Estimating Transmission Loss of Muffler having Two Inlets and Two Outlets

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Estimating Transmission Loss of Muffler having Two Inlets and Two Outlets

Thesis submitted in partial fulfilment

of the requirements of the degree of

Master of Technology

in

Industrial Design

by

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(Roll Number: 214ID1275)

based on research carried out

under the supervision of

Prof. Dibya Prakash Jena

May, 2016

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May 26, 2016

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This is to certify that the work presented in this thesis entitled *Estimating Transmission Loss of Muffler having Two Inlets and Two Outlets* submitted by *Ravi Pal, Roll Number 214ID1275*, is a record of original research carried out by him under my supervision and guidance in partial fulfilment of the requirements of the degree of Master of Technology in Industrial Design. Neither this thesis nor any part of it has been submitted earlier for any degree or diploma to any institute or university in India or abroad.

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Declaration of Originality

I, *Ravi Pal*, Roll Number *214ID1275* hereby declare that this thesis entitled *Estimating Transmission Loss of Muffler having Two Inlets and Two Outlets* presents my original work carried out as a postgraduate student of NIT Rourkela and, to the best of my knowledge, contains no material previously published or written by another person, nor any material presented by me for the award of any degree or diploma of NIT Rourkela or any other institution. Any contribution made to this research by others, with whom I have worked at NIT Rourkela or elsewhere, is explicitly acknowledged in the thesis. Works of other authors cited in this thesis have been duly acknowledged under the sections "Reference". I have also submitted my original research records to the scrutiny committee for evaluation of my thesis.

I am fully aware that in case of any non-compliance detected in future, the Senate of NIT Rourkela may withdraw the degree awarded to me on the basis of the present thesis.

May 26,2016 *Ravi Pal*
NIT Rourkela

Acknowledgement

It is said that curiosity is the mother of all inventions. If so, then I must be her favourite child. I would firstly like to thank my supervisor **Dr. Dibya Prakash Jena** for endowing me with the current project and inculcating his faith in me that I can take it forward from my forerunners. Dr. Jena is one of the few human beings whose passion for experimentation and ardent knowledge regarding the science has sparked the flame of curiosity in me. His keenness towards good and calmness in turmoil commands respect like no other. Besides this, Dr. Jena paved the way for me to visit **IIT Bhubaneswar** with respect to some aspects related to this project. I am very thankful to **Dr. Satyanarayan Panigrahi**, who granted access to software COMSOL Multyphysics in his lab at IIT Bhubaneswar.

I am deeply indebted to each and every classmate of mine for making my two years of post-graduation however they turned out to be. Good or bad, I will take it as a big etch in the corner stone of my soul to remember the lessons I have learned about people, society, culture and ways and rules of life. Special thanks to **Ajay Mishra, Kavindra Singh and Mradul Mishra** for their unconditional support without which this project would not have achieved its completion. I also thank NIT Rourkela for the wonderful infrastructure and beautiful architecture that provides a little bloom and life for this tiny city. For the few provisions that it could provide me with, I did try to make the best use of them during my stay here.

Lastly, but nowhere close to least, I send out a big bouquet of gratitude to my parents, **Mr. Balwant Singh** and **Mrs. Manorama Pal** for being such wonderful parents.

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Abstract

Predicting the acoustics behavior of the exhaust muffler before a prototype model built can save the both substantial amount of time and resources. There are various simulation tools available now a day which can predict the acoustics performance of the muffler. In order to use these tools effectively, it is very important to understand what is the most effective tool for the intended purpose of analysis. as well as how the various elements in the exhaust muffler affects the muffler performance.

This thesis presents how transmission loss for various muffler configuration can be determined through analytical and FEA methods. As extant literature is primarily dedicated to single inlet single outlet exhaust muffler, this work includes acoustics analysis of simple expansion chamber with two inlets one outlet, two inlets two outlets on each end face. Which gave us the advantage of decreasing backpressure. There are both analytical and FEA method used for each muffler configuration for determining transmission loss and the results are compared against each other. The analytical method used is impedance matrix method in which a global impedance matrix is derived in order to get transmission loss. The second and third methods used are the FEA using COMSOL Multiphysics and ANSYS15.0 which also gives the satisfactory result for all the configuration in order to understand which FEA tool is very well suited for predicting the transmission loss results are compared against each other and with analytical method. This thesis also includes the parametric analysis of single inlet single outlet in order to understood the effects of various elements on the muffler performance.

Keywords: *Transmission loss, Backpressure, Impedance matrix, Parametric analysis*.

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Chapter 1

Introduction

1.1 Mufflers

In the development of the transport, the internal combustion occupies a very important position. But with the rise of the internal combustion engine problem of noise pollution arises. The noise from the engine is basically summed up of two factors exhaust noise and noise due to friction occurring inside the engine, and with regard to the sound pressure level the untreated exhaust noise is approximately ten times greater than all the combined noise generated from the vehicle[1].

Muffler plays an important role in reducing the exhaust system noise. The traditional built and test procedure for determining the acoustics performance which is expensive and nowadays can be replace by numerical simulation methods which are capable of predicting the acoustics performance of various designs of the muffing system in a very short time.

1.2 Classification of Mufflers

Muffler is basically classified into two main categories namely reactive and absorptive muffler[2].The two basic classification are shown in figure 1.1

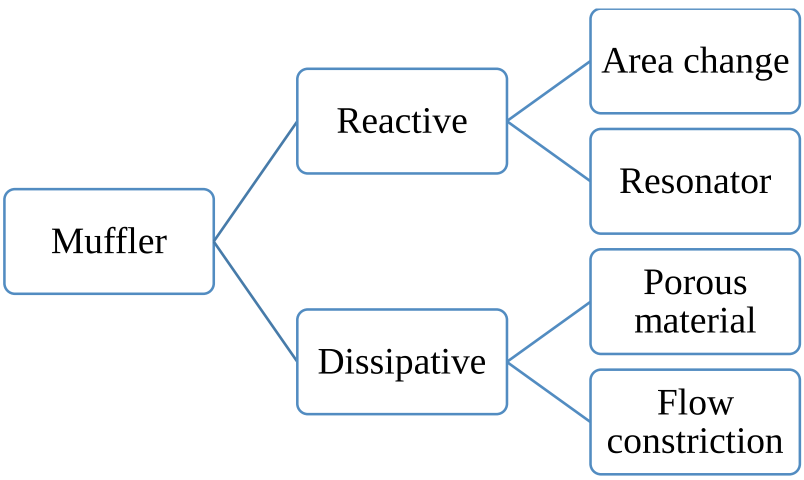


Figure 1.1 Classification of muffler

The reflective or reactive mufflers employs the phenomenon of destructive interference to reduce the sound pressure level. i.e. they are designed in such a way that sound waves which is originating from the engine which is actually a plane wave partially cancel out each other due to the destructive interference between transmitted and reflected wave. Reflected wave are actually produced by employing sudden expansion or contraction or area discontinuity in the geometry, this give rise to two different configuration of reactive muffler simple expansion chamber and resonating chamber [2]. A reflective muffler, as shown in figure 1.2 basically consist of series of expansion and resonating chamber that are designed to muffle sound pressure level at certain frequencies. The only limitation with this type muffler is large backpressure which effects the engine performance

Figure 1.2 Reflective automotive muffler[2]

A dissipative or absorptive muffler as shown in Figure 1.3 is based on the principle of converting the exhaust noise which is generated due to fluctuating pressure into heat in the acoustics material such as perforated tubing and the sound attenuating woven fibres. absorptive muffler in its layout basically consists of straight, circular and perforated pipe that is enclosed in a larger steel housing. Between the housing and the perforated pipe there is a layer of sound absorptive material that absorbs the pressure pulses[2].

Dissipative muffler does not have the limitation of backpressure as that of reactive muffler because in these type of muffler flow path is straight so there being very less flow reversal, twist and turn, due to less restriction to the flow path the pressure drop across the system is relatively low. But they have the limitation over the frequency spectrum as at low frequency range the wavelength is too large to be attenuated by the material so they are insufficient at the low frequency range[3]..

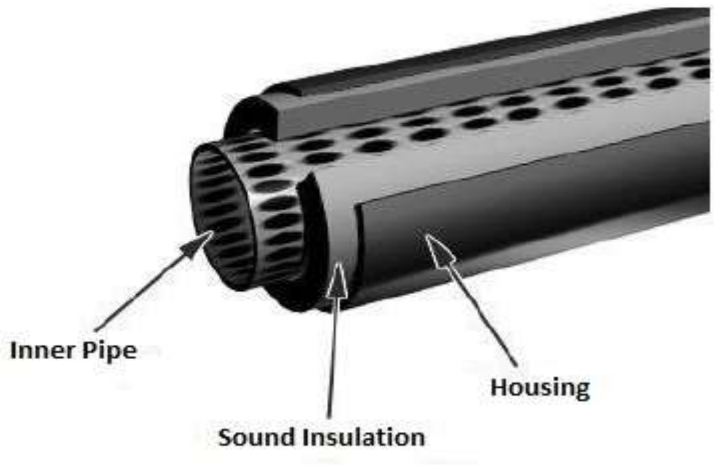


Figure 1.3 Absorptive muffler[2].

1.3 Problem Definition

Backpressure has always been a great concern in designing the muffler geometry, as it can severely affect the muffler performance. In automotive industry basically there are two type of muffler configuration are used absorptive and reactive muffler. Reactive muffler provides a large noise attenuation but also created large backpressure due to its geometry, as whenever there is change in direction of the exhaust gas there will be always additional backpressure. One way to reduce this backpressure is to increase the number of ports without changing the muffler geometry. This thesis involves the acoustics analysis of three different cases single inlet single outlet, two inlets one outlet, two inlets two outlets expansion chamber. Other problem encounters during the designing of exhaust system is to understand how various elements in the exhaust system affects muffler performance. For this there is a parametric analysis is for simple expansion chamber to see how each element in the exhaust system affects the acoustic performance of the muffler. The third problem associated with designing of muffler is to select an appropriate method as there are various methods through which acoustics performance of muffler can be predicted. This thesis includes both analytical and simulation method for each muffler configuration to see by which method we can easily predict the acoustics performance of muffler and also results from both analytical and simulation methods are compared against each other for the purpose of validation.

1.4 Objectives

The objective of the current work is to determine the transmission loss simple expansion chamber with three different configurations having increasing number of port and their parametric studies through both analytical and simulation method.as a part of this study, following activities are performed:

- Determination of acoustics transmission loss of simple expansion chamber having single inlet single outlet through analytical method and finite element methods.
- Comparison of transmission loss curve of analytical method and finite element methods.
- Parametric study of simple expansion chamber having single inlet single outlet.
- Determination of acoustics transmission loss of simple expansion chamber having two inlets one outlet through analytical method and finite element methods.
- Comparison of transmission loss curve of finite element methods and analytical method.
- Determination of acoustics transmission loss of simple expansion chamber having two inlets two outlet through analytical method and finite element methods.
- Comparison of transmission loss curve of finite element methods and analytical method.

1.5 Literature Review

Transmission Loss is the ratio of the sound power of the incident (progressive) pressure wave at the inlet of the muffler to the sound power of the transmitted pressure wave at the outlet of the muffler[3]. The benefit of TL is that it is a parameter of the muffler alone and the source or termination properties are not needed. Because of the simplifications, the TL is the most common and versatile parameter for muffler performance. Transmission loss of a muffler can be determined by three methods now a day experimentally, analytically, and using finite element methods.

There are basically three experimental techniques for the determination of the transmission. These are decomposition method, two source method, two load method[5]. Decomposition method states that if a two microphone random excitation is used, sound pressure may be decomposed into its incident and reflected wave. After the wave is decomposed the sound power of input wave may be calculated. But the drawback of this method is that anechoic termination is required at the termination for calculating transmission loss.

Munjal *et al.* in his proposed the two source-location method for evaluation of four pole parameter of an aeroacoustics elements [6]. In this method there are sound source placed as shown in fig 1.4 and using the transfer matrix method four pole equation for a straight tube between the two microphone 1-2 and 3-4 can be obtained.

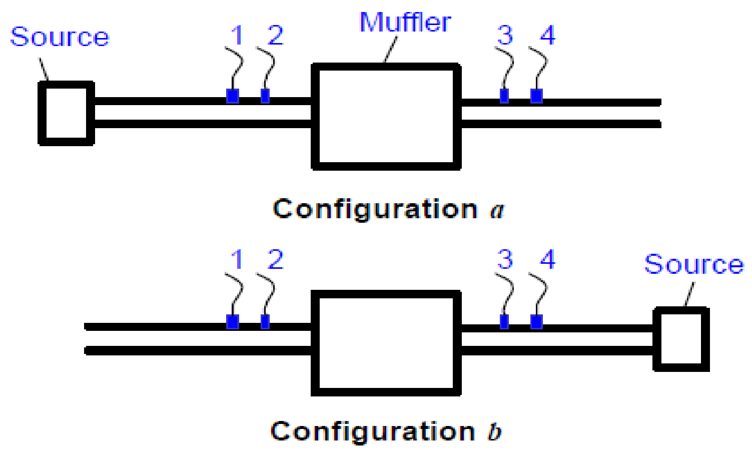


Figure 1.4 Two source location method[5]

T.Y Lung *et al.* proposed the method of two load method for determination of the transmission loss[7]. In this method instead of moving the sound source to the other end in order to the additional two equations the different end conditions are used for getting the same.

The analytical method used for determining the transmission loss is the transfer matrix method. **M.L. Munjal** in his classical book acoustics of duct and muffler [4] describes how the transmission loss of acoustics filter can be determined through transfer matrix method. He basically divides the acoustics filter into acoustic elements in three basic form distributed elements, lumped inline elements, lumped shunt elements and gave the transfer matrix for the same. The overall transfer matrix can be obtained by multiplying transfer matrix of acoustics elements used in acoustics filter. And by using the same we can get the transmission loss for the acoustics filter.

Transfer matrix method are commonly used for determination of transmission loss for single inlet single outlet muffler assuming plane wave propagation. As overall transfer matrix obtained by multiplying the individual matrix is limited only to acoustics elements in series hence it will not be applicable for multiple ports chamber[8].

X Hua *et al* in his work described how transmission loss for multi inlet muffler can be determined through superposition method[8]. In this method sound pressure at the outlet can be divided into two separate contributions one is the sound pressure contributed by the inlet port 1 assuming second inlet anechoic. And other contribution from the second inlet port assuming first inlet port anechoic. So it basically divides the two inlet one outlet muffler into two single inlets single outlet muffler by blocking the block alternately. Then by the use of transfer function transmission loss can be determined.

A.selamet *et al.* in his work consider a two end inlet one side outlet expansion chamber for determining the transmission loss[9]. He presents a one dimensional approach to estimate the transmission loss and determine the same through boundary element method. In order to show the applicability of accuracy of one dimensional solution. He also investigates the effect of geometry and incident wave condition on acoustics performance of muffler.

Z.L.Ji *et al.* in his work consider a single inlet two outlet expansion chamber for determining its attenuation performance using three dimensional analytical approach[9]. This approach includes the continuity conditions of acoustic pressure and particle velocity at the outlets and inlet with the orthogonality relation of Fourier-Bessel function. The results of the analytical method are compared with the boundary element predictions. The effect of expansion chamber length and location of inlet and outlets on acoustics attenuation is also studied.

A.Mimani *et al.* consider an elliptical cylindrical chamber having single side/end inlet and multiple side/end outlet for determining its acoustics behaviour[10]. The analysis used is based on the 3-D semi-analytical formulation which is based on model expansion and the Green’s function. The acoustics pressure response obtained in term of Green’s function is integrated over surface area of side/end ports and upon subsequent division by the port area, which gives the impedance matrix parameters due to uniform piston driven model. The results obtained from the 3-D semi analytical method are then compared with the 3-D FEA(SYSNOISE) simulations. Which are found to be in good agreement and validate the semi analytical method presented in this work. Besides this this paper also includes the parametric studies such as effect of axial and angular location of ports, interchanging the location of outlet and inlet ports, effect of chamber length as well as the addition of outlet port for double outlet muffler on transmission loss is studied.

A.Mimani *et al.* presents the generalized algorithm for analyzing a network of acoustics filter having multiport elements interconnected in arbitrary manner through their respective port[11]. Characterization of multi-port element is done by the impedance matrix and the junction through which these elements are interconnected are characterized by the continuity of mass velocity and acoustics pressure. For entire network a connectivity matrix is written. The acoustics pressure at the network terminations are connected with the mass velocity at the external in order to get the global impedance matrix. Generalized expression are obtained for the determination of transmission loss, level difference, insertion loss for a multi-port system in term of scattering and impedance matrix. Impedance matrix elements are evaluated using the axial plane wave theory. The characterization of impedance matrix is then used to analyse network of multi-port element. And the results are compared with the 3D FEA(SYSNOISE).

D.P.Jena *et al.* in his work presents suitability of finite element method in frequency domain for determining the transmission loss for perforated filters at zero mean flow condition[12]. In order to achieve so a three pole method has been carried out which exactly replicate the experimental transmission loss tube set up. This work resolved the complexity associated with simulating anechoic termination to perform three pole measurement. The desired meshing constraint associated with perforated plate has been quantified. This work also considers the case of external perforation in order to simulate the perforation facing to atmosphere. The methodology is verified by considering a case of evaluating transmission of perforated tube and of a Helmholtz resonator with a leak. The methodology can be used to for acoustics analysis of any shape and size of perforated components and reactive filter with perforated elements.

Chapter 2

Theory of Muffler Acoustics

The theory of acoustics of muffler presented in this chapter is taken from Munjal's classical book 'Acoustics of ducts and muffler'[4] and from the online video lectures of Acoustics by Prof Nachiketa Tiwari. The work of this thesis is a further extension of the work presented by Munjal's and Nachiketa as a result the theory will be covered for the material pertaining to this thesis.

This thesis uses a conventional coordinate system. The rectangular coordinate is represented is represented by x, y, z. the time vector is represented by t, and in frequency domain f is frequency in Hz.

2.1 Wave Equation in One Dimension

Let's take a box of air which has some finite dimensions dx, dy , dz initially there is no sound in the box and then because of some disturbances sound travels in the box and it travels out of the box So firstly we are trying to get how does pressure wave and velocity propagate through this box,

Assumptions:

1. Fluctuations in pressure, density, volume, are very small compared to the case when there was no sound.

2. Constant mass particle because when we have propagation of sound it is not mass transfer rather it has energy transfer so as energy is getting transferred mass is not necessarily transferred.

3. $\frac{\partial}{\partial y} = 0$; $\frac{\partial}{\partial z} = 0$ (one dimensional wave equation). 2.A

Assuming initially there is no sound in the box, conditions can be written as

Initial pressure = P_0

Initial density = ρ_0

Initial volume $= v_0$

Initial velocity $= V_0$

Final conditions

$$p_t(x,t) = p_0 + p(x,t) \tag{2.1}$$

$$\rho_t(x,t) = \rho_0 + \rho(x,t) \tag{2.2}$$

$$v_t(x,t) = v_0 + \tau(x,t) \tag{2.3}$$

$V_t(x,t) = V_0 + u(x,t) = u(x,t)$ (As we assume initial mass is stationary so $\mathrm{V}_0 = 0$) 2.4

Applying momentum equation

At time t, net external force on the air volume is

$$\left[-p_t(x+dx,t) + p_t(x,t)\right]dydz = \frac{d}{dt}(\rho_t v_t u) \tag{2.5}$$

$$\left[p_t(x+dx,t) - p_t(x,t)\right]dydz = -\frac{d}{dt}(\rho_t v_o u) \tag{2.6}$$

Multiplying and dividing by dx in LHS of above equation

$$\frac{[\mathrm{p}_t(x+dx,t) \; - p_t\left(x,t\right)]\mathrm{v}_t}{dx} = -\frac{d}{dt}(\rho_t v_o u) \tag{2.7}$$

Taking limits

$$\frac{\partial p_t}{\partial x} \; v_t \; = -\rho_0 \; v_0 \; \frac{\partial u}{\partial t} \tag{2.8}$$

$$\frac{\partial p}{\partial x} = -\frac{\rho_0 v_0}{v_t} \; \frac{\partial u}{\partial t} \tag{2.9}$$

$$\frac{\partial p}{\partial x} = -\rho_0 \frac{\partial u}{\partial t} \tag{2.10}$$

In sound propagation fluid behaves as isentropic

$$p_t v^{\gamma}{}_t = c \tag{2.11}$$

On differentiating

$$\frac{dp_t}{dt} v_t^{\gamma} + \gamma p_t v_t^{\gamma-1} \frac{dv_t}{dt} = 0 \qquad 2.12$$

$$\frac{\partial p}{\partial t} = -\frac{P_0 \gamma}{v_t} \frac{\partial \tau}{\partial t} \qquad 2.13$$

Now applying continuity equation

Out flow – In flow = change in volume

$$[u(x+\Delta x, t) - u(x,t)]\Delta A dt = \Delta \tau \qquad 2.14$$

Again multiplying by $\frac{\Delta x}{\Delta x}$ in the above equation we got

$$\frac{[u(x+\Delta x, t) - u(x,t)]\mathrm{v}_t}{\Delta x} = \frac{\partial \tau}{\partial t} \qquad 2.15$$

$$\frac{\partial u}{\partial t} v_t = \frac{\partial \tau}{\partial t} \qquad 2.16$$

From equation 2.13 and 2.16

$$\frac{\partial p}{\partial t} = -P_0 \gamma \frac{\partial u}{\partial x} \qquad 2.17$$

Diff. equation 2.17 w.r.t. time

$$\frac{\partial^2 p}{\partial t^2} = -P_0 \gamma \frac{\partial^2 u}{\partial x \partial t} \qquad 2.18$$

Diff. equation 2.10 w.r.t. x

$$\frac{\partial^2 p}{\partial x^2} = -\rho_0 \frac{\partial^2 u}{\partial x \partial t} \qquad 2.19$$

Assuming $\frac{\partial^2 u}{\partial x \partial t} = \frac{\partial^2 u}{\partial t \partial x}$ 2.20

From equation 2.17 and equation 2.19 we got

$$-\frac{1}{p_0 \gamma} \frac{\partial^2 p}{\partial t^2} = \frac{\partial^2 p}{\partial x^2} \frac{1}{\rho_0} \qquad 2.21$$

$$\frac{\partial^2 p}{\partial t^2} = \frac{p_0\gamma}{\rho_0}\frac{\partial^2 p}{\partial x^2} \qquad 2.22$$

Let's take $c^2 = \dfrac{p_0\gamma}{\rho_0}$ (where c = speed of sound) 2.23

$$\frac{\partial^2 p}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}$$ (one dimensional wave equation) 2.24

As $p(x,\mathrm{t})$ is function of space and time

Let $p(x,\mathrm{t}) = p_1(x)p_2(t)$ 2.25

$$c^2\frac{\partial^2 p_1}{\partial x^2}p_2 = p_1\frac{\partial^2 p}{\partial t^2} \qquad 2.26$$

$$c^2 p_1'' p_2 = p_1 p_2'' \qquad 2.27$$

$$c^2\left(\frac{p_1''}{p_1}\right) = \left(\frac{p_2''}{p_2}\right) = -k^2 \qquad 2.28$$

So Equation in space

$$c^2\left(\frac{p_1''}{p_1}\right) = -k^2 \qquad 2.29$$

Equation in time

$$\left(\frac{p_2''}{p_2}\right) = -k^2 \qquad 2.30$$

Now solving equation 2.29 and 2.30 we got

$$p_1 = p_1 e^{\left(\frac{-k}{c}xj\right)} \qquad 2.31$$

$$p_2 = p_2 e^{(\pm kjt)} \qquad 2.32$$

$$p(x,t) = p_1 p_2 \qquad 2.33$$

$$p(x,t) = pe^{\left(\frac{-kx}{c}\pm kt\right)j} \qquad 2.34$$

$$p(x,t) = f_1\left(t - \frac{x}{c}\right) = f_2\left(t + \frac{x}{c}\right) \quad \text{(General solution of wave equation)} \qquad 2.35$$

2.1.1 Transmission Line Equation for Acoustics Waves in Wave Guide

$p(x,t)$ = sum of forward going waves + sum of backward travelling waves

$$p(x,t) = \left[f_1\left(t - \frac{x}{c}\right) + f_2\left(t - \frac{x}{c}\right) + ... \right] + \left[f_1\left(t + \frac{x}{c}\right) + f_2\left(t + \frac{x}{c}\right) + ... \right] \qquad 2.36$$

Let the forward travelling wave is harmonic or sinusoidal in nature

$$p(x,t) = \mathrm{Re}\left[p_+ e^{s\left(t - \frac{x}{c}\right)} + p_- e^{s\left(t + \frac{x}{c}\right)} \right] \qquad 2.37$$

where s is the complex frequency

2.1.2 Velocity Equation

$$\frac{\partial^2 u}{\partial x^2}(x,t) = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} \qquad 2.38$$

By analogy to pressure wave equation velocity equation can be written as

$$u(x,t) = \mathrm{Re}\left[u_+ e^{s\left(t - \frac{x}{c}\right)} + u_- e^{s\left(t + \frac{x}{c}\right)} \right] \qquad 2.39$$

From equation 2.10 we can write

$$\frac{\partial p}{\partial x} = -\rho_0 \frac{\partial u}{\partial t} \qquad 2.40$$

Substituting p and u in the above equation from equation 2.37 and 2.39

$$\mathrm{Re}\left[\left(p_+ e^{\left(-\frac{sx}{c}\right)} + p_- e^{\left(+\frac{sx}{c}\right)} \right) e^{st} \frac{s}{c} \right] = -\rho_0 \, \mathrm{Re}\left[\left(u_+ e^{\left(-\frac{sx}{c}\right)} + u_- e^{\left(+\frac{sx}{c}\right)} \right) s e^{st} \right] \qquad 2.41$$

From above equation we can write

$$p_+ = \rho_0 c u_+ \quad \text{and} \quad p_- = \rho_0 c u_- \qquad 2.42$$

So equation 2.39 can be rewritten as

$$u(x,t) = \mathrm{Re}\left[\frac{p_+}{z_0} e^{s\left(t-\frac{x}{c}\right)} + \frac{p_-}{z_0} e^{s\left(t+\frac{x}{c}\right)}\right] \qquad 2.43$$

where $z_0 = \rho_0 c$

2.1.3 Equation for a Straight Pipe

Assuming one dimensional wave propagation acoustics pressure and the particle velocities can be written as

$$p(x,t) = \left(c_1 e^{-jk_0 x} + c_2 e^{+jk_0 x}\right) \quad \text{(let's take } p_+ = c_1 \text{ and } p_- = c_2\text{)} \qquad 2.44$$

$$u(x,t) = \left(\frac{c_1}{\rho_0 c} e^{-jk_0 x} - \frac{c_2}{\rho_0 c} e^{+jk_0 x}\right) \qquad 2.45$$

Applying boundary conditions at each node in straight pipe (arbitrarily at x=0 and x=L) yields

$$\begin{pmatrix} p_1 \\ \rho_0 c u_1 \end{pmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \qquad 2.46$$

$$\begin{pmatrix} p_2 \\ \rho_0 c u_2 \end{pmatrix} = \begin{bmatrix} e^{-jk_0 x} & e^{+jk_0 x} \\ e^{-jk_0 x} & -e^{+jk_0 x} \end{bmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \qquad 2.47$$

Combining these equations and using Euler's formula provides the following relationship between the two nodes

$$\begin{pmatrix} p_1 \\ \rho_0 c_0 u_1 \end{pmatrix} = \begin{bmatrix} \cos(kL) & j\sin(kL) \\ j\sin(kL) & \cos(kL) \end{bmatrix} \begin{pmatrix} p_2 \\ \rho_0 c_o u_2 \end{pmatrix} \qquad 2.48$$

2.1.4 Equation for Expansion and Contraction

Assuming a one dimensional propagating plane wave cross each discontinuity and at sudden expansion and sudden contraction pressure and velocity is continuous.

$$p_0 = p_1 \tag{2.49}$$

$$v_0 = v_1 \tag{2.50}$$

Therefore, it can be written in matrix form as

$$\begin{pmatrix} p_0 \\ v_0 \end{pmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{pmatrix} p_1 \\ v_1 \end{pmatrix} \tag{2.51}$$

As mass velocity is defined as

$$v_0 = p_0 su \tag{2.52}$$

Applying definition of mass velocity in equation 18

$$\begin{pmatrix} p_0 \\ \rho_0 c_0 u_o \end{pmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{s_1}{s_0} \end{bmatrix} \begin{pmatrix} p_1 \\ \rho_0 c_0 u_1 \end{pmatrix} \tag{2.53}$$

2.2 Transfer Matrix Method

An acoustics filter or muffler geometrically can be defined as combination of different acoustics elements which are placed in the path of source and receiver.in this thesis we do not consider the effect of mass flow rate and temperature gradient effect to fully understand how these individual element behaves. Therefore, idealized case of no mass flow and temperature gradient will be presented in this section.

In order to understand acoustic, behaviour of any element we have to define the relationship between two state variable acoustics pressure (p) and mass velocity (v) both at the upstream and downstream of the elements. Most effective approach to define this relationship is Transfer matrix method (TMM). TMM method basically describe the relationship between p and v both at upstream and downstream through the use of 4 pole parameters. Each muffler element has their own 4 pole parameter, major benefit of this method is that if we have the combine the elements we just have to multiply the transfer matrix of different elements through matrix multiplication so that a global matrix having 4 pole parameters is formed which can be used to fully quantify the acoustics properties of the filter formed by the elements. As almost all the elements used in acoustics filter can be categorized into three different basic elements [4]: a distributed element, in line lumped element, shunt lumped elements. A distributed element represents a uniform tube, as in case of simple expansion chamber there are three distributed elements inlet pipe, expansion chamber, tail pipe. An in line lumped element represents a sudden area change as in the case of simple expansion chamber sudden expansion into the chamber and sudden contraction into the tailpipe are the inline lumped elements. A shunt lumped element represents a Helmholtz resonator or the quarter wave resonator. The transfer matrix for these basic are shown in equation given below:

For Distributed Elements:

$$\begin{bmatrix} \cos(k_0 l) & jY\sin(k_0 l) \\ \dfrac{j}{y}\sin(k_0 l) & \cos(k_0 l) \end{bmatrix} \quad 2.54$$

For in-Line Shunt Elements:

$$\begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \quad 2.55$$

For Shunt Elements:

$$\begin{bmatrix} 1 & 0 \\ \dfrac{1}{Z} & 1 \end{bmatrix} \quad 2.56$$

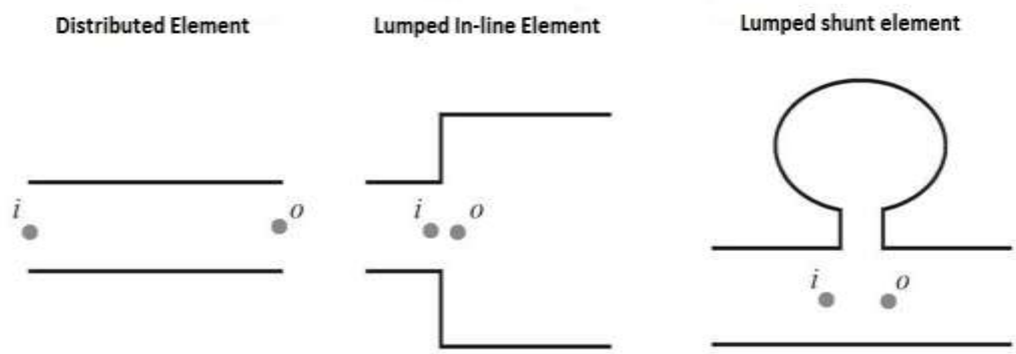


Figure 2.1 The three basic type of elements in the exhaust system[4]

The acoustic filter can be converted into an equivalent electric circuit by using the relationship of acoustic transmission networking to electric circuit [4]. The analogy can be represented as

Table 2-1 Comparison of acoustics elements with corresponding electric circuit elements

Acoustics Network		**Electric Circuit Network**	
Acoustics Pressure	p	Voltage	v
Mass Velocity	v	Current	i
Acoustics Impedance	Z	Electric Impedance	Z_{elec}
Resistance	R	Resistance	R_{elec}
Interance	M	Inductance	L
Compliance	C	Capacitance	C_{elec}

2.3 Muffler Performance Parameter

There are several acoustics parameters which determines acoustics performance of muffler. These includes Transmission loss (TL), Insertion loss (IL), Attenuation (ATT), Backpressure. However proper selection of performance parameter is essential for to properly draw conclusion on overall effectiveness of the exhaust system.

2.3.1 Transmission Loss (TL)

Transmission loss is defined as the difference between the acoustics power of the forward travelling wave at the inlet of the muffler to the forward travelling transmitted wave at the outlet [4]. Transmission loss requires an anechoic termination at the outlet which means there is no reflected wave in the outlet tube as shown in fig 2.2. below

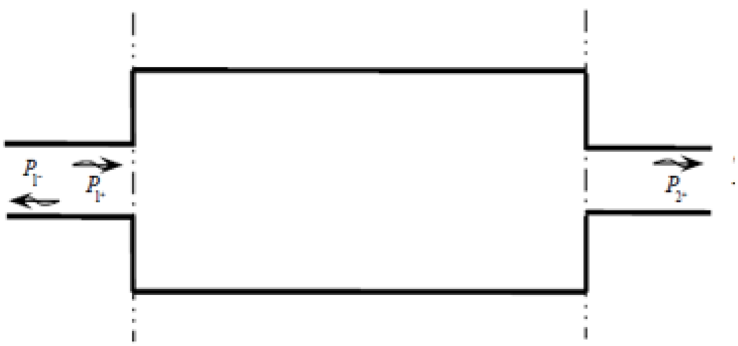


Figure 2.2 Anechoic termination[2]

Transmission loss of above simple expansion chamber can be written as:

$$TL = 20\log_{10}\left(\frac{p_{1+}}{p_{2+}}\right) + 10\log_{10}\left(\frac{S_0}{S_i}\right) \quad 2.57$$

Where S_0 and S_i represents the cross-section area of outlet and inlet respectively.

Generally, the cross-section area of inlet and outlet are same so the second term of the TL equation can be removed. Transmission loss using FEM techniques can be easily determined as the formulation of equation is very easy. The particle velocity at the inlet can be easily defined by giving boundary condition. Also the anechoic termination at the outlet can be modelled very easily. But this is not true experimentally as creation of anechoic termination is not feasible.

2.3.2 Insertion Loss (IL)

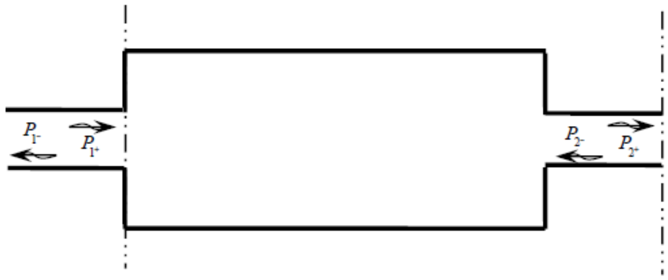
Insertion loss of the exhaust system can be defined as difference between the acoustics power radiated with and without muffler fitted. The equation of insertion loss can be written as[11]

$$IL = L_{w1} - L_{w2} = 10\log_{10}\left(\frac{w_1}{w_2}\right) \quad 2.58$$

Where L_w represents the sound pressure level in decibels and w represents the sound pressure level in Pascal’s (Pa). the subscript 1 is used for exhaust system with a straight pipe and the subscript 2 is used for exhaust system with a muffler.

2.3.3 Attenuation (ATT)

Attenuation can be defined as the difference between sound power incident at the muffler inlet and the sound power radiated at the outlet. Here the termination need not be anechoic.it predict the actual behaviour of muffler[2].



$$ATT = 20\log_{10}\left(\frac{P_{1^+}}{P_{2^+}}\right)$$

Figure 2.3 ATT of simple expansion chamber[2]

2.3.4 Backpressure

It is the extra static pressure exerted by the muffler configuration on the engine generated because of restriction in the passage of the exhaust gases created by the muffler geometry. Generally higher is the attenuation of sound greater is the backpressure generated[2]. Reactive muffler which are very good in attenuation impose large backpressure because in these type of muffler exhaust gas has to pass through the complex muffler geometry. Whenever there is change in direction of the exhaust gas additional backpressure will be created. Therefore, to limit the backpressure geometry changes has to be kept minimum.in passenger car backpressure is not a great concern but in the performance vehicle backpressure is of very great concern because it adversely affects the engine performance.

One of the possible solution of reducing backpressure is increasing the number of ports in the expansion chamber. However existing literature is limited only to single inlet single outlet muffler.

Chapter 3

Methodology

3.1 Analytical Methods

This section covers the analytical method used for single inlet single outlet, two inlets one outlet and two inlets two outlets expansion chamber in detail.

3.1.1 Transfer Matrix Method (TMM) for Single Inlet and Single Outlet Muffler

The TMM method basically separates the simple expansion chamber in individual components consisting of straight pipes, an expansion and a contraction. As we have already derived the 2×2 matrices in term of pressure and particle velocity for these elements in the chapter 2. So we can easily implement those equations here by dividing simple expansion chamber into sections or subsystems as shown in fig 3.1

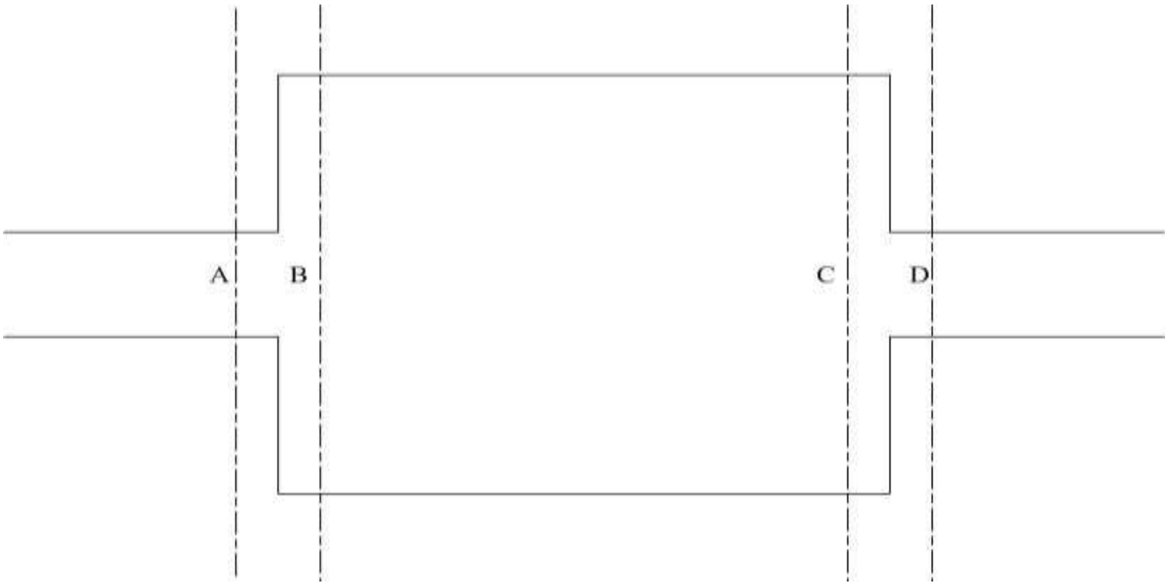


Figure 3.1 Sections in the simple expansion chamber

The transfer matrices for area discontinuity A-B and C-D can be derived by taking two assumptions

(1) pressure is continuous across the area discontinuity

(2) velocity is continuous across the area discontinuity

These assumptions can be written as

$$p_r = p_{r-1} \tag{3.1}$$

$$v_r = v_{r-1} \tag{3.2}$$

Equations can be written in the matrix form

$$\begin{bmatrix} p_r \\ v_r \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p_{r-1} \\ v_{r-1} \end{bmatrix} \tag{3.3}$$

For section A-B the transfer matrix can be written as

$$\begin{pmatrix} p_a \\ v_a \end{pmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{pmatrix} p_b \\ v_b \end{pmatrix} \tag{3.4}$$

For section C-D:

$$\begin{pmatrix} p_c \\ v_c \end{pmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{pmatrix} p_d \\ v_d \end{pmatrix} \tag{3.5}$$

For continuous area i.e. for distributed element matrix can be written as

$$\begin{pmatrix} p_b \\ v_b \end{pmatrix} = \begin{bmatrix} \cos(kl_{bc}) & jY_{bc} \sin(kl_{bc}) \\ \frac{j}{Y_{bc}} \sin(kl_{bc}) & \cos(kl_{bc}) \end{bmatrix} \begin{pmatrix} p_c \\ v_c \end{pmatrix} \tag{3.6}$$

The overall transfer matrix can be obtained by multiplying each expansion chamber subsystem matrices, so final matrix can be written as

$$\begin{pmatrix} p_a \\ \rho_0 c_0 u_a \end{pmatrix} = [I][II][III] \begin{pmatrix} p_f \\ \rho_0 c_0 u_f \end{pmatrix} \tag{3.7}$$

Therefore, overall transfer matrix can be defined as,

$$\begin{pmatrix} p_a \\ \rho_0 c_0 u_a \end{pmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{pmatrix} p_f \\ \rho_0 c_0 u_f \end{pmatrix} \tag{3.8}$$

Transmission loss of the muffler can be defined as

$$TL = 20\log_{10}\left[\left(\frac{Y_1}{Y_3}\right)^{\frac{1}{2}}\left|\frac{T_{11}+\dfrac{T_{12}}{Y_1}+Y_3T_{21}+\left(\dfrac{Y_3}{Y_1}\right)T_{22}}{2}\right|\right] \quad 3.9$$

3.1.2 Impedance Matrix Method for Two Inlets One Outlet Muffler

This section contains the impedance matrix method for determining transmission loss of muffler. Impedance matrix method presented in this section is underlying theory of muffler acoustics derived from the work of Mimani [11]. This muffler configuration consists of two inlet port on the one end face and one outlet port on the other end face as shown in fig 3.2

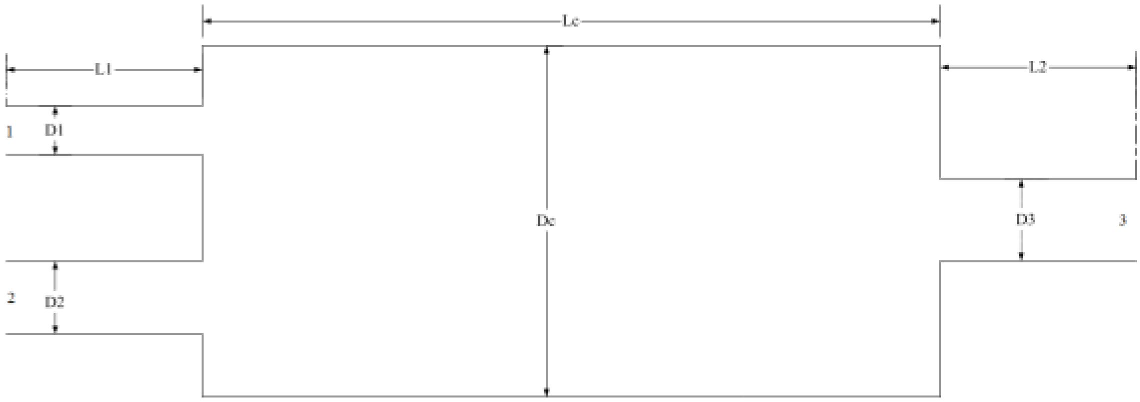


Figure 3.2 Two inlet one outlet expansion chamber

The impedance matrix of this system can be written as:

$$\begin{Bmatrix} p_1 \\ p_2 \\ p_3 \end{Bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{bmatrix} \begin{Bmatrix} v_1 \\ v_2 \\ v_3 \end{Bmatrix} \quad 3.10$$

Let us take the incident progressive wave directed into the system as A_i, while the reflective progressive wave in the outward direction as B_i .thus we can rewrite the above matrix as

$$\begin{Bmatrix} A_1 + B_1 \\ A_2 + B_2 \\ A_3 + B_3 \end{Bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{bmatrix} \begin{Bmatrix} \dfrac{A_1 - B_1}{Y_1} \\ \dfrac{A_2 - B_2}{Y_2} \\ \dfrac{A_3 - B_3}{Y_3} \end{Bmatrix} \tag{3.11}$$

we can relate the reflective progressive wave B_i to the incident progressive waves A_i by the use of scattering matrix in the following form:

$$\begin{Bmatrix} B_1 \\ B_2 \\ B_3 \end{Bmatrix} = \left[C_1^{-1} D_1 \right] \begin{Bmatrix} A_1 \\ A_2 \\ A_3 \end{Bmatrix} \tag{3.12}$$

Where C_1 and D_1 can be written as

$$C_1 = \begin{bmatrix} \dfrac{Z_{11}}{Y_1} + 1 & \dfrac{Z_{12}}{Y_2} & \dfrac{Z_{13}}{Y_3} \\ \dfrac{Z_{21}}{Y_1} & \dfrac{Z_{22}}{Y_2} + 1 & \dfrac{Z_{23}}{Y_3} \\ \dfrac{Z_{31}}{Y_1} & \dfrac{Z_{32}}{Y_2} & \dfrac{Z_{33}}{Y_3} + 1 \end{bmatrix} \tag{3.13}$$

$$D_1 = \begin{bmatrix} \dfrac{Z_{11}}{Y_1} - 1 & \dfrac{Z_{12}}{Y_2} & \dfrac{Z_{13}}{Y_3} \\ \dfrac{Z_{21}}{Y_1} & \dfrac{Z_{22}}{Y_2} - 1 & \dfrac{Z_{23}}{Y_3} \\ \dfrac{Z_{31}}{Y_1} & \dfrac{Z_{32}}{Y_2} & \dfrac{Z_{33}}{Y_3} - 1 \end{bmatrix} \tag{3.14}$$

Characterization of Impedance Matrix

Characterization of impedance matrix ([Z] matrix) is done using axial plane wave theory. To obtain the [Z] parameters we first excite the system at one port and block the others. For example, to get the [Z] matrix parameter for first column we block the port marked as 2 and 3.1.2 as shown in the fig. and excite the system at port 1. The Z matrices can be written as

$$Z_{11} = -jY_0 \frac{\cos(k_0 L)}{\sin(k_0 L)} \tag{3.15}$$

$$Z_{12} = -jY_0 \frac{\cos(k_0 L)}{\sin(k_0 L)} \tag{3.16}$$

$$Z_{13} = -jY_0 \frac{1}{\sin(k_0 L)} \tag{3.17}$$

$$Z_{21} = -jY_0 \frac{\cos(k_0 L)}{\sin(k_0 L)} \tag{3.18}$$

$$Z_{22} = -jY_0 \frac{\cos(k_0 L)}{\sin(k_0 L)} \tag{3.19}$$

$$Z_{23} = -jY_0 \frac{1}{\sin(k_0 L)} \tag{3.20}$$

$$Z_{31} = -jY_0 \frac{1}{\sin(k_0 L)} \tag{3.21}$$

$$Z_{32} = -jY_0 \frac{1}{\sin(k_0 L)} \tag{3.22}$$

$$Z_{33} = -jY_0 \frac{\cos(k_0 L)}{\sin(k_0 L)} \tag{3.23}$$

We can see that $\left[C_1^{-1}D_1\right]_{3\times3}$ is actually $[S]_{(3\times3)}$ matrix, so we can write again the relation between $[B]$ and $[A]$ as

$$\begin{Bmatrix} B_1 \\ B_2 \\ B_3 \end{Bmatrix} = [S]_{(3\times3)} \begin{Bmatrix} A_1 \\ A_2 \\ A_3 \end{Bmatrix} \tag{3.24}$$

We can also define relation between $[S]$ and $[Z]$ as

$$[S]_{(4\times4)} = \left[Z[Y]^{-1} + I\right]^{-1}\left[Z[Y]^{-1} - I\right] \tag{3.25}$$

Where, $[I]$ is the identity matrix, $[Y]$ is the diagonal matrix consisting of characteristic impedance of the pipes having size equal to the $[Z]$ matrix.

Now reflected wave at the outlets can be express as a linear combination of ratio of the incident wave amplitude at the inlets

$$B_3 = A_1\left\{S_{31} + \lambda_{12}S_{32}\right\} \tag{3.26}$$

Now total acoustics pressure associated with the incident wave is written as

$$E_{total\_incident} = \frac{A_1^2}{2\rho_0}\left\{\frac{1}{Y_1} + \frac{\lambda_{12}^2}{Y_2}\right\} \tag{3.27}$$

Total acoustics pressure associated with transmitted wave is written as

$$E_{total\_\mathrm{transmitted}} = \frac{1}{2\rho_0}\left\{\frac{B_3^2}{Y_3}\right\} \tag{3.28}$$

And Transmission loss can be defined as

$$TL = 10\log_{10}\left(\frac{E_{total\_incident}}{E_{total\_transmitted}}\right) \tag{3.29}$$

3.1.3 Impedance Matrix Method for Two Inlets Two Outlets Muffler

This section contains the impedance matrix method for determining transmission loss of muffler. Impedance matrix method presented in this section is underlying theory of muffler acoustics derived from the work of Mimani [11]. This muffler configuration consists of two inlet port on the one end face and one outlet port on the other end face as shown in fig 3.2

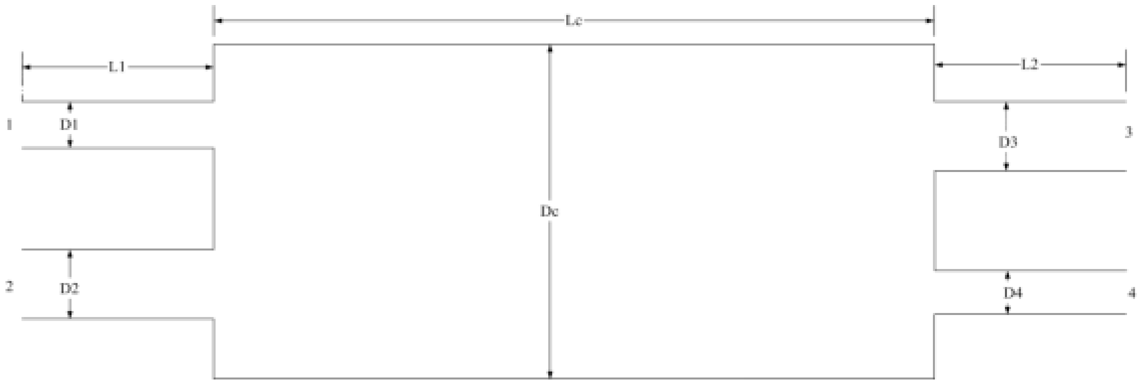


Figure 3.3 Two inlet two outlet expansion chamber

The global impedance matrix can be written as

$$\begin{Bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{Bmatrix} = \begin{bmatrix} z_{11} & z_{12} & z_{13} & z_{14} \\ z_{21} & z_{22} & z_{23} & z_{24} \\ z_{31} & z_{32} & z_{33} & z_{34} \\ z_{41} & z_{42} & z_{43} & z_{44} \end{bmatrix} \begin{Bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{Bmatrix} \qquad 3.30$$

Let us take the incident progressive wave directed into the system as A_i, while the reflective progressive wave in the outward direction as B_i .thus we can rewrite the above matrix as

$$\begin{Bmatrix} A_1 + B_1 \\ A_2 + B_2 \\ A_3 + B_3 \\ A_4 + B_4 \end{Bmatrix} = \begin{bmatrix} z_{11} & z_{12} & z_{13} & z_{14} \\ z_{21} & z_{22} & z_{23} & z_{24} \\ z_{31} & z_{32} & z_{33} & z_{34} \\ z_{41} & z_{42} & z_{43} & z_{44} \end{bmatrix} \begin{Bmatrix} \dfrac{A_1 - B_1}{Y_1} \\ \dfrac{A_2 - B_2}{Y_2} \\ \dfrac{A_3 - B_3}{Y_3} \\ \dfrac{A_4 - B_4}{Y_4} \end{Bmatrix} \qquad 3.31$$

After a little algebra we can relate the reflective progressive wave B_i to the incident progressive waves A_i in the following form

$$\begin{Bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \end{Bmatrix} = \left[C_1^{-1} D_1 \right]_{(4\times 4)} \begin{Bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{Bmatrix} \qquad 3.32$$

Where C_1 and D_1 can be written as:

$$C_1 = \begin{bmatrix} \dfrac{Z_{11}}{Y_1} + 1 & \dfrac{Z_{12}}{Y_2} & \dfrac{Z_{13}}{Y_3} & \dfrac{Z_{14}}{Y_4} \\ \dfrac{Z_{21}}{Y_1} & \dfrac{Z_{22}}{Y_2} + 1 & \dfrac{Z_{23}}{Y_3} & \dfrac{Z_{24}}{Y_4} \\ \dfrac{Z_{31}}{Y_1} & \dfrac{Z_{32}}{Y_2} & \dfrac{Z_{33}}{Y_3} + 1 & \dfrac{Z_{34}}{Y_4} \\ \dfrac{Z_{41}}{Y_1} & \dfrac{Z_{42}}{Y_2} & \dfrac{Z_{43}}{Y_3} & \dfrac{Z_{44}}{Y_4} + 1 \end{bmatrix} \qquad 3.33$$

$$D_1 = \begin{bmatrix} \frac{Z_{11}}{Y_1} - 1 & \frac{Z_{12}}{Y_2} & \frac{Z_{13}}{Y_3} & \frac{Z_{14}}{Y_4} \\ \frac{Z_{21}}{Y_1} & \frac{Z_{22}}{Y_2} - 1 & \frac{Z_{23}}{Y_3} & \frac{Z_{24}}{Y_4} \\ \frac{Z_{31}}{Y_1} & \frac{Z_{32}}{Y_2} & \frac{Z_{33}}{Y_3} - 1 & \frac{Z_{34}}{Y_4} \\ \frac{Z_{41}}{Y_1} & \frac{Z_{42}}{Y_2} & \frac{Z_{43}}{Y_3} & \frac{Z_{44}}{Y_4} - 1 \end{bmatrix} \quad 3.34$$

Characterization of Impedance Matrix

This system is then characterize using impedance matrix characterization [Z] with axial plane wave theory. To obtain the [Z] parameters we excite the system at one port and block the others. For example, to get the [Z] matrix parameter for first column we block the port marked as 2,3, and 4 in the fig. and excite the system at port 1. the elements of impedance matrix can be written as

$$z_{11} = -jY_0 \frac{\cos(k_0 L)}{\sin(k_0 L)} \quad 3.35$$

$$z_{12} = -jY_0 \frac{\cos(k_0 L)}{\sin(k_0 L)} \quad 3.36$$

$$z_{13} = -jY_0 \frac{1}{\sin(k_0 L)} \quad 3.37$$

$$z_{14} = -jY_0 \frac{\cos(k_0 l_3)}{\sin(k_0 L)} \quad 3.38$$

$$z_{21} = -jY_0 \frac{\cos(k_0 L)}{\sin(k_0 L)} \quad 3.39$$

$$z_{22} = -jY_0 \frac{\cos(k_0 L)}{\sin(k_0 L)} \quad 3.40$$

$$z_{23} = -jY_0 \frac{1}{\sin(k_0 L)} \quad 3.41$$

$$z_{24} = -jY_0 \frac{1}{\sin(k_0 L)} \quad 3.42$$

$$z_{31} = -jY_0 \frac{1}{\sin(k_0 L)} \tag{3.43}$$

$$z_{32} = -jY_0 \frac{1}{\sin(k_0 L)} \tag{3.44}$$

$$z_{33} = -jY_0 \frac{\cos(k_0 L)}{\sin(k_0 L)} \tag{3.45}$$

$$z_{34} = -jY_0 \frac{\cos(k_0 L)}{\sin(k_0 L)} \tag{3.46}$$

$$z_{41} = -jY_0 \frac{1}{\sin(k_0 L)} \tag{3.47}$$

$$z_{42} = -jY_0 \frac{1}{\sin(k_0 L)} \tag{3.48}$$

$$z_{43} = -jY_0 \frac{\cos(k_0 L)}{\sin(k_0 L)} \tag{3.49}$$

$$z_{44} = -jY_0 \frac{\cos(k_0 L)}{\sin(k_0 L)} \tag{3.50}$$

We can see that $\left[C_1^{-1} D_1\right]_{(4\times4)}$ is actually the scattering matrix [S].so we can again write

$$\begin{Bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \end{Bmatrix} = [S]_{(4\times4)} \begin{Bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{Bmatrix} \tag{3.51}$$

We can also define relation between $[S]$ and $[Z]$ as

$$[S]_{(4\times4)} = \left[Z[Y]^{-1} + I \right]^{-1} \left[Z[Y]^{-1} - I \right] \tag{3.52}$$

Where

$[I]$ is the identity matrix, $[Y]$ is the diagonal matrix consisting of characteristic impedance of the pipes having size equal to the $[Z]$ matrix.

Now reflected wave at the outlets can be express as a linear combination of ratio of the incident wave amplitude at the inlets

$$B_3 = A_1\left\{S_{31} + \lambda_{12}S_{32}\right\} \tag{3.53}$$

$$B_4 = A_1\left\{S_{41} + \lambda_{12}S_{42}\right\} \tag{3.54}$$

Where $\lambda_{12} = \dfrac{A_2}{A_1}$ = Ratio of incident wave amplitude at the inlet.

Now total acoustics pressure associated with the incident wave is written as

$$E_{total\_incident} = \frac{A_1^2}{2\rho_0}\left\{\frac{1}{Y_1} + \frac{\lambda_{12}^2}{Y_2}\right\} \tag{3.55}$$

Total acoustics pressure associated with transmitted wave is written as

$$E_{total\_\mathrm{transmitted}} = \frac{1}{2\rho_0}\left\{\frac{B_3^2}{Y_3} + \frac{B_4^2}{Y_4}\right\} \tag{3.56}$$

where Y_1, Y_2, Y_3 and Y_4 represents the characteristic impedance and is equals to $\dfrac{C_0}{S_{cross}}$ where

C_0 is the speed of sound and S_{cross} represents cross-sectional area.

And Transmission loss can be defined as

$$TL = 10\log_{10}\left(\frac{E_{total\_incident}}{E_{total\_transmitted}}\right) \tag{3.57}$$

3.2 Simulation

This section is about finite element analysis of the various muffler configuration that has been described in the chapter 2. results obtained will be validated with the analytical methods

3.2.1 Finite Element Model for Acoustics Analysis

(a) The General Acoustics Equation

Considering enclosed volume of fluid, which is in-viscous, compressible, with uniform density and pressure, without mean flow and also enclosure wall is assumed to be ideally rigid. The acoustics wave equation may be written as

$$\nabla^2 p = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} \tag{3.58}$$

Where c is the speed of sound in medium, p is acoustics pressure, t is time and ∇^2 is Laplace operator[4]. As this equation does not consider viscous dissipation, above expression can also be considered loss-less wave equation for sound propagation. Then, for a small change in pressure δp, over a finite fluid volume, the wave equation can be represented in the integral form as

$$\int_v \frac{1}{c^2}\partial p \frac{\partial^2 p}{\partial t^2} dV + \int_v \left(\{L\}^T \delta p\right)\left(\{L\} p\right) dV - \int_s \{n\}^T \delta p \left(\{L\} p\right) dS = \{0\} \tag{3.59}$$

Where, the matrix operator can be defined as

$$\{L\}^T = \begin{bmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{bmatrix} \text{ and, } \{L\} = \begin{bmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{bmatrix}^T \tag{3.60}$$

In the above equation, V is the volume of the fluid domain, S is the surface here the derivative of the pressure normal to the surface is applied and $\{n\}$ is the unit vector normal to the surface S.

In simulating fluid-structure interaction problem, the surface S is treated as fluid solid interface. The normal pressure gradient of the fluid and the normal acceleration of the structure at the fluid-structure interface S can be expressed as

$$\{n\}\cdot\{\nabla p\} = -\rho_0 \{n\} \frac{\partial^2 \{u\}}{\partial t^2} \tag{3.61}$$

Where $\{u\}$ is the displacement vector of the structure at the interface. Now Eq. 3.61 can be rewritten as

$$\int_v \frac{1}{c^2}\delta p \frac{\partial^2 p}{\partial t^2} dV + \int_v \left(\{L\}^T\right)\left(\{L\} p\right) dV + \rho_0 \int_s \delta p \{n\}^T \left(\frac{\partial^2}{\partial t^2}\{u\}\right) dS = \{0\} \tag{3.62}$$

In finite element modelling, using element shape function $\{N\}$ for pressure, element shape function $\{N^{'}\}^T$ for displacement, and nodal pressure vector $\{\ddot{p}_e\}$, the second order derivative can be represented as

$$\frac{\partial^2 p}{\partial u^2} = \{N\}^T \{\ddot{p}_e\} \tag{3.63}$$

$$\frac{\partial^2}{\partial u^2}\{u\} = \{N^{'}\}^T \{\ddot{u}_e\} \tag{3.64}$$

Where $p = \{N\}^T \{p_e\}$ and $u = \{N\}^T \{u_e\}$. Now, the discretized wave equation can be written as

$$[M_F]\{\ddot{p}_e\} + [K_F]\{p_e\} + \rho_0 [R]^T \{\ddot{u}_{F,e}\} = \{0\} \tag{3.65}$$

Where,

$$[M_F] = \frac{1}{c^2}\int_v \{N\}\{N\}^T dV \tag{3.66}$$

$$[K_F] = \int_v [\nabla N]^T [N] dV \tag{3.67}$$

$$[R]^T = \int_s \{N\}\{n\}^T \{N^{'}\}^T dS \tag{3.68}$$

The $[M_F], [K_F], [R]^T$ are the acoustics fluid mass matrix, acoustics fluid stiffness matrix and acoustics fluid coupling matrix respectively. Assuming time harmonic input, $p = pe^{j\omega t}$. The Eq. 3.58 can be re-written as

$$\nabla^2 p + k^2 p = 0 \tag{3.69}$$

Where $k = (\omega / c)$ is wave number, ω is the angular frequency $(\omega = 2\pi f / c)$.

3.2.2 Determination of Transmission Loss Using ANSYS 15.0

(a) Single Inlet Single Outlet Expansion Chamber

- **Solid model creation**

The first step with any finite element analysis always started with a solid model generation. In this work solid model of simple expansion chamber is created by using design modeller ANSYS. Solid model is shown in the figure.3.4

Figure 3.4 Solid model of single inlet single outlet expansion chamber

- **Selecting mesh element**

For proper analysis it is very important to define proper element. FLUID30 is used for modelling the fluid medium and the interface in the in fluid/structure interaction problems. The geometry configuration includes a brick(hexahedral), wedge, pyramid and tetrahedral. Each node has one pressure degree of freedom, and three optional translational degree of freedom along the x, y, and z axes. Figure 3.5 shows the Fluid 30 element.

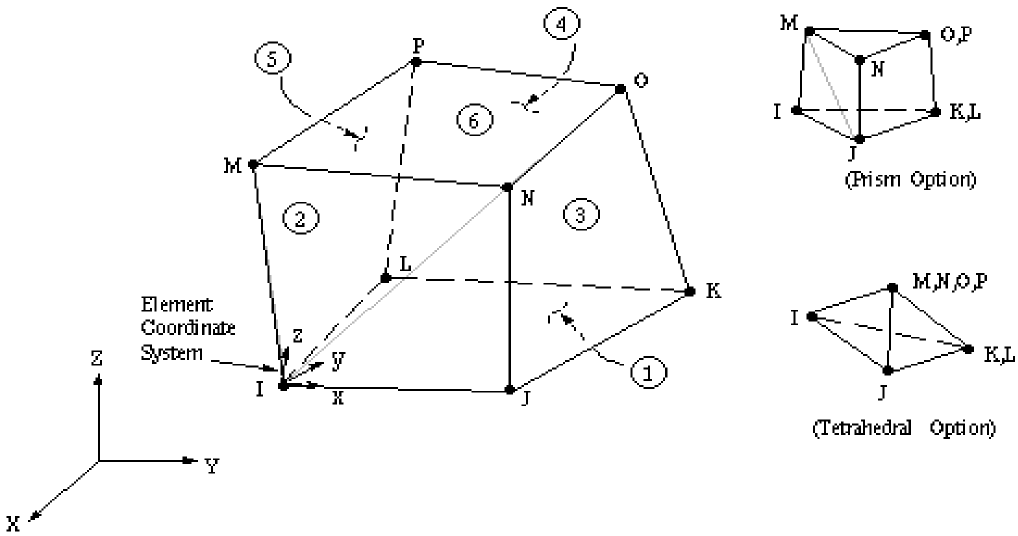


Figure 3.5 Fluid 30 element[13]

The element is capable of modelling sound absorbing material on the boundary and an impedance sheet inside a fluid can also be defined. The element can also be used with other 3-D structural elements to perform unsymmetrical or damped model, harmonic and transient method analyses.

- **Meshing**

For generating proper meshing, wavelength determination is very important as we are doing analysis using FEA method. Normally to calculate the transmission loss 8 element per wavelength of FLUID30 is sufficient.

For wavelength calculation,

$$\lambda = \frac{c}{f} \tag{3.60}$$

Since the range of frequency of muffler is from 0-2000 Hz

Maximum frequency (f) = 2000 Hz.

Sonic velocity (c) = $343\,\text{mt/sec.}$

$\lambda = \frac{343}{2000} = 0.1715\,\text{mt}$

As there has to be 8 elements per wavelength,

Then the Mesh density or the maximum element size = $\frac{0.1715}{8} = 0.0214\,\text{mt.}$

Fig 3.6 shows a good quality mesh is generated with a mesh density of 10mm. which is more than the mesh density of 28.5mm. this will give room for more accurate solution. The generated mesh gives a total number of 6050 elements and 10444 nodes. The mesh used is hybrid mesh which is good enough for this type of analysis.

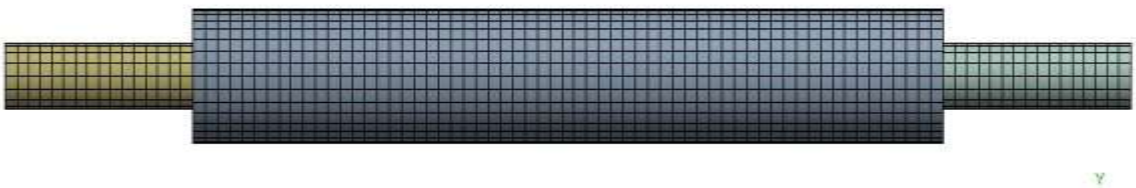


Figure 3.6 Meshing of single inlet single outlet chamber

- **Boundary Conditions**

ANSYS can understand the nature of the problem only if the boundary conditions are defined. So for starting any analysis we need to define the boundary conditions.in this analysis first thing we have to define the ports which is nothing but defining inlet and outlet ports which make the problem visible to the ANSYS. Inlet port is the port where sound source comes and the outlet port is the port where sound goes out of the muffler. Next thing is to define defining the body as acoustics body. When the acoustics body is inserted into an analysis, bodies can be designated as acoustics domain to be meshed with acoustics elements. ANSYS workbench will mesh bodies with structural solid elements by default and Acoustics Body option will transform the body into acoustics fluid elements. Next boundary to define is the acoustics normal surface velocity at the inlet of the muffler. An acoustics source can be simulated by applying a normal surface velocity to a face on the exterior of the acoustics domain. The vibrating surface causes acoustics particle adjacent to the surface to move and therefore will generate an acoustics pressure. In this analysis amplitude of the normal surface velocity is given (4.8 mm sec^-1). Now in order to estimate the transmission loss it is mandatory to define anechoic condition at inlet and outlet. This can be simulated by applying radiation boundary condition at the exterior face of the inlet and outlet.

- **Solution**

To obtain transmission loss curve a we need transmission loss values at several frequencies. For this we have performed a detailed harmonic analysis at several frequencies. The harmonic analysis is done in step of 100Hz each like 0Hz,100Hz…2000Hz.

(b) Finite Element Analysis of Two Inlets One Outlet Expansion Chamber

- **Solid model creation**

The first step with any finite element analysis always started with a solid model generation. In this work solid model of simple expansion chamber with two inlets one outlet is created by using design modeller ANSYS. Solid model is shown in the figure.3.7

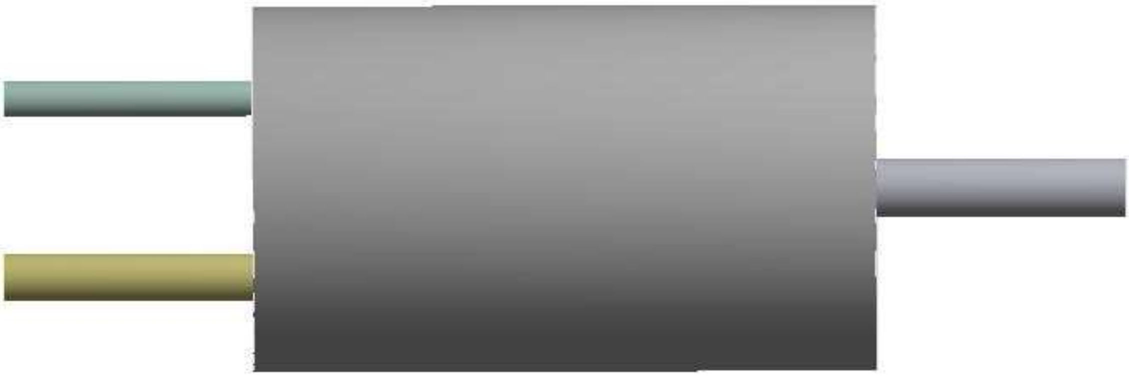


Figure 3.7 Solid model of two inlet one outlet expansion chamber

- **Meshing**

For this design procedure for mesh creation is same as for single inlet single outlet. A good quality mesh is generated with a mesh density of 10mm. which is more than the mesh density of 28.5mm. this will give room for more accurate solution. The generated mesh gives a total number of 6050 elements and 10444 nodes. The mesh used is hybrid mesh which is good enough for this type of analysis.

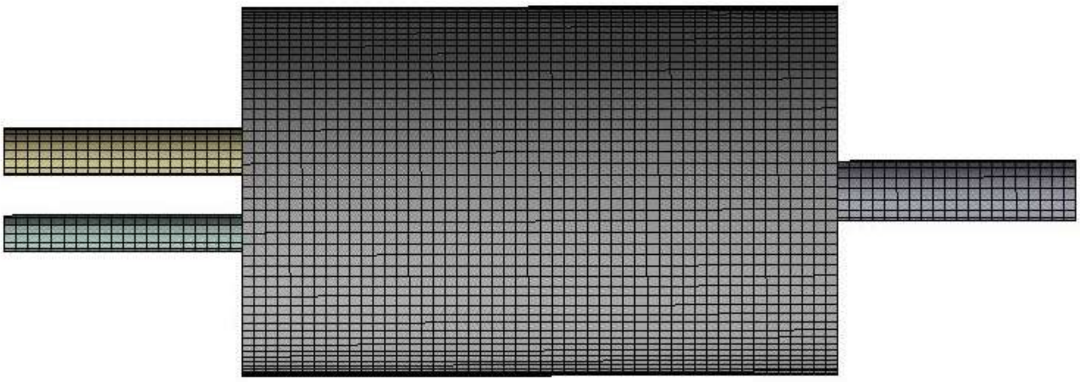


Figure 3.8 Meshing of two inlet one outlet (ANSYS)

- **Boundary Condition**

The boundary conditions are same as for single inlet single outlet expansion chamber.

- **Solution**

To obtain transmission loss curve a we need transmission loss values at several frequencies. For this we have performed a detailed harmonic analysis at several frequencies. The harmonic analysis is done in step of 100Hz each like 0Hz,100Hz…2000Hz.

(c) Finite Element Analysis of Two Inlets Two Outlets Expansion Chamber

- **Solid model creation**

The first step with any finite element analysis always started with a solid model generation. In this work solid model of simple expansion chamber with two inlets two outlets is created by using design modeller ANSYS. Solid model is shown in the figure.3.9

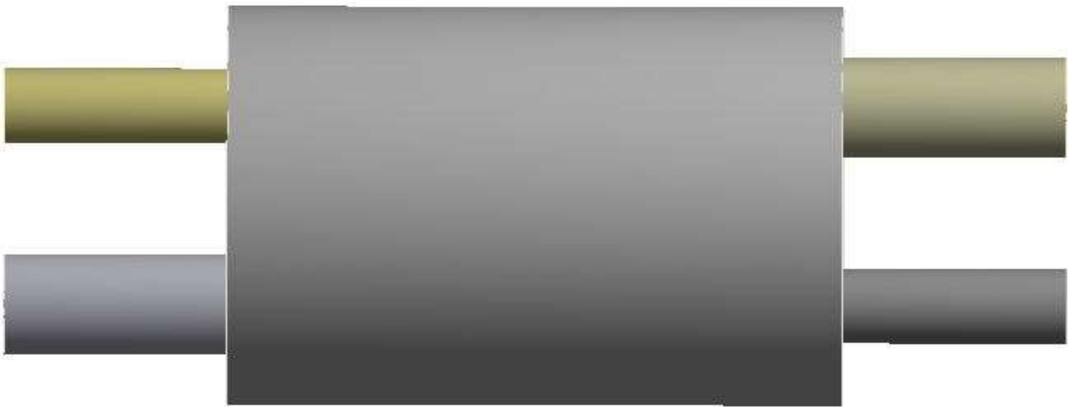


Figure 3.9 Solid model of two inlet two outlet expansion chamber

- **Meshing**

For this design procedure for mesh creation is same as for single inlet single outlet. A good quality mesh is generated with a mesh density of 10mm. which is more than the mesh density of 28.5mm. this will give room for more accurate solution. The generated mesh gives a total number of 6050 elements and 10444 nodes.

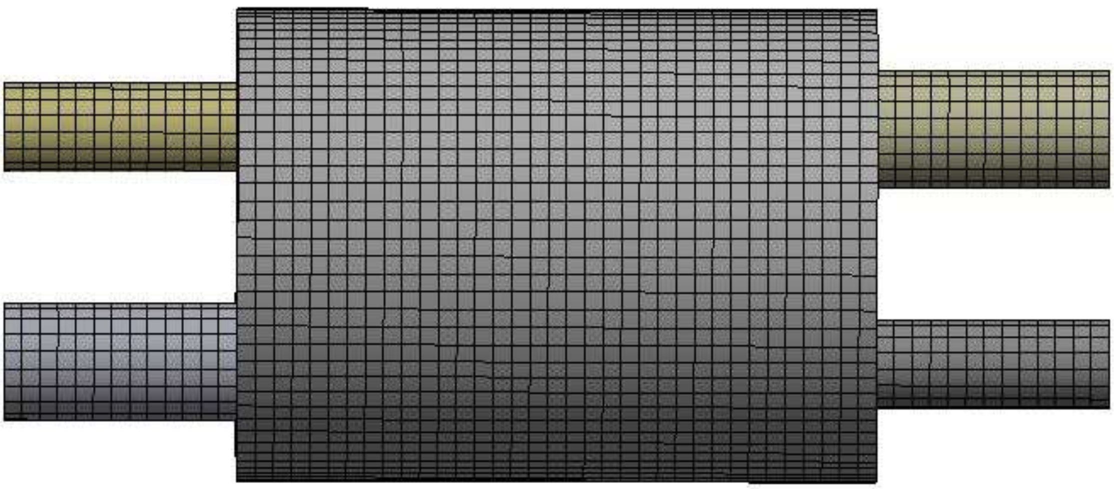


Figure 3.10 Meshing of two inlets two outlet expansion chamber (ANSYS)

- **Boundary Condition**

The boundary conditions are same as for single inlet single outlet expansion chamber.

Solution:

To obtain transmission loss curve a we need transmission loss values at several frequencies. For this we have performed a detailed harmonic analysis at several frequencies. The harmonic analysis is done in step of 100Hz each like 0Hz,100Hz…2000Hz.

3.2.2 Determination of Transmission Loss using BEM

(a) Single Inlet Single Outlet Expansion Chamber

The pressure acoustics module of COMSOL Multiphysics was used to create the muffler system under study.

- **Solid Model Generation**

The COMSOL built-in features are used to develop the solid model of muffler system. Solid model of muffler having single inlet single outlet is shown in the fig.3.11.

Figure 3.11 Solid model of single inlet single outlet (COMSOL)

- **Boundary Conditions**

A sound hard boundary is simulated along the exterior wall of the muffler system. The sound hard boundary wall adds a boundary condition for a wall at which the normal component of the acceleration is zero. Next boundary condition applied is the plane wave radiation at the inlet. This radiation condition allows an outgoing plane wave to leave the modelling domain with minimum reflections, when angle of incidence is near the normal. At the exterior surface of the port impedance boundary conditions is given. Impedance given in this analysis is 1.2[kg/m^3] *343[m/s] Pa-s/m.

- **Meshing**

A custom mesh size is generated and a triangular mesh was applied to the model. The element size is defined as 10epw. It is given using a user defined value 13536[in/s]/2000[Hz]/10. Where 13536 in/s is the sonic speed in air, 2000Hz is the maximum frequency value of the study and obviously 10 is the number of element per wavelength. There are 6 elements per wavelength is sufficient for frequency dependent studies in order to ensure that there are sufficient elements to characterize shape of the highest frequency wavelength. The mesh of the single inlet single outlet muffler system can be shown in the fig 3.12.

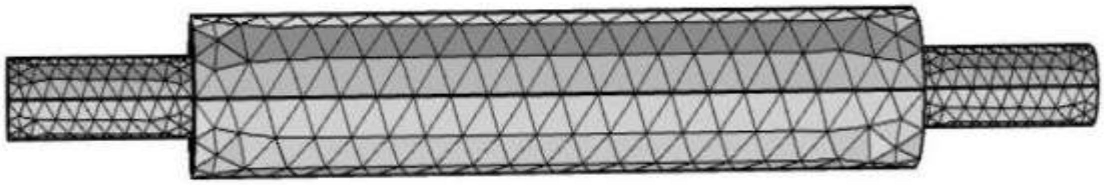


Figure 3.12 Meshing of single inlet single outlet expansion chamber (COMSOL)

For determining transmission loss in finite element model two variable [14] power of incoming and outgoing wave are defined at the inlet and outlet of the muffler respectively. These equations were defined in COMSOL are as follows:

Power of the incoming wave:

$$W_{in} = \mathrm{intopinlet}\left(\frac{{p_0}^2}{2} * \left(2 * acpr.rho * acpr.c\right)\right) \qquad 3.61$$

Power of the outgoing wave:

$$W_{out} = \mathrm{intopoutlet}\left(\frac{abs\left(p\right)^2}{2} * \left(2 * acpr.rho * acpr.c\right)\right) \qquad 3.62$$

Transmission loss:

$$TL = 10\log_{10}\left(\frac{W_{in}}{W_{out}}\right) \qquad 3.63$$

- **Solution:**

To obtain transmission loss curve a we need transmission loss values at several frequencies. For this we have performed a frequency dependent study is performed at several frequencies. The analysis is done in step of 100Hz each like 0Hz,100Hz…2000Hz.

(b) Two Inlet One Outlet Expansion Chamber:

- **Solid Model Generation:**

The COMSOL built-in features are used to develop the solid model of muffler system. Solid model of muffler having two inlets single outlet is shown in the fig.3.13.

Figure 3.13 Solid model of two inlets single outlet expansion chamber (COMSOL)

- **Boundary Condition**

The boundary condition applied are same as for single inlet single outlet.

- **Meshing**

Meshing is done in the same way as for single inlet single outlet muffler. Meshing of two inlets one outlet system can be shown in the fig.3.14

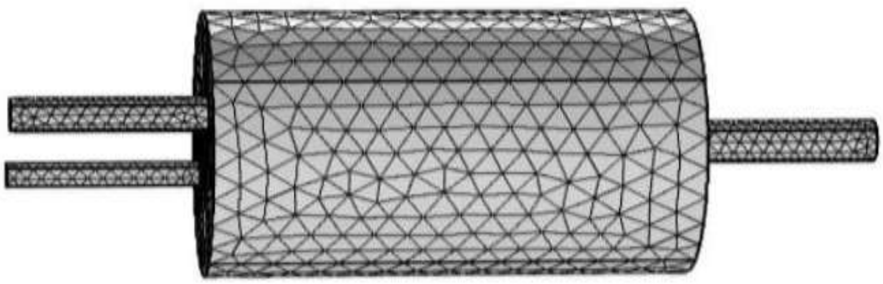


Figure 3.14 Meshing model of two inlets one outlet expansion chamber (COMSOL)

(c) Two Inlet Two Outlets Expansion Chamber

- **Solid Model Generation**

The COMSOL built-in features are used to develop the solid model of muffler system. Solid model of muffler having two inlets two outlets is shown in the fig 3.15.

Figure 3.15 Solid model of two inlets two outlets expansion chamber (COMSOL)

- **Boundary Condition**

The boundary condition applied are same as for single inlet single outlet.

- **Meshing**

Meshing is done in the same way as for single inlet single outlet muffler. Meshing of two inlets two outlets system can be shown in the fig.3.16

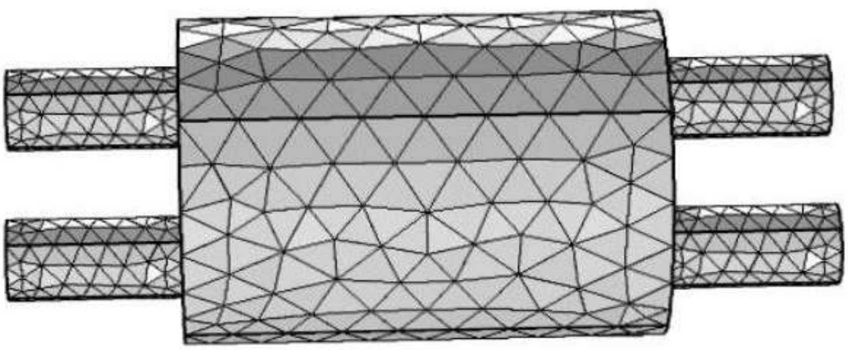


Figure 3.16 Meshing model of two inlets two outlet expansion chamber (COMSOL)

Chapter 4

Results and Discussions

4.1 Single Inlet Single Outlet

This section contains results obtained from all the three methods for single inlet single outlet expansion chamber.

4.1.1 Transfer Matrix Method Result

TMM method was performed as described in section 3.1.1 and iterated from 0Hz to 2000Hz with 2 Hz resolution using the MATLAB. The MATLAB code input parameters are provided below in the Table 4-1

Table 4-1 MATLAB code input parameter (SISO)

MATLAB VARIABLE	MATLAB Value	Unit
Max frequency	2000	Hz
res	2	Hz
frequency	0:2:2000	Hz
c	13536	in/s
density	0.00004335	lb/in^3
LC	24	in
LI	6	in
LO	6	in
RI	2	in
RO	4	in

- **Geometrical Parameter of Single Inlet Single Outlet Expansion Chamber**

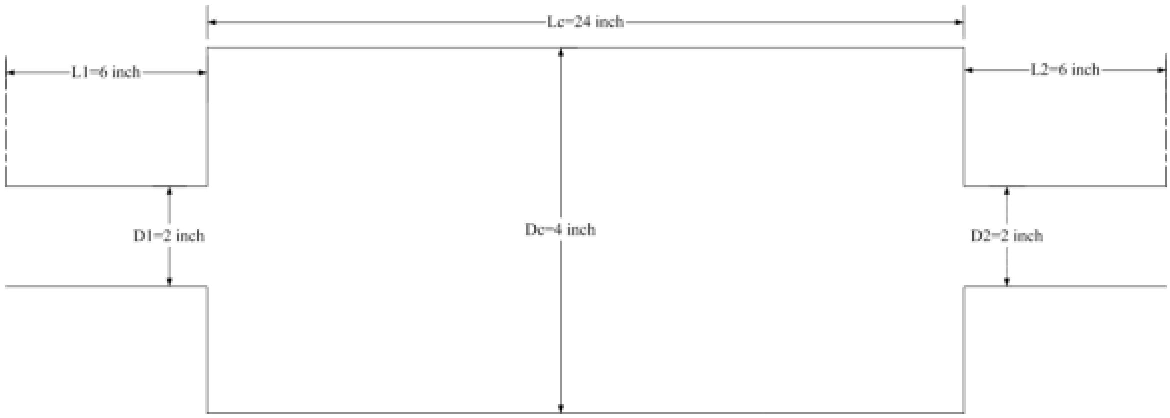


Figure 4.1 Geometrical parameter of single inlet single outlet expansion chamber

4.1.2 Validation of the Analytical Result Against Finite Element Analysis

The transmission loss curve obtained from analytical method (TMM) for single inlet single outlet expansion chamber is compared with the finite element methods ANSYS and COMSOL. Results obtained from all three methods are in agreement up to a certain value of frequency in the frequency spectrum. The value up to which all result is in agreement is the cut off frequency as below the cut off frequency only planer wave exists. And impedance matrix method is based on the theory of planer wave propagation. The result of finite elements methods and analytical method are not in agreement above cut off frequency because of existence of 3D waves. Result from all three methods is shown in Figure 4.2.

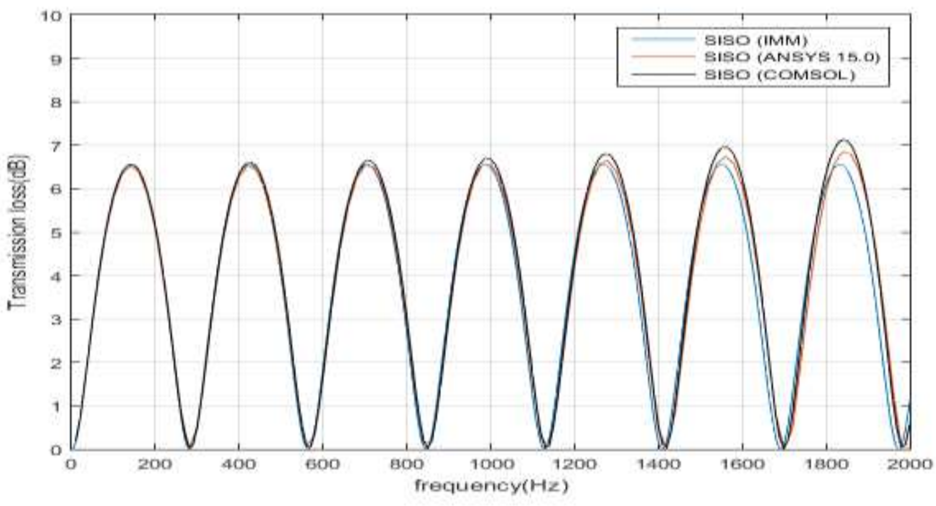


Figure 4.2 Validation of the analytical result against finite element method

4.1.3 Parametric Analysis

This section includes parametric analysis of simple expansion chamber in order to show how the transmission loss depends on geometrical parameter of muffler

(a) Change Expansion Chamber Diameter

In this case the overall diameter is changed to 7inch from 4inch.from the figure 4.3. it can be easily seen that with increasing the diameter of the expansion chamber amplitude of the peak of the transmission loss also get increases. but the downside of the increasing diameter is the existence of the 3-D propagation wave occurs earlier in the frequency spectrum. The result using the plane wave method (TMM) are identical with the finite element methods ANSYS and COMSOL up to the cut-off frequency. Because above the cut off frequency the higher order mode starts to propagate and 3-D wave comes into existence. This occurs at a frequency of for 7-inch diameter.

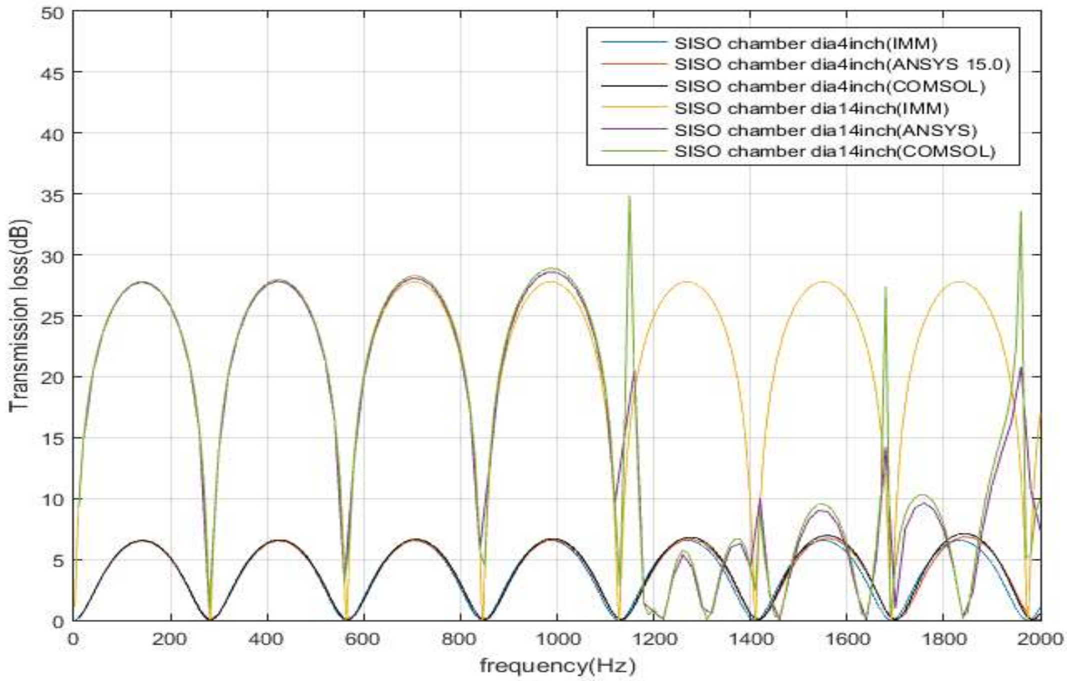


Figure 4.3 Effect of chamber diameter on transmission loss curve

(b) Change Port Diameter

This study includes the change in diameter of the inlet and outlet port of the muffler. The diameter of the inlet and outlet port of the muffler are changed from 4 inch to 3 inch. It can be seen from the figure 4.4 with decrease the diameter of the port the transmission loss gets increases.so decrease in diameter of inlet and outlet port has two major advantages one is the space saving aspect and other 3-D wave does not get shifted to the lower frequency. But the downside of decreasing the port diameter is increase in backpressure which reduces the power output of the engine. The inlet and outlet port in case are taken equal in dimension.

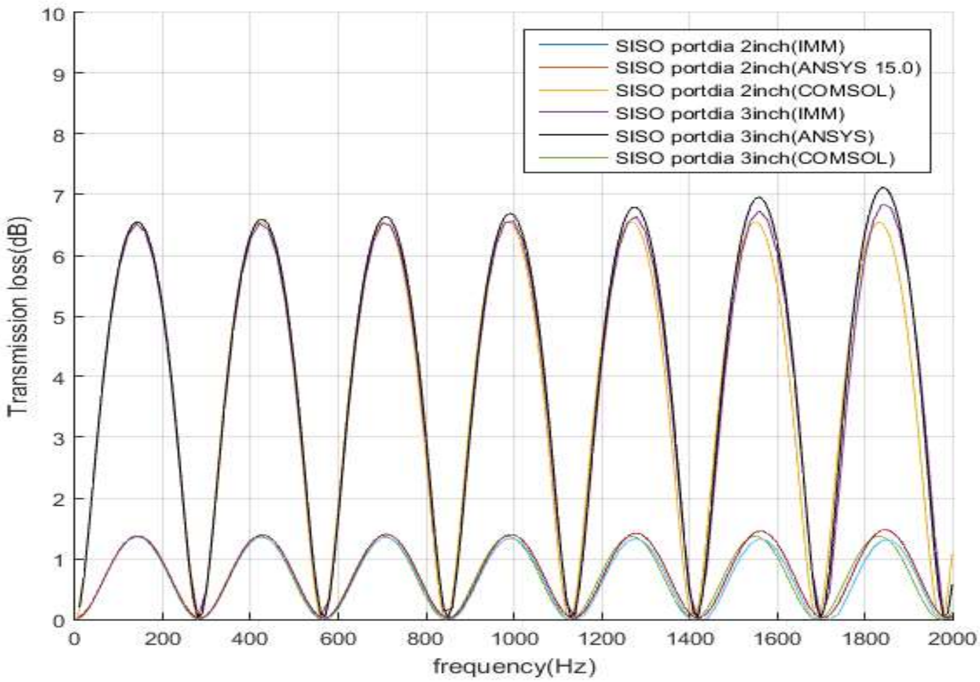


Figure 4.4 Effect of port diameter on transmission loss curve

4.2 Two Inlet One Outlet Expansion Chamber

This section contains results obtained from all the three methods for two inlets single outlet expansion chamber.

4.2.1 Impedance Matrix Method Input Parameter

TMM method was performed as described in section and iterated from 0Hz to 2000Hz with 2 Hz resolution using the MATLAB. The MATLAB code input parameters are provided below in the Table 4.2

Table 4-2 MATLAB code input parameter (TISO)

MATLAB VARIABLE	MATLAB Value	Unit
Max frequency	2000	Hz
res	2	Hz
frequency	0:2:2000	Hz
c	13536	in/s
density	0.00004335	lb/in^3
LC	20	in
LI	2	in
LO	2	in
RI	0.6	in
RO	0.984	in
RC	6.102	in

- **Geometrical Parameter of Two Inlets One Outlet Expansion Chamber**

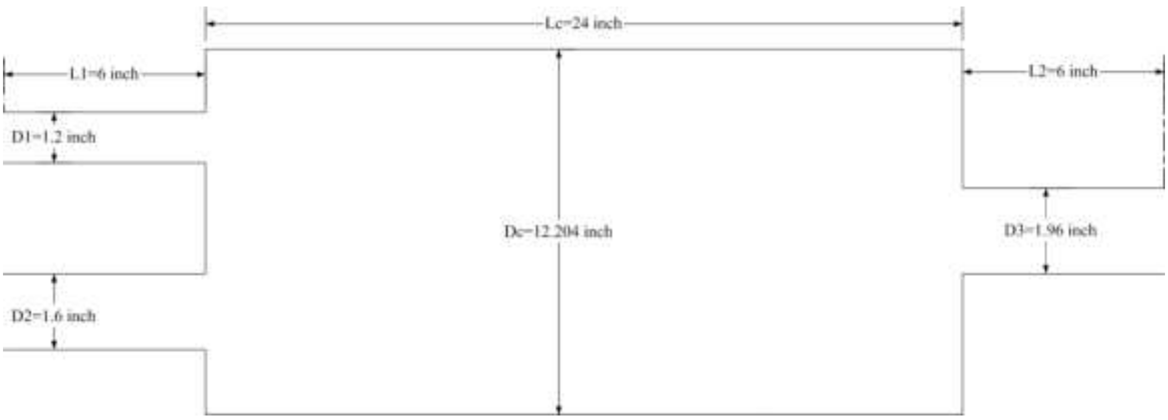


Figure 4.5 Geometrical parameter of two inlets one outlet expansion chamber

4.2.2 Validation of the Analytical Result Against Finite Element Method Results

The transmission loss curve obtained from analytical method (IMM) for two inlets single outlet expansion chamber is compared with the finite element methods ANSYS and

COMSOL. Results obtained from all three methods are in agreement up to a certain value of frequency in the frequency spectrum. The value up to which all result is in agreement is the cut off frequency as below the cut off frequency only planer wave exists. And impedance matrix method is based on the theory of planer wave propagation. The result of finite elements methods and analytical method are not in agreement above cut off frequency because of higher order modes. Results from all three methods is shown in Figure 4.6.

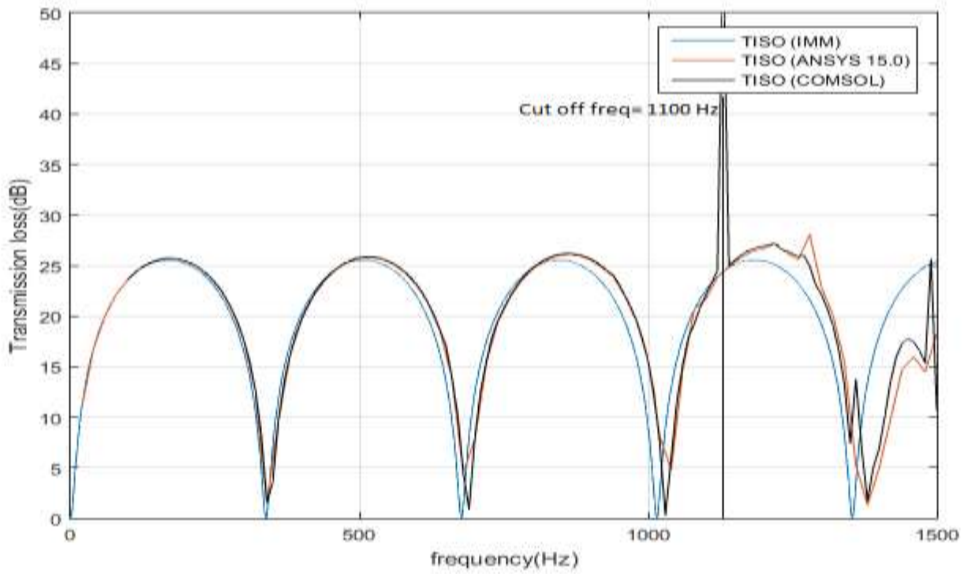


Figure 4.6 Validation of the analytical result against finite element methods

4.3 Two Inlet Two Outlet Expansion Chamber

This section contains results obtained from all the three methods for two inlets two outlets expansion chamber

4.3.1 Impedance Matrix Method Parameters

IMM method was performed as described in section and iterated from 0Hz to 2000Hz with 2 Hz resolution using the MATLAB. The MATLAB code input parameters are provided below in the Table 4.3

Table 4-3 MATLAB code input parameters (TITO)

MATLAB VARIABLE	MATLAB Value	Unit
Max frequency	2000	Hz
res	2	Hz
frequency	0:2:2000	Hz
c	13536	in/s
density	0.00004335	lb/in^3
LC	10.827	in
L1	3.937	in
L2	3.937	in
D1=D4	1.5	in
D2=D3	2	in
DC	8	in

- **Geometrical Parameters Used in Finite Element Methods**

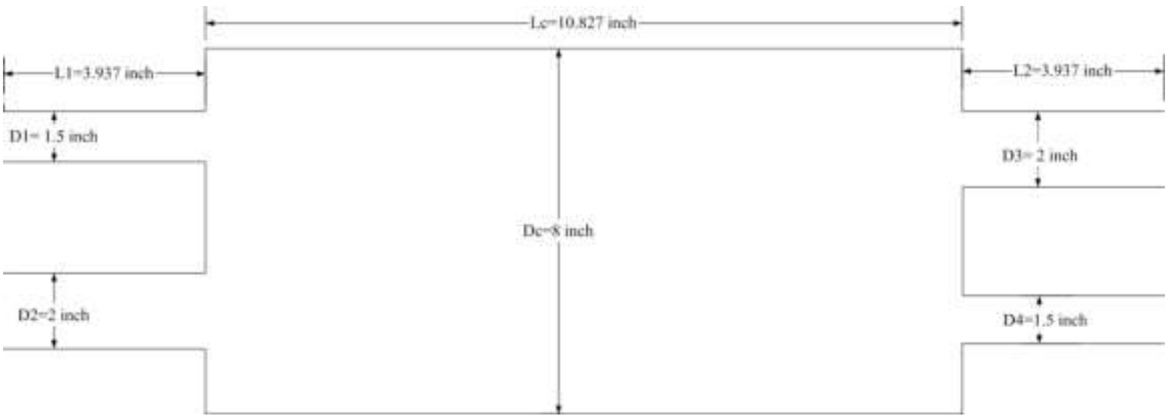


Figure 4.7 Geometrical parameters used in finite element methods

4.3.2 Validation of The Analytical Method Result Against Finite Element Methods

The transmission loss curve obtained from analytical method (IMM) for two inlets two outlet expansion chamber is compared with the finite element methods ANSYS and COMSOL. Results obtained from all three methods are in agreement up to a certain value of frequency in the frequency spectrum. The value up to which all result is in agreement is the cut off frequency as below the cut off frequency only planer wave exists. And impedance matrix method is based on the theory of planer wave propagation. The result of finite elements methods and analytical method are not in agreement above cut off frequency because of existence higher order modes. Results from all three methods is shown in Figure 4.7

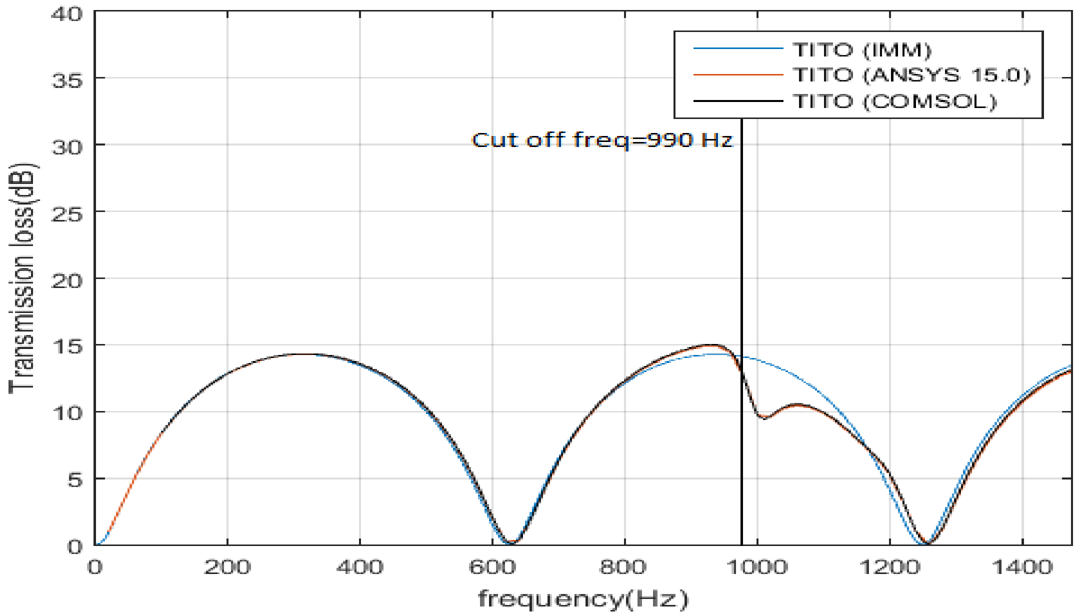


Figure 4.8 Validation of the analytical result against finite element Methods

Chapter 5

Conclusion

The result obtained from analytical method impedance matrix method are compared with finite element methods ANSYS and COMSOL for every design of muffler. Results shows that below the cut-off frequency IMM is giving the accurate results for all the three designs single inlet single outlet, two inlets one outlet, two inlets two outlet. This is because IMM is based on the theory of plane wave propagation, so above the cut-off frequency it can provide the erroneous results. Based on the results it can be says that IMM provide effective and easy way to compute the transmission loss but it is frequency limited based on the cut off frequency of the system. Parametric studies done in this work concluded that peak of the transmission is governed by the overall diameter of the expansion chamber. however, with larger diameter 3D wave propagation occur more early in frequency spectrum which can be detrimental. Apart from this decreasing port diameter have two advantages space saving aspect and 3D waves will not shift to lower frequency. But the downside is the increasing backpressure which can reduce the power output of the engine.

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