user equipment. The pre-coding matrix is synthesized in such a way that on zero forcing the aggregated signal received by UE over two time slots which is a composite of desired and interfering signals, the desired signals are aligned at respective receivers and interference is completely eliminated.

In the first time slot $[0, T_0]$, relay successfully decode the messages transmitted by BS and then applies a mathematical transformation to the conjugated of the decoded message, then the relay transmits the transformed signal in the second time slot $[T_0, T]$ and the transmitted message is

$$\boldsymbol{X_R} = \begin{bmatrix} t_{11}X_1^* + t_{12}X_2^* \\ t_{21}X_1^* + t_{22}X_2^* \end{bmatrix} \tag{3.6}$$

where $T = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix}$ is the mathematical transformation performed at relay. The received signals at User equipment UE_i in second time slot in represented as $Y_1^{'}$, $Y_2^{'}$ and written as follows

$$\begin{aligned} Y_1^{'} &= (h_{R1,1}t_{11} + h_{R1,2}t_{21})X_1^* + (h_{R1,1}t_{12} + h_{R1,2}t_{22})X_2^* + Z_1^{'} && (3.7) \\ Y_2^{'} &= (h_{R2,1}t_{11} + h_{R2,2}t_{21})X_1^* + (h_{R2,1}t_{12} + h_{R2,2}t_{22})X_2^* + Z_2^{'} && (3.8) \end{aligned}$$

Over the duration of two time slots, destination nodes receive signals from BS s as shown in equations (3.1 - 3.3) and from relay in (3.4 - 3.5). The motive is to design a mathematical transformation T such that the combination of signals over two time slots at destinations eliminate interference completely. In order to satisfy above requirement, the pre-coding matrix at relay should satisfy the following condition

$$\begin{bmatrix} h_{R1,1} & h_{R1,2} \\ h_{R2,1} & h_{R2,2} \end{bmatrix} \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{t22} \end{bmatrix} = k \begin{bmatrix} h_{21} & -h_{11} \\ h_{22} & h_{12} \end{bmatrix} \tag{3.9}$$

where is the variable intended to satisfy the power constraint. The received signals at UE s in second time slot can be written as

$$Y_1^{'} = kt_{21}X_1^* - kh_{11}X_2^* + Z_1^{'} \quad (3.10)$$

$$Y_2^{'} = kh_{22}X_1^* - kh_{12}X_2^* + Z_2^{'} \quad (3.11)$$

On combination of equations (3.2), (3.3), (3.10), (3.11), the composite signal received in two time slots can be written as

$$\begin{bmatrix} Y_1 \\ Y_1^{'} \end{bmatrix} = \begin{bmatrix} X_1 & X_2 \\ -kX_2^* & kX_1^* \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{21} \end{bmatrix} + \begin{bmatrix} Z_1 \\ Z_1^{'} \end{bmatrix} \quad (3.12)$$

$$\begin{bmatrix} Y_2 \\ Y_2^{'} \end{bmatrix} = \begin{bmatrix} X_1 & X_2 \\ -kX_2^* & kX_1^* \end{bmatrix} \begin{bmatrix} h_{12} \\ h_{22} \end{bmatrix} + \begin{bmatrix} Z_2 \\ Z_2^{'} \end{bmatrix} \quad (3.13)$$

Detection of Symbols at UEs

Before using transmit filter over the overall signal, the UEs apply conjugate operation on the signal received in second time slot and therefore the combined signal for two time slots can be written as

$$\begin{bmatrix} Y_1 \\ Y_1^{'*} \end{bmatrix} = \begin{bmatrix} h_{11} & h_{21} \\ kh_{21}^* & -kh_{11}^* \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} Z_1 \\ Z_1^{'*} \end{bmatrix} \quad (3.14)$$

$$\begin{bmatrix} Y_2 \\ Y_2^{'*} \end{bmatrix} = \begin{bmatrix} h_{12} & h_{22} \\ kh_{22}^* & -kh_{12}^* \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} Z_2 \\ Z_2^{'*} \end{bmatrix} \quad (3.15)$$

From the above equations X_1 and X_2 can be extracted at UE_1 and UE_2 respectively by performing following operations

$$\begin{bmatrix} kh_{11}^* & h_{21} \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_1^{'*} \end{bmatrix} = k(|h_{11}|^2 + |h_{21}|^2)X_1 + kh_{11}^*Z_1 + h_{21}Z_1^* \quad (3.16)$$

$$\begin{bmatrix} kh_{22}^* & -h_{12} \end{bmatrix} \begin{bmatrix} Y_2 \\ Y_2^{'*} \end{bmatrix} = k(|h_{22}|^2 + |h_{12}|^2)X_2 + kh_{11}^*Z_2 + h_{21}Z_2^* \quad (3.17)$$

From the above equations it can be observed that interfering signals are eliminated completely at both the receivers and only desired signal component and noise component are present after applying filtering operation at receiver.

The achievable throughput for the channel can be written as follows

$$R_1^{UE}(k) \leq 0.5 * \log\left(1 + \frac{|k|^2(|h_{11}|^2 + |h_{21}|^2)P_1}{|kh_{11}|^2 + |h_{21}|^2}\right) \quad (3.18)$$

$$R_2^{UE}(k) \leq 0.5 * \log\left(1 + \frac{|k|^2(|h_{22}|^2 + |h_{12}|^2)P_2}{|kh_{22}|^2 + |h_{12}|^2}\right) \quad (3.19)$$

The objective here is to maximize the sum rate of the network subject to the power spent at relay. While Sum-rate maximisation is considered, the first constraint arises due to the power constraint at relay given by

$$(|t_{11}|^2P_1 + |t_{12}|^2P_2 + |t_{21}|^2P_1 + |t_{22}|^2P_2) \leq P_R \tag{3.20}$$

and the second constraint is due to innate nature of transformation to eliminate interference

$$\begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} = k \begin{bmatrix} h_{R1,1} & h_{R1,2} \\ h_{R2,1} & h_{R2,2} \end{bmatrix}^{-1} \begin{bmatrix} h_{21} & -h_{11} \\ h_{22} & -h_{12} \end{bmatrix} \tag{3.21}$$

We can obtain a closed form solution for the pre-coding matrix satisfying the above constraints. From equations (3.20) and (3.21), we can express the elements of transformation matrix T as function of k and use those in equation (3.20) to obtain the optimal value of k which will maximise the sum rate capacity of the network.

3.3.3 Conventional Scheme: Standard relaying

In this scheme, we assume that relay is not equipped with global channel state information and relies on a fixed pre-coding matrix and not the optimized one. Therefore a fixed pre coding matrix is applied at relay

$$T_{SR} = 0.5 \begin{bmatrix} \sqrt{\frac{P_R}{P_1}} & \sqrt{\frac{P_R}{P_1}} \\ \sqrt{\frac{P_R}{P_2}} & \sqrt{\frac{P_R}{P_2}} \end{bmatrix} \tag{3.22}$$

3.3.4 Simulation Environment

The Monte Carlo simulations for the above scenario has been performed in **MATLAB ™** till the number of error bits received are 1000 by each user and the relay power is varied for P_R = 1, 10, 100 units of Power. The results of investigated scheme for 2 User downlink cellular network has been compared with Standard relaying scheme and the channel considered for the simulations is Rayleigh fading channel with $h_{ij} = \sqrt{d_{ij}^{-\alpha}}A_{ij}, i, j \in \{1, 2\}$.

The value for factor α is taken 4 and a symmetric system is considered with distances all the nodes is taken as stated below.

$$\begin{aligned} d_{11} &= d_{22} &= 1 \\ d_{1R,1} &= d_{2R,2} &= 0.5 \\ d_{1R,2} &= d_{2R,1} &= 0.5 \\ d_{R1,1} &= d_{R2,2} &= 0.5 \\ d_{R1,2} &= d_{R2,1} &= 0.7 \\ P_1 &= P_2 &= 10 \end{aligned}$$

3.4 Results and Discussion

From the results it is straight forward that the novel relaying scheme of interference alignment outperforms the standard relaying scheme. This is due to the fact that Relay exploits the global channel state information present in aligning the interference orthogonal to desired signal subspace. From Figure (3.3),Using this scheme we could also establish the fact that Sum rate of network increases even with the increase in amplitude of interference signal as long as there is no limitation on relay power.

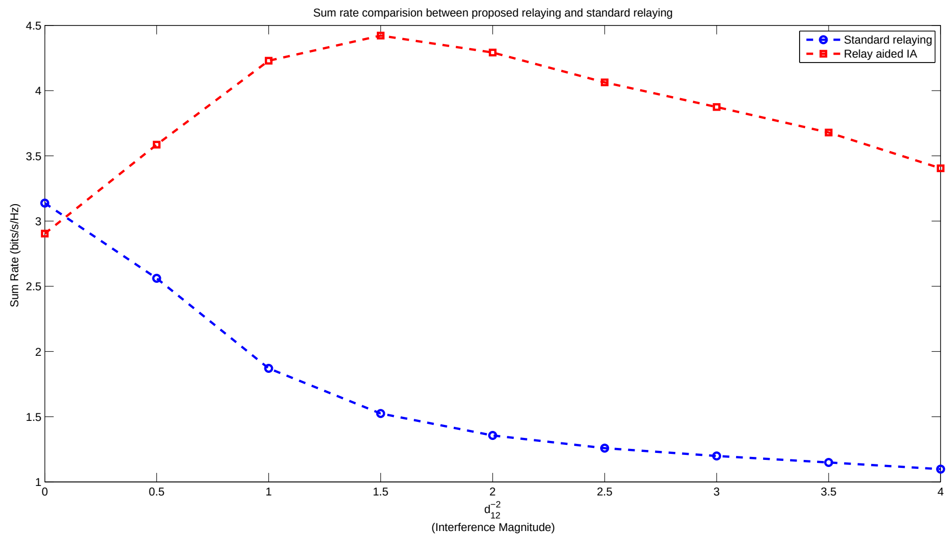

Figure 3.2: Comparison of Distributed interference alignment vs Standard Relaying for relay power 10

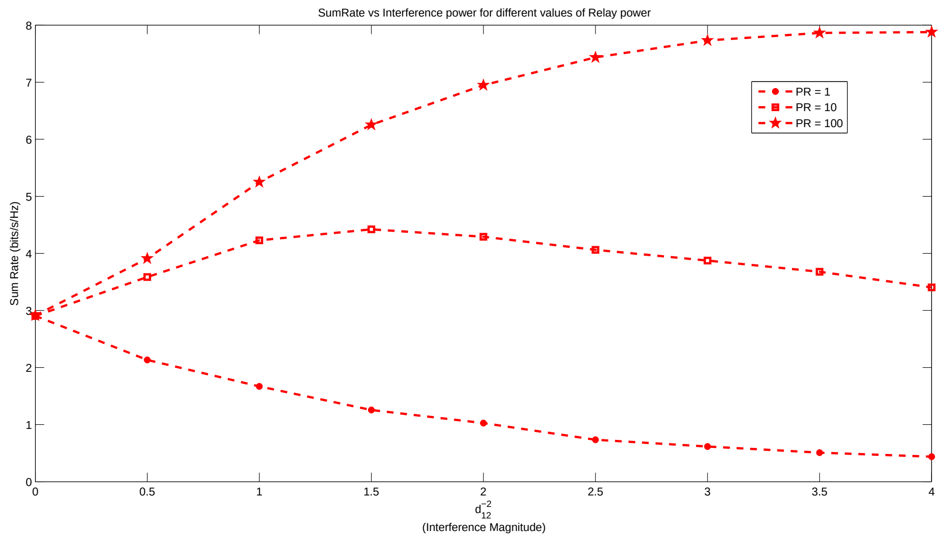

Figure 3.3: Comparison of Distributed interference alignment for different relay power 10

3.5 Conclusion

The need for Multi-hop Interference alignment scheme is explained and its advantages over conventional multi-hop schemes have been proved with the numerical results. The important observation made from simulations is that Sum capacity of network is not interference limited as long as there is no limitation at relay power. Moreover, all the discussion so far considered the channel to be full rank channel which is not always feasible in practical. Future sections will address this problem and devising proper strategies in increasing throughput even in rank deficient environment.

Chapter 4

Interference Alignment for rank deficeint channels

4.1 Introduction

Interference mitigation in wireless networks has always been a complicated problem with no general approach that fits all the different scenarios. Major reason contributing to this problem is, in wireless networks the undesired signals from the other users can be heard in all the neighbouring nodes. As a result, the channel capacity in Multiple Access Channels (MAC) decreases drastically. The condition gets worse as the number of users in the network increases. There are a lot of strategies employed in practice that minimize or eliminate the effect of interference by effective distribution of the available wireless resources among the users and the conventional schemes in practice are resource allocation in time known as Time Division Multiple Access (TDMA) and resources allocation in frequency known as Frequency Division Multiple Access (FDMA), treating interference as noise or successive decoding of strong interference before the desired signal. All these share a common idea of dividing the available resources between users resulting in overall Degrees of Freedom (DoF) summing up to one. Recently, a novel concept Interference Alignment (IA) was proposed which gave a new perspective to interference management problem with two communication pairs [9]. Later on the idea of IA has been extended for K-user Interference Channel (IC) and it was shown that IA scheme can achieve the optimal DoF of K/2 in [5]. With the ground breaking scope of IA, it has been widely applied to different wireless networks like multiple-input multiple-output (MIMO) interference networks, X-networks, Cellular networks and device-to-device (D2D) networks.

All the IA techniques assume perfect global channel state information (CSI) availability at all nodes and most importantly requirement of richly scattered channel environment in mathematical terms a full rank channel matrix between source and destination pairs. In a rich scattering environment, it has been shown that the Capacity of MIMO point-to-point (P2P) channel scale linearly with the number of transmit and receive antennas. In a poorly scattered environment, increase in antennas cannot guarantee any improvement in channel capacity because of the possibility of the rank deficiency of the channel matrix. When a network is operated in poor scattering environment, channel coefficients tend to become more and more correlated to each other as carrier frequency increase and line-of-sight (LoS) component becomes stronger. In the absence of scatters, the Channel matrix become rank deficient resulting in reduced DoF or the Multiplexing gain and in very poorly scattered channel it might even reduce to one thus restraining from the benefits of MIMO system. This situation also arises in a rich scattering environment when mobile user equipment moves away from the base station. As the separation between Base Station (BS) and Mobile Station (MS) increases, the Doppler Effect results in reducing the frequency received and increasing the effective wavelength received. As a result an additional separation between the antennas is required to observe independent channels by the antennas and to exploit the rich channel scattering environment otherwise the channel observed by antennas becomes highly correlated resulting in rank deficiency of Channel matrix. But since the antennas and their relative separation is fixed in practical scenario, we lose Multiplexing gain offered by MIMO system even in presence of richly scattered environment. (Normal separation between antennas in MIMO systems is to be maintained at a minimum of $\lambda/2$ where λ is the wavelength is used for communication in order to observe independent channels by the respective antennas). Therefore rank deficiency of Channel matrix reduces the Overall Throughput of the system and there is a strong necessity to address this problem. The present chapter presents solution to the problems addressed in the above paragraphs by providing two novel relaying schemes for rank deficient MIMO channels.

4.2 Scheme - 1 : Cooperative Relaying for rank deficient channels

The scheme proposed considers a MIMO rank deficient interference channel. The network is composed of multiple source-destination node pairs communication with each other with the aid of relays. A poorly scattered channel is considered in which channel matrices are considered to be rank deficient due to domination of Line of sight component and also due to absence of multipath. Because of this rank deficient nature of channel could considerably decrease the DoF of the network as compared to rich scattering environment with full rank channel matrix.

This novel relaying scheme proves us that with the help of relay, a significant improvement in DoF can be achieved in rank deficient channels irrespective of whether the forward channel state information is present at relay or not. This scheme uses relay not only to increase the DoF but also nullifies the inter user interference.

4.2.1 System Model

A general K-user Interference channel is considered with all the users fully connected transmitter-receiver pairs and R relays. Transmitter i wants to sends s_i independent data symbols to receiver i with the assistance of N relays. m_s, m_d and m_r denotes the number of antennas at each transmitter , receiver and the relay respectively. Let κ be the set of indices for users in the network {1,...,K} and R be the set of indices for relays in the network $\{1, ...,$R$\}$. $H_{j,i}(\forall i, j \in \kappa)$

is the channel matrix from Tr_i to Rx_j . $F_{r,i}(\forall i \in \kappa$ and $r \in R)$is the channel matrix from Tr_i to Relay r. $G_{i,r}(\forall i \in \kappa$ and $r \in R)$is the channel matrix from Relay r to Rx_i . All links are subjected to frequency flat fading and since the focus is on rank deficient channel environment, channel coefficients are assumed to be not independent and the rank of the channel matrix with dimensions $m \times n$ is always less than or equal to the full rank of the matrix which is $min(m, n)$. Let r_{sr}, r_{rd}, r_{sd} be the ranks of Channel matrices from source to relay link, relay to destination link and source to destination link full rank of the respective matrices being R_{sr}, R_{rd}, R_{sd} . The relay employs two hop Amplify and forward (AF) protocol and operates in half duplex mode. At first time slot received signal at relay is given by

$$Y_{r,j}(t) = \sum_{i \in \kappa} \boldsymbol{F_{ji}} X_i(t) + Z_{r,j}(t) \tag{4.1}$$

where $y_{r,j}(t)$ is the $m_r \times 1$ received signal vector at relay r $\in R$. $z_{r,j}(t)$ is $m_r \times 1$ noise vector at relay r in time instant t independent and identically distributed (i.i.d) gaussian noise with $\mathcal{N}(0_{m_r}, I_{m_r})$. Here $\mathcal{N}(0_{m_r}, \boldsymbol{I}_{m_r})$ denotes zero mean circularly symmetric gaussian noise with covariance matrix $\boldsymbol{I}_n$. the $m_d \times 1$ signal received at destination node is given by

$$Y_j(t) = \sum_{i=1}^{N} \boldsymbol{G}_{ji} X_{r,i}(t) + Z_j(t) \tag{4.2}$$

$z_{r,j}(t)$ is $m_r \times 1$ noise vector at relay r in time instant t independent and identically distributed (i.i.d) Gaussian noise with $\mathcal{N}(0_{m_r}, I_{m_r})$. Here $\mathcal{N}(0_{m_r}, \boldsymbol{I}_{m_r})$ denotes zero mean circularly symmetric Gaussian noise with covariance matrix $\boldsymbol{I}_n$. All the nodes in the network should satisfy the power constraint i.e., $\mathbb{E}[|x_i(t)|^2] \leq P_S$ $\forall i \in \{1, ..., K\}$ at source and $\mathbb{E}[|x_{r,i}(t)|^2] \leq P_R$ $\forall i \in \{1, ..., N\}$. $r = min(r_{rs}, r_{dr}, r_{ds})$.

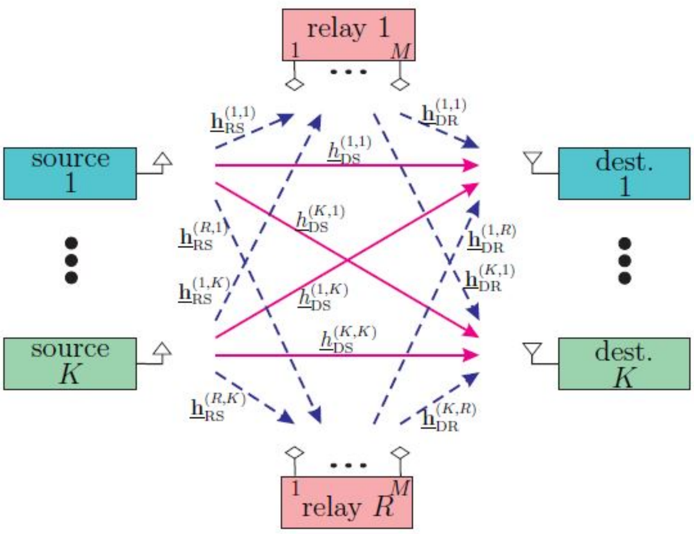

Figure 4.1: K User Interference Channel with R relays

4.2.2 Cooperative Relaying

Forward Channel State Information at Relay

We investigate this scheme for $m_r = Kr$. This schemes operates over two time slots, where in first time slot source nodes transmit independent signals $X_i(t) \forall i \in \kappa$ to relays and in the second time slot relays amplify and forward the signals received in previous time slot to destination $X_{r,i}(t) = \sum_{i=1}^{K} U_{ij} V_{ij}^{\dagger} Y_{r,i}(1) \; \forall i \in \{1, 2..R\}$ where $Kr \times r$ matrices U_{ij} and V_{ij} are to satisfy the following conditions.

$$\boldsymbol{G}_{ki} U_{ij} \begin{cases} = \mathbf{0}_{m_d \times r}, & \forall k \in \{1, ...K\} k \neq j \\ \neq \mathbf{0}_{m_d \times r}, & k = j \end{cases} \tag{4.3}$$

$$\boldsymbol{V}_{ij}^{\dagger} F_{ik} \begin{cases} = \mathbf{0}_{r \times m_s}, & \forall k \in \{1, ...K\} k \neq j \\ \neq \mathbf{0}_{r \times m_s}, & k = j \end{cases} \tag{4.4}$$

From the above equations we can be sure that U_{ij} exists as the rank of $\boldsymbol{G}_{ki}$ is $\geq r$ and the matrix $[\boldsymbol{G}_{1i}^T, ..., \boldsymbol{G}_{Ki}^T]^T$ is always a full rank matrix $\forall i$. There are many iterative methods to calculate the matrices U_{ij} and V_{ij} but in this thesis, null space concept is employed to find them. Similarly, V_{ij} is also found out and Moreover there is flexibility in scaling these matrices to satisfy the power constraints at their respective nodes.

$$U_{ij} \in \text{nullspace } [\boldsymbol{G}_{1i}^T, ..., \boldsymbol{G}_{Ki}^T]^T \tag{4.5}$$

From the above equations received signals at destination node in second time slot can be written as follows

$$\begin{aligned} Y_j(2) &= \sum_{i=1}^{N} \boldsymbol{G}_{ji} X_{r,i}(2) + Z_j(2) \\ &= \sum_{i=1}^{N} \boldsymbol{G}_{ji} \sum_{k=1}^{K} U_{ik} V_{ik}^{\dagger} \left(\sum_{l \in \kappa} \boldsymbol{F_{il}} X_l(1) + Z_{r,i}(1) \right) + Z_j(2) \\ &= \sum_{i=1}^{N} \boldsymbol{G}_{ji} \sum_{k=1}^{K} U_{ik} V_{ik}^{\dagger} \boldsymbol{F_{ik}} X_k(1) + Z_{r,i}(1) + Z_j(2) \\ &= \bar{\boldsymbol{H}}_{\boldsymbol{jj}} X_j(1) + \bar{\boldsymbol{z}}_{\boldsymbol{j}} \end{aligned} \tag{4.6}$$

where $\bar{\boldsymbol{H}}_{\boldsymbol{jj}} = \sum_{i=1}^{N} \boldsymbol{G}_{ji} U_{ij} V_{ij}^{\dagger} \boldsymbol{F_{ij}}$ and $\bar{\boldsymbol{z}}_{\boldsymbol{j}} = \sum_{i=1}^{N} \boldsymbol{G}_{ji} U_{ij} V_{ij}^{\dagger} Z_{r,i}(1) + Z_j(2)$ and from the equation (4.6) we can write the Capacity at each destination node as

$$R_j(P) = \frac{1}{2} \log \left(\left| \frac{P}{m_s} \bar{\boldsymbol{H}}_{\boldsymbol{jj}} \bar{\boldsymbol{H}}^{\dagger}{}_{jj} + R_{\bar{\boldsymbol{z}}_{\boldsymbol{j}}} \right| / R_{\bar{\boldsymbol{z}}_{\boldsymbol{j}}} \right) \tag{4.7}$$

where $R_{\bar{z}_j} = E[\bar{z}_j \bar{z}_j^{\dagger}]$ and the factor 1/2 is because the scheme operates in two time slots. The DoF achieved by each user in this scheme can be given by

$$\begin{aligned} \lim_{P\to\infty} \frac{R_j(P)}{\log(P)} &= \lim_{P\to\infty} \frac{\frac{1}{2}\log\left(\left|\frac{P}{m_s}\bar{\boldsymbol{H}}_{\boldsymbol{jj}}\bar{\boldsymbol{H}}^{\dagger}{}_{jj}\right|\right)}{\log(P)} \\ &= \frac{\min(m_s, m_d, Nr)}{2}. \end{aligned} \quad (4.8)$$

and therefore the total DoF is given by $\Gamma = \frac{K\min(m_s,m_d,Nr)}{2}$. In the case where $m_r < Kr$ then only $\lfloor \frac{m_r}{r} \rfloor$ out of K source destination pairs can be feasible to participate in the communication. Therefore SDoF for general case is given by

$$\Gamma = \frac{\min(K, \lfloor \frac{m_r}{r} \rfloor)\min(m_s, m_d, Nr)}{2} \quad (4.9)$$

No Forward Channel state Information at relay

In this scenario, the strategy in first time slot remains same as in the previous section, But because of absence of channel state information at relay , it follows dedicated channel communication in the second phase. At $(j+1)th$ time slot, relay AF the signal received in first time slot to destination node j $\forall j \in \{1,...,K\}$. The pre coding at relay is fixed and can be given by $X_{r,i}(j+1) = (\mathbf{1}_K \otimes \boldsymbol{I}_r)V_{ij}^{\dagger}Y_{r,i}(1)$ and therefore received signal vector at destination node j in $(j+1)$ timeslot can be given by

$$\begin{aligned} Y_j(j+1) &= \sum_{i=1}^{N} \boldsymbol{G}_{ji}X_{r,i}(j+1) + Z_j(j+1) \\ &= \bar{\boldsymbol{H}}_{\boldsymbol{jj}}X_j(1) + \bar{\boldsymbol{z}}_{\boldsymbol{j}} \end{aligned} \quad (4.10)$$

where $\bar{\boldsymbol{H}}_{\boldsymbol{jj}} = \sum_{i=1}^{N} \boldsymbol{G}_{ji}(\mathbf{1}_K \otimes \boldsymbol{I}_r)V_{ij}^{\dagger}\boldsymbol{F}_{\boldsymbol{ij}}$ and $\bar{\boldsymbol{z}}_{\boldsymbol{j}} = \sum_{i=1}^{N} \boldsymbol{G}_{ji}(\mathbf{1}_K \otimes \boldsymbol{I}_r)V_{ij}^{\dagger}Z_{r,i}(1) + Z_j(j+1)$. Then by similar analogy as above we can state the total DoF in this case as

$$\Gamma = \frac{\min(K, \lfloor \frac{m_r}{r} \rfloor)\min(m_s, m_d, Nr)}{K+1} \quad (4.11)$$

. The only difference here being the second phase of communication takes place in K slots with first phase in 1 slot and total of $K+1$ time slots.

Source to Destination Direct Transmission

If we consider direct transmission from source to destination without any relay in rank deficient MIMO interference channel, the rank of direct channel matrices $[[\boldsymbol{H}_{11},..,\boldsymbol{H}_{1K}]^T,...,[\boldsymbol{H}_{K1},..,\boldsymbol{H}_{KK}]^T]^T$ is Kr_{ds} and the maximum DoF it could offer is bounded by $\min(Kr_{ds}, m_d)$.

Time Division

In this method only one of the source communicate with relay and then relay forwards to desired destination in the next time slot. Therefore the SDoF offered by this scheme is given by $\frac{\min(m_s,m_d,Nr)}{2}$

4.2.3 Simulation Environment

The Monte Carlo simulations for the above scenario has been performed in **MATLAB™**till the number of error bits received are 1000 by each user and the transmitted power is varied from -10 dB to 50 dB. The parameters are as follows

$$\begin{aligned} m_s &= m_d &= N = 6 \\ m_r &= K &= 2 \\ r_{ds} &= r &= 1 \end{aligned}$$

4.3 Results and Discussion

The first graph shows us the DoF bounds offered by the proposed scheme over two base line schemes when $m_r = Kr > 1$ and $r_{ds} = r$.

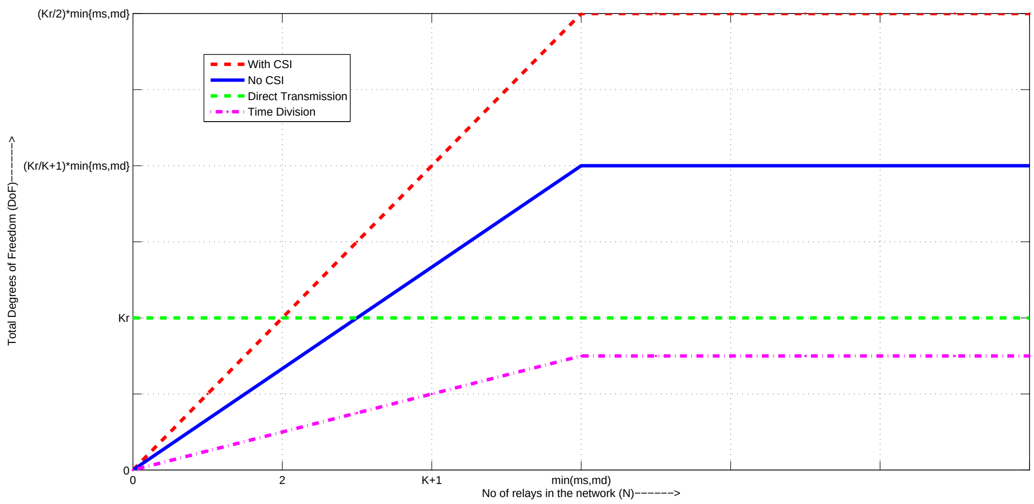

Figure 4.2: DoF bounds for K User Interference Channel with N relays

The results of the proposed scheme were compared with Time Division scheme and from the graph it is evident that the proposed outperforms the conventional scheme and the performance gap between them increases in high SNR regime. This is due to novel relay scheme in which relays acts as external scatters in a rank deficient channel and improves the overall DoF of the network. The DoF offered by the investigated scheme is 6,4 where conventional time division scheme could offer only 3 DoF.

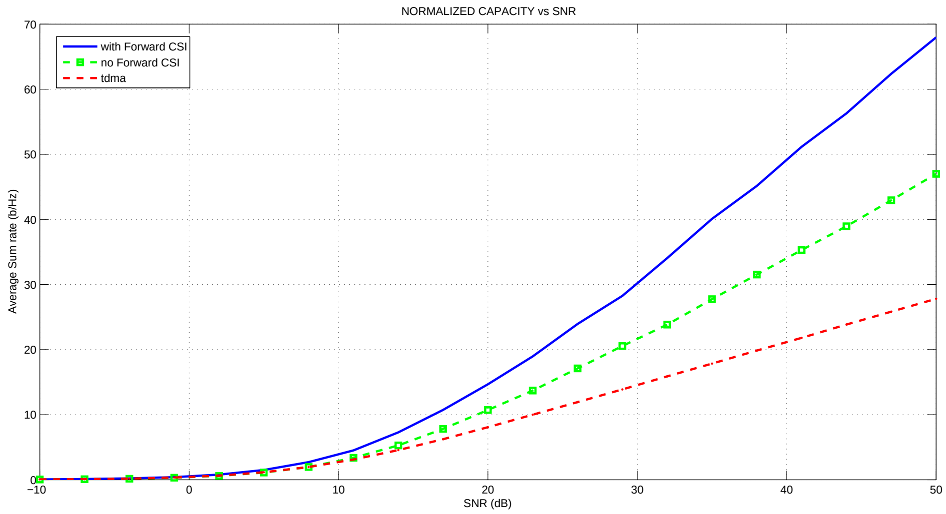

Figure 4.3: Sum Capcity of the network for K User Interference Channel with N relays

4.4 Scheme - 2: Opposite Direction Interference Alignment Scheme for Rank deficient channels

The present scheme is inspired from novel interference alignment scheme proposed by [30]. The actual scheme was given only for full rank K- user Gaussian interference channel whereas present scheme focuses on rank deficient general K user Gaussian interference channel with half duplex MIMO relay for any arbitrary number of antennas at source and destination nodes. The present scheme increase rank of the effective channel exploiting the additional spatial dimensions offered by the relay i.e., relay acting as external scatter and increases the Sum DoF of the network close to full rank channel environment. Moreover, this scheme doesn't require Global Channel state information at all nodes in the network unlike other IA schemes but requires only at relay node. Relay uses the GCSI in such a way that relay link interference is used to cancel the interference from direct link. The Sufficient and Feasibility of this scheme is validated by the solvability of linear system of equations, Kruskal rank of Khatri Rao product, and finally present a closed form solution for transformation at relay. The advantages of these scheme offered in terms of DoF have been verified by simulation results.

4.4.1 System Model

A general K-user Interference channel is considered with all the users fully connected transmitter-receiver pairs and a MIMO relay. Transmitter i wants to sends s_i independent data symbols to Receiver i with the assistance of MIMO relay. M_i , N_i and N_R Denotes the number of antennas at transmitter i, Receiver i and the relay respectively. Let K be the set of indices for users in the network $\{1...K\}$. $H_{j,i}(\forall i, j \in K)$ is the channel matrix from Tr_i to Rx_j . $G_{R,i}(\forall i \in K)$ is the channel matrix from Tr_i to Relay. $F_{i,R}(\forall i \in K)$ is the channel matrix from Relay to Rx_i . All links are subjected to frequency flat fading and since the focus is on rank deficient channel environment, channel coefficients are assumed to be not independent and the rank of the channel matrix with dimensions $m \times n$ is always less than or equal to the full rank of the matrix which is $\min(m, n)$. Let r_{sr}, r_{rd}, r_{sd} be the ranks of Channel matrices from source to relay link, relay to destination link and source to destination link full rank of the respective matrices being R_{sr}, R_{rd}, R_{sd} . the relay employs two hop Amplify and forward (AF) protocol and operates in half duplex mode. At first time slot received signal at Rxj and relay are respectively given by

$$Y_j(1) = \sum_{i=1}^{K} \boldsymbol{H}_{j,i} X_i + Z_j(1), \forall j \in K \tag{4.12}$$

$$Y_R = \sum_{i=1}^{K} \boldsymbol{G}_{j,i} X_i + Z_R \tag{4.13}$$

Where X_i is the $M_i \times 1$ transmit signal vector at Txi and satisfying the power constraint $E[X_i^* X_i] \leq P$. $Z_j(1)$ and Z_R are the additive white Gaussian noise components at Rxi and relay respectively. Each noise component has Gaussian distribution with 0 mean and σ^2 variance represented by $N(0, \sigma^2)$. In the second time slot all the Txs remain silent, while the relay amplifies and forward the received signal. The received signal in second time slot at Rxj is

$$Y_j(2) = \boldsymbol{H}_{j,R} \boldsymbol{T} Y_R + Z_j(2), \forall j \in K \tag{4.14}$$

$\boldsymbol{T}$ is the Interference nullifying matrix obtained at the relay and $Z_j(2)$ is additive white Gaussian noise component. In the proposed algorithm the signals received in time slot 1 and time slot 2 are combined at Rxj

$$\begin{aligned} \tilde{Y_j} &= \left(\boldsymbol{H}_{j,j} X_j + \boldsymbol{F}_{i,R} \boldsymbol{T} \boldsymbol{G}_{R,j} X_j \right) \\ &+ \left(\sum_{i=1, i \neq j}^{K} \boldsymbol{H}_{j,i} X_i + \boldsymbol{F}_{i,R} \sum_{i=1, i \neq j}^{K} \boldsymbol{T} \boldsymbol{G}_(R,i) X_i \right) \\ &+ \left(\boldsymbol{F}_{i,R} \boldsymbol{T} Z_R + Z_j(1) + Z_j(2) \right), \forall j \in K \end{aligned} \tag{4.15}$$

The first line in equation (4.15) is the desired signal with message X_j and the second line is the summation of interfering signals from direct link and relay link. As per the proposed Interference nullifying scheme, Interference from relay link is exactly of the same magnitude as the Interference from direct link but with opposite sign. For this to happen, Transformation at relay T is performed such that interference signals with same data vector are expected to cancel each other nullifying the undesired signal at respective receiver and at the same time the transformation ensures that rank of effective channel matrix with desired signal vector is increases to full rank. Thus this Interference nullifying scheme doesn't require availability of Global CSI at all nodes except at relay, only local CSI is required at Rx to extract the symbols on the contrary to previous IA schemes which requires Global CSI at all nodes to minimize interference and extract the desired symbols. The mathematical formulation of the scheme is stated below

$$\boldsymbol{F}_{i,R}\boldsymbol{T}\boldsymbol{G}_{R,j} = -\boldsymbol{H}_{j,i}, i \neq j, \forall i, j \in K \tag{4.16}$$

$$s_j = rank\bigg(\boldsymbol{H}_{jj} + \boldsymbol{F}_{j,R}\boldsymbol{T}\boldsymbol{G}_{R,j}\bigg) \leq \min M_j, N_j \tag{4.17}$$

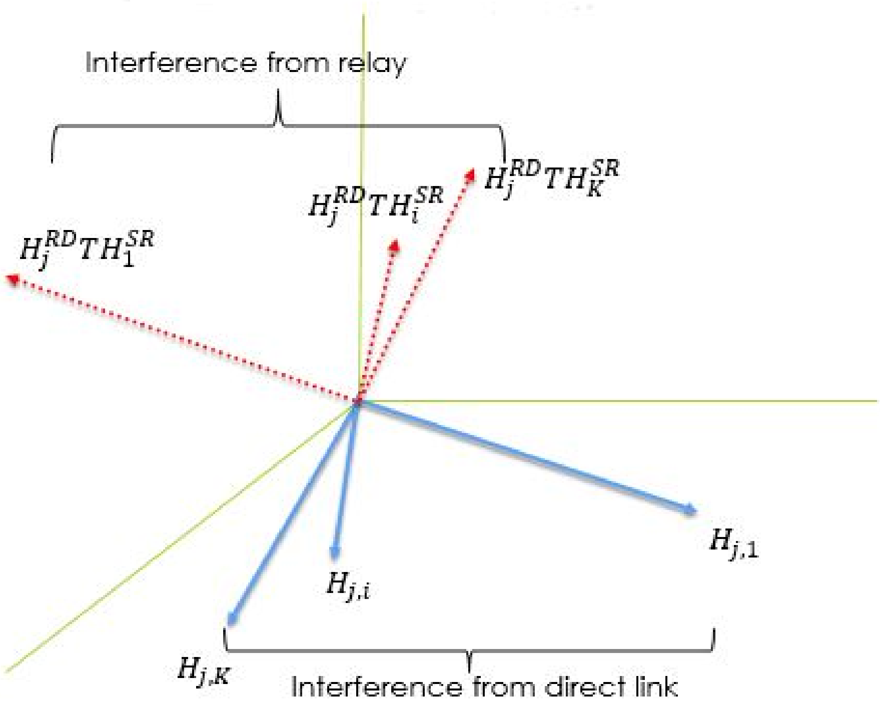

Figure 4.4: Graphical representation of Opposite direction IA

4.4.2 Feasibility Conditions

In this section, we emphasize on the mathematical feasibility and necessary constraints required to be satisfied for the proposed scheme. We define

$$\bar{\boldsymbol{H}}_j = \left[\boldsymbol{H}_{j,1}...\boldsymbol{H}_{j,j-1}, \boldsymbol{H}_{j,j+1}....\boldsymbol{H}_{j,K}\right] \tag{4.18}$$

$$\bar{\boldsymbol{G}}_{R,j} = \left[\boldsymbol{G}_{R,1}...\boldsymbol{G}_{R,j-1}, \boldsymbol{G}_{j,j+1}....\boldsymbol{G}_{R,K}\right] \tag{4.19}$$

From the above equations eq (4.16) can be rewritten as

$$\boldsymbol{F}_{j,R}\boldsymbol{T}\bar{\boldsymbol{G}}_{R,j} = -\bar{\boldsymbol{H}}_j, \forall j \in K \tag{4.20}$$

From [31], it is known that

$$vec(\mathbf{ABC}) = (\mathbf{C}^T \otimes \mathbf{A})vec(\mathbf{B}) \tag{4.21}$$

. Using vectorization transformation on equation (4.20) we get

$$({\bar{\boldsymbol{G}}_{\boldsymbol{R,j}}}^T \otimes \boldsymbol{F}_{j,R})vec(\mathbf{T}) = vec(-\bar{\boldsymbol{H}}_j) \tag{4.22}$$

Combining the above equations for all values of $j \in K$, we get a composite equation of the following form

$$\underbrace{\begin{pmatrix} {\bar{\boldsymbol{G}}_{\boldsymbol{R,1}}}^T \otimes \boldsymbol{F}_{1,R} \\ {\bar{\boldsymbol{G}}_{\boldsymbol{R,2}}}^T \otimes \boldsymbol{F}_{2,R} \\ . \\ . \\ . \\ {\bar{\boldsymbol{G}}_{\boldsymbol{R,K}}}^T \otimes \boldsymbol{F}_{K,R} \end{pmatrix}}_{\mathbf{A}} \underbrace{vec(\mathbf{T})}_{\mathrm{t}} = \underbrace{\begin{pmatrix} vec(-\bar{\boldsymbol{H}}_1) \\ vec(-\bar{\boldsymbol{H}}_2) \\ . \\ . \\ . \\ vec(-\bar{\boldsymbol{H}}_K) \end{pmatrix}}_{\mathrm{h}} \tag{4.23}$$

Here $\mathbf{A}$ is the coefficient matrix with size $\big(\sum\limits_{j=1}^{K} N_j \sum\limits_{i\neq j,i=1}^{K} M_i\big) \times N^2{}_R$ and t is the $N^2{}_R \times 1$ variable matrix. For the above linear system of equations to be have solution, from the properties Kruskal rank of Khatri-Rao product, the following condition is necessary

$$N_R \geq \max\left(\sum_{j=1}^{K} N_i, \big(\sum_{i\neq j,j=1}^{K} M_i\big)\Big|_{j=1}^{K}\right) \tag{4.24}$$

On satisfying the above feasibility condition, the least norm solution to $\mathbf{A}t = h$ can be given by

$$t = \mathbf{A}^H(\mathbf{A}\mathbf{A}^H)^{-1}h \tag{4.25}$$

By using above equation, we could construct precoding matrix T at relay which satisfies equation (4.20) i.e., $\boldsymbol{F}_{j,R}\boldsymbol{T}\bar{\boldsymbol{G}}_{R,j} = -\bar{\boldsymbol{H}}_j, \forall j \in K$. Since all elements in channel matrices are independent and identically distributed, $\bar{\boldsymbol{G}}_{R,j}$ is of almost full rank even though individual channel matrices of which it is composed of are rank deficient.

$$\boldsymbol{F}_{j,R}\boldsymbol{T} = -\bar{\boldsymbol{H}}_j(\bar{\boldsymbol{G}}_{R,j})^{\dagger} \quad (\forall j \in K) \tag{4.26}$$

where $(\bar{\boldsymbol{G}}_{R,j})^{\dagger}$ is the pseudo inverse of $\bar{\boldsymbol{G}}_{R,j}$, On substituting equation (4.26) in equation (4.17) and as all elements of $\boldsymbol{H}_{jj}$ are independent of $\bar{\boldsymbol{H}}_j(\bar{\boldsymbol{G}}_{R,j})^{\dagger}\boldsymbol{G}_{R,j}$ we get

$$\begin{aligned} s_j &= rank\big(\boldsymbol{H}_{jj} + \boldsymbol{F}_{j,R}\boldsymbol{T}\boldsymbol{G}_{R,j}\big) \\ &= rank\big(\boldsymbol{H}_{jj} - \bar{\boldsymbol{H}}_j(\bar{\boldsymbol{G}}_{R,j})^{\dagger}\boldsymbol{G}_{R,j}\big) \\ &= \min(M_j, N_j) \end{aligned}$$

As this scheme makes rank deficient K-User interference channel into full rank K parallel MIMO channels in two time slots, The achievable DoF in this scheme is given by

$$\begin{aligned} \Gamma_{\sum} &= \sum_{j=1}^{K} \tau_j \\ &= \sum_{j=1}^{K} (rank(SignalSpace) - rank(Interferencespace)) \\ &= \sum_{j=1}^{K} \big(\min(M_j, N_j) - 0\big) \quad \text{(Since Interference space is eliminated completely)} \\ &= \sum_{j=1}^{K} \big(\min(M_j, N_j)\big) \end{aligned}$$

For K-User symmetric case, TDoF = $KM/2$ which is same as the results shown in [].

4.4.3 Simulation Environment

The Monte Carlo simulations for the above scenario has been performed in **MATLAB**™ till the number of error bits received are 1000 by each user and the transmitted power is varied from 0 dB to 50 dB. The simulations were performed for 3 User symmetric system with $M_i = 6, N_i = 6$and $N_R = \{18, 24, 30\}$. The simulations were performed for three different scenarios. For seeing the performance efficiency of the proposed scheme, it is compared is for the conventional TDMA scheme for the same channel conditions as in the proposed scheme.

- Only direct link channel is rank deficient for ranks $\{1, 3, 5\}$
- Varying no of antennas at relay with direct link channel is rank deficient for rank $\{3\}$
- Both direct link (rank = 3)and Source - Relay link is rank deficient for ranks $\{1, 3, 5\}$
- Both direct link (rank = 3) and Relay-Destination link is rank deficient for ranks $\{1, 3, 5\}$

$*$ Full rank for all channels is 6.

4.5 Results and Discussion

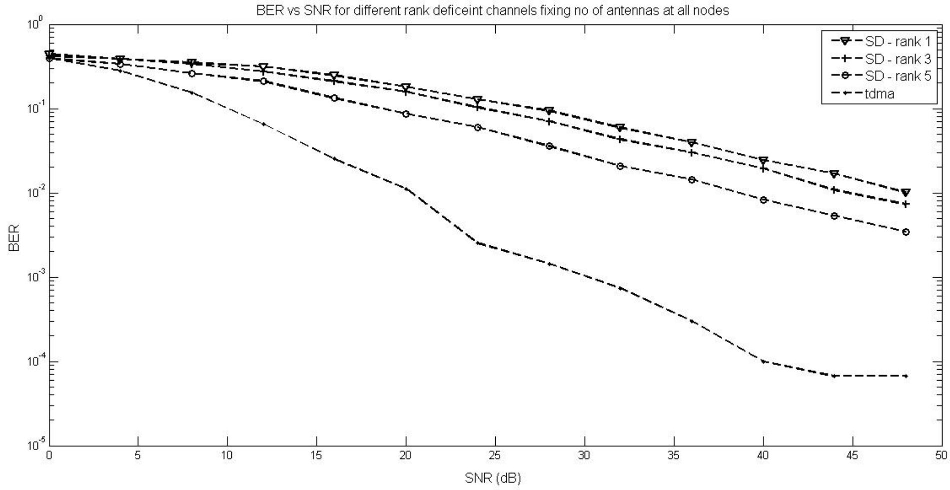

Figure 4.5: Comparison of BER performance of proposed scheme, tdma scheme Proposed scheme :for different source destination links rank deficient fixing no of antennas at relay

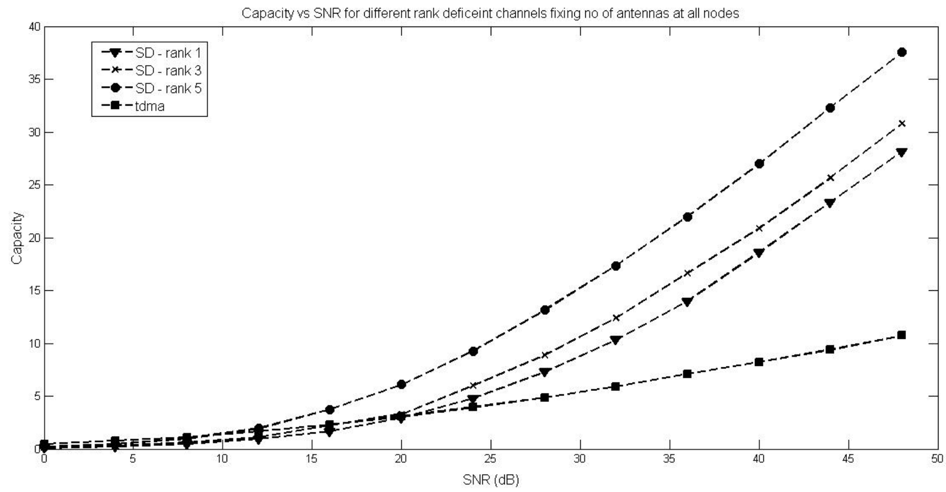

Figure 4.6: Comparison of Capacity performance of proposed scheme, tdma scheme Proposed scheme :for different source destination links rank deficient fixing no of antennas at relay

From Fig (4.5) and (4.6), it is evident that the proposed scheme performs exceptionally good offering us Capacity much higher than conventional tdma scheme even when source destination channel links are rank deficient. The proposed scheme's BER and capacity performance increases as the rank deficiency of the direct link channel decreases.

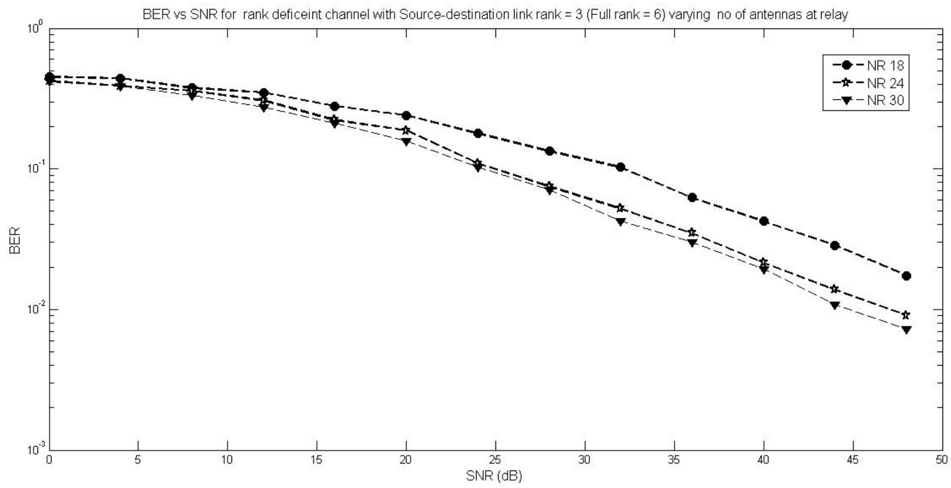

Figure 4.7: Comparison of BER performance of proposed scheme, tdma scheme
Proposed scheme : Varying no of antennas at relay for Sourced destination channel rank = 3

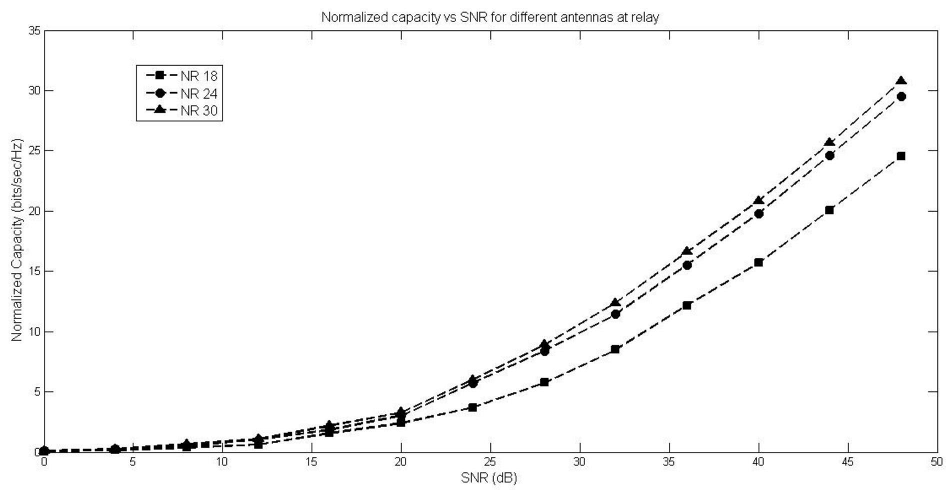

Figure 4.8: Comparison of Normalized Capacity performance of proposed scheme vs tdma
Proposed scheme : Varying no of antennas at relay for Sourced destination channel rank = 3

From Fig (4.7) and (4.8), we can see that the proposed scheme BER performance increases as the no of antennas at relay increases. This is due to the additional antennas (no of antennas at relay greater than the no of antennas required for feasibility of the scheme) are involved in transmitter diversity inherently improving the quality of the communication. On the other hand, graph also suggest us that on further increase in no of antennas at relay would lead to saturation in performance.

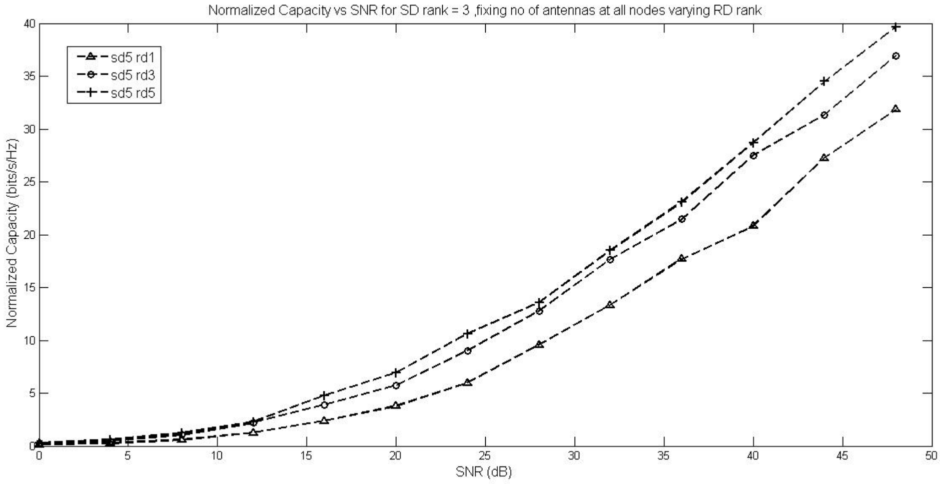

Figure 4.9: Comparison of Normalized Capacity performance of proposed scheme for relay destination link channel rank {1,3,5} and source destination link channel rank = 5

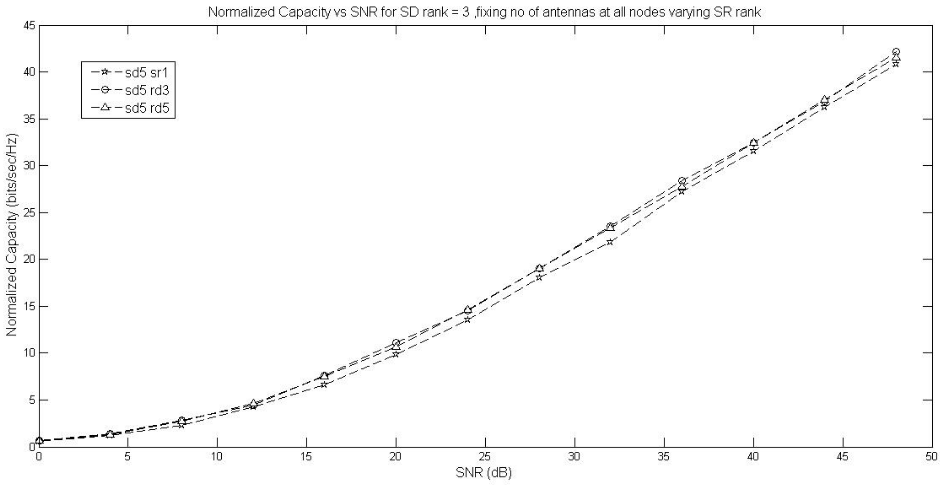

Figure 4.10: Comparison of Normalized Capacity performance of proposed scheme for source relay link channel rank {1,3,5} and source destination link channel rank = 5

From figure (4.9), we can see that Capacity performance of the proposed scheme improves as the relay destination channel rank approaches full rank for a particular rank deficient source destination channel. where as figure (4.10) shows us that Capacity performance of the proposed scheme remains almost same as the source relay channel rank is varied for a particular rank deficient source relay channel.

4.6 Conclusion

In the entire discussion of the chapter, more practical scenarios are considered, where channels are rank deficient. The idea of exploiting relay to act as external scatterer in rank deficient channels is discussed. Two relaying schemes were proposed addressing the rank deficiency problem of channel. The schemes proposed are distributed, doesn't require global channel state information at all nodes, doesn't assume channel to richly scattered but still could perform exceptionally great in comparison with conventional TDMA schemes.

Chapter 5

Summary

5.1 Overview

Interference Alignment scheme is the epicentre of the entire research work carried out. In the beginning of the thesis, a brief idea of the growing cellular networks and the demand to provide each user quality communication and high data rate at the same time is addressed. The major problems meeting the demands of the user are finite channel resources, channel fading and interference. Interference being major problem in Multi user MIMO network, the existing interference mitigation techniques and their drawbacks have been discussed. First chapter ends with a need for a novel interference mitigation scheme.

In the second chapter the idea of Interference alignment, a novel interference mitigation technique is been introduced. Origins of interference alignment, implementation for different scenarios has been discussed at stretch and through IA technique each user can communicate with its intended receiver simultaneously at a data rate equal to half of the Channel capacity when the channel is free from interference irrespective of the number of users involved in the network i.e., the total DoF offered in a $M \times M$ symmetric K user network = $KM/2$. The numerical results supporting the performance advantage of IA scheme is verified for different scenarios. Further, the need to extend this scheme for practical wireless networks is considered. Later on, interference alignment for multi hop networks is discussed. A practical multi user multi hop wireless cellular network scenario is considered and distributed interference alignment scheme for such a network is implemented. The results infer us that Sum rate capacity of the network can be increased even in high interference cases as long as there is no limitation of relay power. Al the above implementations assumed the channel matrices to be full rank and presence of global channel state information at all nodes.Moreover, most of IA schemes are centralized schemes which are far from practically feasible. Therefore new interference scheme which addresses drawbacks of all above schemes is needed.

In chapter 4, practical scenarios are considered, where channels are rank deficient. The idea of exploiting relay to act as external scatterer in rank deficient channels is discussed. Two relaying schemes were proposed addressing the rank deficiency problem of channel. The schemes proposed are distributed, doesn't require global channel state information at all nodes, doesn't assume channel to richly scattered but perform exceptionally great on comparison with conventional TDMA scheme make them more practically feasible than existing schemes.

5.2 Scope for Future work

The idea of Interference alignment and its realization is done under ideal conditions such as very high SNR, rich scattering environment, Perfect Global Channel state information at all nodes, Significant delays. The present thesis tried to minimize the assumption that channels are always richly scattered. There is lot of scope to take it even further by realizing Interference alignment for even more practical scenarios such as in low SNR regime, imperfect and delayed Channel state information and low computational complexity.

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